

COLLEGE ALGEBRA
AND
TRIGONOMETRY
WITH APPLICATIONS
SECOND EDITION

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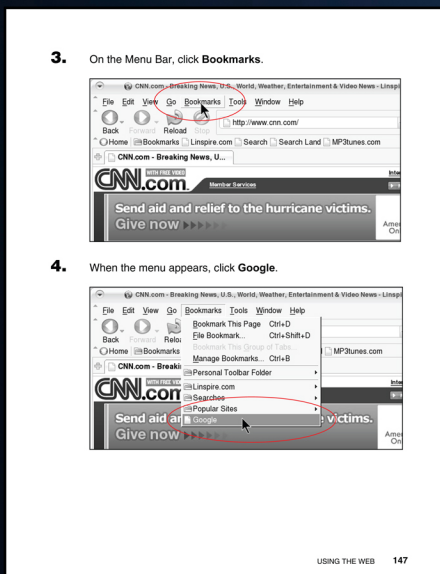
To Thomas Stewart Wesner

Dad

To Margot

Phil

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Appendix A

Development of Several Formulas

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

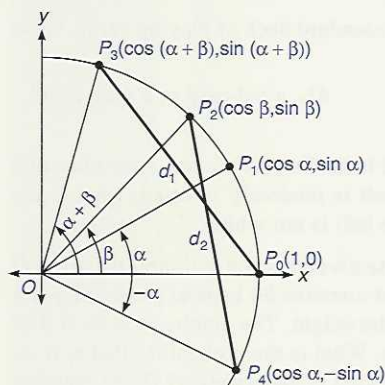


Figure A-1

A proof that this identity, which is discussed in section 7-2, is true is beyond the scope of this text, but an argument for its correctness can be obtained in the following way.

Let α and β be two angles in standard position (see figure A-1). Let P_1 be the point where the terminal side of α intersects the unit circle, and let P_2 be the point where angle β intersects the unit circle. Let P_3 be the point where the angle $\alpha + \beta$ (the sum of the angles α and β) intersects the circle. Let P_0 be the point (1,0). Finally, let P_4 be the point where the terminal side of angle $-\alpha$ intersects the unit circle.

On the unit circle the x - and y -coordinates of a point are the cosine and sine values for the appropriate angle. Thus P_1 has coordinates $(\cos \alpha, \sin \alpha)$. The coordinates for the other points are shown in the figure.

Angle $\alpha + \beta$, or angle P_0OP_3 in standard position, has the same measure as angle P_4OP_2 . It is a geometric property that central angles of a circle having equal measure have chords of equal length. Thus, the chords P_3P_0 and P_2P_4 have the same length. The length of a line segment with end points (x_1, y_1) and (x_2, y_2) is given by the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We apply this to the chords P_3P_0 and P_2P_4 .

Let d_1 = length of P_3P_0 , and let d_2 = length of P_2P_4 .

$$d_1 = d_2$$

$$\begin{aligned} & \sqrt{(\cos(\alpha + \beta) - 1)^2 + (\sin(\alpha + \beta) - 0)^2} \\ &= \sqrt{(\cos \beta - \cos \alpha)^2 + (\sin \beta - (-\sin \alpha))^2} \end{aligned}$$

Square both sides.

$$(\cos(\alpha + \beta) - 1)^2 + (\sin(\alpha + \beta))^2 = (\cos \beta - \cos \alpha)^2 + (\sin \beta + \sin \alpha)^2$$

Performing the indicated operations we obtain

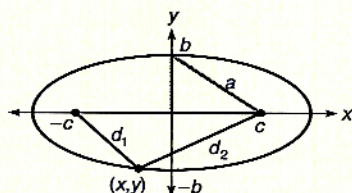
$$\begin{aligned} \cos^2(\alpha + \beta) - 2 \cos(\alpha + \beta) + 1 + \sin^2(\alpha + \beta) &= \cos^2 \beta - 2 \cos \alpha \cos \beta \\ &+ \cos^2 \alpha + \sin^2 \beta + 2 \sin \alpha \sin \beta + \sin^2 \alpha \end{aligned}$$

$$\begin{aligned} [\cos^2(\alpha + \beta) + \sin^2(\alpha + \beta)] - 2 \cos(\alpha + \beta) + 1 &= (\cos^2 \beta + \sin^2 \beta) \\ &+ (\cos^2 \alpha + \sin^2 \alpha) + 2 \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta \end{aligned}$$

Using the fundamental identity $\sin^2 \theta + \cos^2 \theta = 1$, we obtain

$$\begin{aligned} 1 - 2 \cos(\alpha + \beta) + 1 &= 1 + 1 + 2 \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta \\ -2 \cos(\alpha + \beta) &= 2 \sin \alpha \sin \beta - 2 \cos \alpha \cos \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \end{aligned}$$

Equation of the ellipse



The figure shows an ellipse (section 11-2) placed so its center is at the origin. The foci are placed on the x -axis equidistant for the origin; they are at $(-c, 0)$ and $(c, 0)$. The point (x, y) represents any point on the ellipse. The y -intercept is labeled b . We call the distance from the y -intercept to the focus a . The right triangle shown illustrates that $a^2 = b^2 + c^2$. We can develop an analytic description of this ellipse as follows.

The sum of d_1 and d_2 is a constant. If we consider (x, y) to be at $(0, b)$ (one of the y -intercepts) we can see that this constant is $2a$. We thus proceed algebraically from the statement $d_1 + d_2 = 2a$.

$$d_1 = \sqrt{(x - (-c))^2 + (y - 0)^2} \quad \text{Distance formula with } (-c, 0) \text{ and } (x, y)$$

$$= \sqrt{(x + c)^2 + y^2}$$

$$d_2 = \sqrt{(x - c)^2 + (y - 0)^2} \quad \text{Distance formula with } (c, 0) \text{ and } (x, y)$$

$$= \sqrt{(x - c)^2 + y^2}$$

$$d_1 + d_2 = 2a$$

Definition of ellipse

$$\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$$

Replace d_1 and d_2 in $d_1 + d_2 = 2a$ by the values above

$$\sqrt{(x + c)^2 + y^2} = 2a - \sqrt{(x - c)^2 + y^2}$$

$$[\sqrt{(x + c)^2 + y^2}]^2 = [2a - \sqrt{(x - c)^2 + y^2}]^2 \quad \text{Square both members}$$

$$(x + c)^2 + y^2 = 4a^2 - 4a\sqrt{(x - c)^2 + y^2} + (x - c)^2 + y^2$$

$$x^2 + 2cx + c^2 + y^2 = 4a^2 - 4a\sqrt{(x - c)^2 + y^2} + x^2 - 2cx + c^2 + y^2$$

$$4cx = 4a^2 - 4a\sqrt{(x - c)^2 + y^2}$$

Simplify terms

$$cx = a^2 - a\sqrt{(x - c)^2 + y^2}$$

Divide each term by 4

$$a\sqrt{(x - c)^2 + y^2} = a^2 - cx$$

Rearrange terms

$$[a\sqrt{(x - c)^2 + y^2}]^2 = (a^2 - cx)^2$$

Square both members

$$a^2[(x - c)^2 + y^2] = a^4 - 2a^2cx + c^2x^2$$

$$a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2 = a^4 - 2a^2cx + c^2x^2$$

$$a^2x^2 + a^2c^2 + a^2y^2 = a^4 + c^2x^2$$

$$a^2x^2 - c^2x^2 + a^2y^2 = a^4 - a^2c^2$$

$$(a^2 - c^2)x^2 + a^2y^2 = a^2(a^2 - c^2)$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

$$a^2 = b^2 + c^2, \text{ so } a^2 - c^2 = b^2$$

$$\frac{b^2x^2}{a^2b^2} + \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2}$$

Divide each term by a^2b^2

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Also, solving for c in $a^2 + b^2 = c^2$ we find that $c = \sqrt{a^2 - b^2}$.

Thus, an analytic description of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $c = \sqrt{a^2 - b^2}$.

Study Group.

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Appendix B

Answers and Solutions

Chapter 1

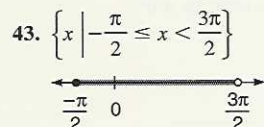
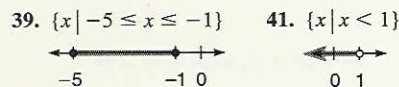
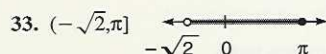
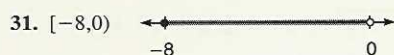
Exercise 1-1

Answers to odd-numbered problems

1. {4, 5, 6, 7, 8, 9, 10, 11}
 3. {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}
 5. {-6, -3, 0, 3, 6, 9, 12}
 7. 0.4, terminating
 9. 0.230769230769, repeating
 11. -276 13. $\frac{31}{40}$ 15. $\frac{48}{23}$ 17. $-\frac{1}{16}$

19. $\frac{bx - ay}{ab}$ 21. $\frac{-xy - 3y^2 - 8x^2}{12xy}$

23. $-14x^7$ 25. $\frac{3x^3}{25y^3}$ 27. $\frac{3a^2 + 6b^2}{10a^2}$



45. $\{x | -\frac{1}{2} < x \leq 1\frac{1}{2}\}$, $(-\frac{1}{2}, 1\frac{1}{2}]$
 47. $\{x | 5\frac{1}{2} \leq x < 7\}$, $[5\frac{1}{2}, 7)$
 49. $\{x | -2 < x < \frac{1}{2}\}$, $(-2, \frac{1}{2})$
 51. $\{x | x > 5\}$, $(5, \infty)$ 53. 4 55. -2
 57. $\sqrt{10} + 3$ 59. $\frac{7}{4}$ 61. 25
 63. $\sqrt{2} - 3$ 65. $2x^4$

67. $\frac{x^2y^6}{z^8}$ 69. $-5x^2$

71. $\frac{5|x|}{2y^2}$ 73. $(x - 2)^2 |x + 1|$

75. if $x > 0$, $\frac{x^2}{|x|} = \frac{x^2}{x} = x$;
 if $x < 0$, $\frac{x^2}{|x|} = \frac{x^2}{-x} = -x$

77. $-\frac{3}{5}$ or -0.6

Solutions to skill and review problems

1. $2 \cdot 3(x^2 \cdot x^3) = 6x^{2+3} = 6x^5$

2. $2n + (-2n) = 0$
 $8 - (-3) = 8 + 3 = 11$

4. $2 \cdot 1,000,000,000 = 2 \cdot 10^9$, b

5. $0.3 = \frac{3}{10}$, $0.03 = \frac{3}{100}$,
 $0.003 = \frac{3}{1000}$, $0.0003 = \frac{3}{10,000}$,
 $0.00003 = \frac{3}{100,000}$; c

6. $-3[2(4[\frac{1}{2}(2 - 3) + 2] - 1) + 7] + 4$
 $-3[2(4[\frac{1}{2}(-1) + 2] - 1) + 7] + 4$
 $-3[2(4[-\frac{1}{2} + \frac{4}{2}] - 1) + 7] + 4$
 $-3[2(4[\frac{3}{2}] - 1) + 7] + 4$
 $-3[2(6 - 1) + 7] + 4$
 $-3[10 + 7] + 4$
 $-51 + 4$
 -47

7. $2a(2a - 2b - ac)$
 $2a(2a) - 2a(2b) - 2a(ac)$
 $4a^2 - 4ab - 2a^2c$

8. $(2a - c)(3a + 2c)$
 $2a(3a + 2c) - c(3a + 2c)$
 $6a^2 + 4ac - 3ac - 2c^2$
 $6a^2 + ac - 2c^2$

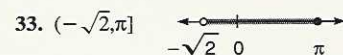
Solutions to trial exercise problems

3. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}, $x \in \mathbb{N}$ and $x < 21$
 {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}, x is odd
 {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

4. $\frac{178}{185} = 0.9621621621 \dots$;
 {1, 2, 6, 9}

16. $\left(\frac{3}{7} - \frac{7}{12}\right) \div \left(\frac{3}{7} + \frac{7}{12}\right)$
 $\left[\frac{3(12) - 7(7)}{7(12)}\right] \div \left[\frac{3(12) + 7(7)}{7(12)}\right]$
 $\left(-\frac{13}{84}\right) \div \left(\frac{85}{84}\right)$
 $\left(-\frac{13}{84}\right) \cdot \frac{84}{85}$
 $-\frac{13}{85}$

21. $\frac{x - y}{4x} - \frac{2x + y}{3y}$
 $\frac{3y(x - y) - 4x(2x + y)}{4x(3y)}$
 $\frac{3xy - 3y^2 - 8x^2 - 4xy}{12xy}$
 $\frac{-xy - 3y^2 - 8x^2}{12xy}$



58. $-\frac{|\sqrt{10} - 6|}{-(6 - \sqrt{10})}$ 62. $-\frac{|\sqrt{2}|}{-(\sqrt{2})}$
 $\frac{\sqrt{10} - 6}{\sqrt{10} - 6}$ $\frac{-\sqrt{2}}{-\sqrt{2}}$

70. $\left|\frac{3x^2}{2y}\right|$
 $\frac{3|x^2|}{2|y|}$
 $\frac{3x^2}{2|y|}$



b. (0, 2], (2, 3], (3, 5], (5, 10], (10, ∞)

c. $\frac{15}{2} = 7.5¢/\text{oz}$, $\frac{20}{3} = 6.7¢/\text{oz}$,

$\frac{30}{5} = 6¢/\text{oz}$, $\frac{40}{10} = 4¢/\text{oz}$, $3.5¢/\text{oz}$

Exercise 1-2

Answers to odd-numbered problems

1. $2x^{11}$ 3. -32 5. $6a^7b^3$ 7. $128x$
 9. $\frac{3x^4}{y^3}$ 11. $8x^9y^{15}$ 13. $\frac{81a^4}{b^6}$ 15. $\frac{y^3}{x^2}$
 17. $-\frac{1}{27}$ 19. $\frac{4}{x^4}$ 21. $-\frac{6x^7}{y}$ 23. 1
 25. $\frac{3y^5}{x^5}$ 27. $\frac{8b^{21}}{a^{12}}$ 29. $\frac{-8x^{15}}{125y^6}$
 31. $\frac{b^6c^{16}}{9a^{10}}$ 33. $\frac{2}{x^4}$ 35. $\frac{9}{16x^2y^8}$ 37. x^{4n}
 39. x 41. $\frac{x^{8n}}{y^{8n-16}}$ 43. 3.65×10^{15}
 45. -1.9002×10^{13} 47. -2.92×10^{-14}
 49. 3.502×10^{-12}
 51. 25,020,000,000,000
 53. $-0.000\ 000\ 000\ 138\ 4$
 55. 9,230,000 57. 9.1×10^{-28} grams
 59. trinomial, degree is 2
 61. polynomial, degree is 3
 63. trinomial, degree is 6
 65. not a polynomial because of the \sqrt{x}
 67. -557 69. $36\frac{1}{3}$ 71. $\frac{1}{4}$
 73. $2x^2 - 4x + 6$ 75. $6a - 7b + c$
 77. $-2x^2y + 2xy$ 79. $9x - 2y$
 81. $-16a + 7b$ 83. $10x^5 - 4x^4 + 14x^2$
 85. $-10a^4b^2 - 6a^4b^3 + 4a^3b^4$
 87. $25a^2 - 9$ 89. $15x^2 + 2xy - y^2$
 91. $2a^2 + 2b^2 - 5ab + 2ac - bc$
 93. $10x^4 - 19x^3 + 25x^2 - 23x + 7$
 95. $5b^4 + 3b^3 - 14b^2 + 9b - 9$
 97. $x^3 - 7xy^2 - 6y^3$
 99. $9a^3 + 21a^2b + 4ab^2 - 4b^3$
 101. $6a^2 - 4ab + 2ac - 9a + 6b - 3c$
 103. $x^3 + 2x^2y - 4xy^2 - 8y^3$
 105. $8x^3 + 60x^2 + 150x + 125$
 107. $\frac{2x^3y}{3}$ 109. $3a^2 - 4b^2 + 6b^4$
 111. $\frac{2}{3}x^2z^2 + xz - \frac{4}{3}y^2$ 113. $x - 2$
 115. $x^3 - 5x^2 + 10x - 20 + \frac{48}{x+2}$
 117. $3x^2 - 4x + 1 + \frac{2}{2x+3}$
 119. $4x^2 + 7x + 17 + \frac{38}{x-2}$
 121. $4x + 3 + \frac{-x+2}{x^2-x+1}$
 123. $3x^2 + 7 + \frac{-x+22}{x^2-3}$
 125. a. $2t_1 - 2t_2 + 3t_3$
 b. $3t_1^2 + 7t_1t_2 - 9t_1t_3 + 4t_2^2 - 12t_2t_3$
 c. $-24x_1^5x_2^7$

127. First show that $(a^2 + b^2)(c^2 + d^2)$
 $= (ac + bd)^2 + (ad - bc)^2$
 $= (a^2 + b^2)(c^2 + d^2)$
 $= a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$
 and $(ac + bd)^2 + (ad - bc)^2$
 $= (a^2c^2 + 2abcd + b^2d^2)$
 $+ (a^2d^2 - 2abcd + b^2c^2)$
 $= a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$

Now show that

$$(a^2 + b^2)(c^2 + d^2)$$

$$= (ac - bd)^2 + (ad + bc)^2$$

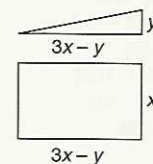
$$= (a^2c^2 + b^2d^2 - 2abcd) + (a^2d^2 + b^2c^2 + 2abcd)$$

$$= a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2$$

129. 77.7

131. Square: $(x - 2y)(x - 2y)$
 $= x^2 - 4xy + 4y^2$
 Rectangle: $(a + b)(a + 2b)$
 $= a^2 + 3ab + 2b^2$

133. Area = Triangle + Rectangle
 $= \frac{1}{2}(3x - y)(y) + (3x - y)(x)$
 $= \frac{1}{2}(3xy - y^2) + (3x^2 - xy)$
 $= \frac{3}{2}xy - \frac{1}{2}y^2 + 3x^2 - xy$
 Area = $3x^2 + \frac{1}{2}xy - \frac{1}{2}y^2$



135. $\frac{1}{2}(a + c) \cdot \frac{1}{2}(b + d)$
 $= \frac{1}{2} \cdot \frac{1}{2}(a + c)(b + d)$
 $= (\frac{1}{2} \cdot \frac{1}{2})[(a + c)(b + d)]$
 $= \frac{1}{4}[ab + ad + bc + bd]$
 137. a. 7, 23 b. 22, 27 c. 7, 17
 d. 8, 35 e. 13, 87

139. $(5 \times 10^{12}) \div (2.5 \times 10^{-10})$
 2×10^{22}

$$5 \text{ [EXP] } 12 \text{ [÷] } 2.5 \text{ [EXP] } 10 \text{ [+/-] [=]}$$

$$\text{TI-81: } 5 \text{ [EE] } 12 \text{ [÷] } 2.5 \text{ [EE] } (-) 10 \text{ [ENTER]}$$

141. $\sqrt{4 \times 10^{18}}$
 2×10^9

$$4 \text{ [EXP] } 18 \text{ [\sqrt{x}]}$$

$$\text{TI-81: } 2\text{nd} \text{ [x^2] } 4 \text{ [EE] } 18 \text{ [ENTER]}$$

Solutions to skill and review problems

1. $360 = 10 \cdot 36$
 $= 2 \cdot 5 \cdot 6 \cdot 6$
 $= 2 \cdot 5 \cdot 2 \cdot 3 \cdot 2 \cdot 3$
 $= 2^3 \cdot 3^2 \cdot 5$
 2. $3x^2y^3(x^3y - 4x + 2)$
 3. $(x - 4)(x + 4)$ 4. $(x + 2)(x + 4)$
 5. $(x - 2)(x + 8)$ 6. $(2x - 3)(3x + 1)$
 7. $x^3 - 1$

Solutions to trial exercise problems

8. $3a^{-1}b^4(3^{-1}a^2b)$
 $3^{1-1}a^{-1+2}b^{4+1}$
 $3^0a^1b^5$
 ab^5
 15. $\frac{1}{x^2y^{-3}}$
 $\frac{y^3}{x^2}$
 21. $(2x^4y)(-3x^3y^{-2})$
 $-6x^7y^{-1}$
 $-\frac{6x^7}{y}$
 26. $\frac{2x^{-2}y^0z^3}{-2^{-3}x^2y^2z^{-1}}$
 $-\frac{2(2^3)z^3z^1}{x^2x^2y^2}$
 $-\frac{16z^4}{x^4y^2}$

27. $\left(\frac{9a^5b^4}{18ab^{11}}\right)^{-3}$
 $\left(\frac{a^4}{2b^7}\right)^{-3}$
 $\frac{a^{-12}}{2^{-3}b^{-21}}$
 $\frac{2^3b^{21}}{a^{12}}$
 $\frac{8b^{21}}{a^{12}}$

35. $\left(\frac{x^2y}{x^2y^2}\right)^2 \left(\frac{3x^0y^{-2}}{4xy}\right)^2$
 $\left(\frac{1}{y}\right)^2 \left(\frac{3}{4xy \cdot y^2}\right)$
 $\frac{1}{y^2} \cdot \frac{9}{16x^2y^6}$
 $\frac{9}{16x^2y^8}$

36. $\left(\frac{a^5bc^2}{2abc}\right)^3 \left(\frac{2^2a^3b^2c}{ab^{-2}c^3}\right)^{-1}$
 $\left(\frac{a^4c}{2}\right)^3 \left(\frac{4a^2b^4}{c^2}\right)^{-1}$
 $\frac{a^{12}c^3}{8} \cdot \frac{4^{-1}a^{-2}b^{-4}}{c^{-2}}$
 $\frac{a^{12}c^3}{8} \cdot \frac{c^2}{4a^2b^4}$
 $\frac{a^{10}c^5}{32b^4}$

41. $\left(\frac{x^n y^{2-n}}{x^{-n} y^{n-2}}\right)^4$
 $\left(\frac{x^n x^n}{y^{n-2} y^{-(2-n)}}\right)^4$
 $\left(\frac{x^{2n}}{y^{2n-4}}\right)^4$
 $\frac{x^{8n}}{y^{8n-16}}$

111. The following is a program for a TI-81 programmable calculator which will compute the greatest common factor of two integers. The integers must be in the variables F and S .

Note: All program lines are terminated with the

ENTER key, which is not shown.

Display	Keystrokes—use ENTER at the end of each line.
Prgm3:GCF	PRGM 3 TAN PRGM COS
:If F>S	PRGM 3 ALPHA COS 2nd
	MATH 3 ALPHA LN
:Goto 1	PRGM 2 1
:F→T	ALPHA COS STO 4
:S→F	ALPHA LN STO COS
:T→S	ALPHA 4 STO LN
:Lbl 1	PRGM 1 1
:!Part(F/S)→T	MATH 2 (ALPHA COS ÷
	ALPHA LN) STO 4
:F - TS→R	ALPHA COS - ALPHA 4
	ALPHA LN STO ×
:If R=0	PRGM 3 ALPHA × 2nd
	MATH 1 0
:Goto 2	PRGM 2 2
:S→F	ALPHA LN STO COS
:R→S	ALPHA × STO LN
:Goto 1	PRGM 2 1
:Lbl 2	PRGM 1 2
:abs S→S	2nd x⁻¹ ALPHA LN STO
	LN 2nd CLEAR

To use this program to find the GCF of 140 and 196, do the following:

140 **STO** **COS** **ENTER** 196 **STO** **LN** **ENTER** **PRGM** **3** **ENTER** **ALPHA** **LN** **ENTER**

and the result, 28, appears. This program could be made more user friendly, but it is used as is in problem 112.

Solutions to skill and review problems

- $2x$ must be 5, so x must be $2\frac{1}{2}$.
- Never, since 4 is positive and x^2 is positive or zero. Adding positive values gives positive results.
- $-\frac{1}{2}$ is the same as $\frac{1}{2}$. Think of $-(-0.5)$ to help see this.
- $\frac{3x^3}{6x^6} = \frac{3}{6} \cdot \frac{xxx}{xxxxxx} = \frac{1}{2} \cdot \frac{xxx}{xxxxxx} = \frac{1}{2x^3}$
- $\frac{3}{4} \cdot \frac{12}{5} = \frac{3}{1} \cdot \frac{3}{5} = \frac{9}{5}$
- $\frac{2}{3} + \frac{1}{4} = \frac{2 \cdot 4 + 3 \cdot 1}{3 \cdot 4} = \frac{11}{12}$
- $(2x - 3)(x + 1) - (x - 2)(x - 1)$
 $= (2x^2 - x - 3) - (x^2 - 3x + 2)$
 $= 2x^2 - x - 3 - x^2 + 3x - 2$
 $= x^2 + 2x - 5$
- $\frac{5}{8} \div 2 = \frac{5}{8} \div \frac{2}{1} = \frac{5}{8} \cdot \frac{1}{2} = \frac{5}{16}$

Solutions to trial exercise problems

- $2m(n + 5) - 1(n + 5) - p(n + 5)$
 $(n + 5)(2m - 1 - p)$
- $(xy - 3z)[(xy)^2 + 3z(xy) + (3z)^2]$
 $(xy - 3z)(x^2y^2 + 3xyz + 9z^2)$
- $9(3x + 1)^2 - (x - 3)^2$
 $9a^2 - b^2$
 Replace $3x + 1$ by a , $x - 3$ by b
 $(3a - b)(3a + b)$
 $[3(3x + 1) - (x - 3)][3(3x + 1) + (x - 3)]$
 Replace a by $3x + 1$, b by $x - 3$
 $(8x + 6)(10x)$
 $2(4x + 3)(10x)$
 $20x(4x + 3)$
- $x^2(x^2 - 9) + 2x(x^2 - 9) - 15(x^2 - 9)$
 $(x^2 - 9)(x^2 + 2x - 15)$
 $(x - 3)(x + 3)(x + 5)(x - 3)$
- $2x^2(a^2 - b^2) + x(a^2 - b^2) - 3a^2 + 3b^2$
 $2x^2(a^2 - b^2) + x(a^2 - b^2)$
 $- 3(a^2 - b^2)$
 $(a^2 - b^2)(2x^2 + x - 3)$
 $(a - b)(a + b)(2x + 3)(x - 1)$
- $(x + y)^2 - 8(x + y) - 9$
 $z^2 - 8z - 9$ $z = x + y$
 $(z + 1)(z - 9)$
 $(x + y + 1)(x + y - 9)$
- $a^2(9 - x^2) - 6a(9 - x^2) - 9(x^2 - 9)$
 $a^2(9 - x^2) - 6a(9 - x^2) + 9(9 - x^2)$
 $(9 - x^2)(a^2 - 6a + 9)$
 $(3 - x)(3 + x)(a - 3)(a + 3)$
 $(3 - x)(3 + x)(a - 3)^2$
- (See page 608.)

Exercise 1-4

Answers to odd-numbered problems

- $\frac{4p^4q^4}{3}$
- $\frac{4}{3}$
- $\frac{a - 3}{4}$
- $-8 - 7p$
- $\frac{6(a^2 + ab + b^2)}{a + b}$
- $\frac{-2}{a^2 + 4a + 16}$
- $\frac{a + 6}{a + 3}$
- $\frac{15x^2 - 4y^2}{10xy}$
- $\frac{2x^2 - 3x - 3}{x^2 - 1}$
- $\frac{-7}{x - 4}$
- $\frac{45a + 6ab - 20b}{10ab}$
- $\frac{12a^2}{b^2}$
- $\frac{2x + 5}{x(x - 3)}$
- $\frac{-a(13a + 9)}{(a + 5)(a + 2)(a - 3)}$
- $\frac{3a - 5}{2a - 3}$

112. The following TI-81 program will compute the required values a , b , c , and d . It uses the program GCF of problem 111.

Display	Keystrokes—use ENTER at the end of each line.	Display	Keystrokes—use ENTER at the end of each line.
Prgm5:QTRI	PRGM 5 9 4 × x²	:Stop	PRGM 8
:Input A	PRGM 2 ALPHA MATH	:Lbl 5	PRGM 1 5
:Input B	PRGM 2 ALPHA MATRX	:(B/abs B)N → N	(ALPHA MATRX ÷ 2nd
:Input C	PRGM 2 ALPHA PRGM		x⁻¹ ALPHA MATRX) ALPHA
:AC → Z	ALPHA MATH ALPHA PRGM		LOG STO LOG
	STO 2	Z/N → M	ALPHA 2 ÷ ALPHA LOG
:Z/abs Z → D	ALPHA 2 ÷ 2nd x⁻¹ ALPHA		STO ÷
	2 STO x⁻¹	:A → F	ALPHA MATH STO COS
:1 → M	1 STO ÷	:M → S	ALPHA ÷ STO LN
:abs AC → N	2nd x⁻¹ ALPHA MATH	:Prgm3	PRGM 3 3
	ALPHA PRGM STO LOG	:S → D	ALPHA LN STO x⁻¹
:Lbl 3	PRGM 1 3	:N → F	ALPHA LOG STO COS
:If MN ≠ abs Z	PRGM 3 ALPHA ÷ ALPHA	:C → S	ALPHA PRGM STO LN
	LOG 2nd MATH 2 2nd x⁻¹	:Prgm3	PRGM 3 3
	ALPHA 2	:(N/abs N)S → E	(ALPHA LOG ÷ 2nd x⁻¹
:Goto 6	PRGM 2 6		ALPHA LOG) ALPHA LN
:If abs	PRGM 3 2nd x⁻¹ (ALPHA		STO SIN
(M + DN)=abs B	÷ + ALPHA x⁻¹ ALPHA	:A/D → F	ALPHA MATH ÷ ALPHA x⁻¹
	LOG) 2nd MATH 1		STO COS
	2nd x⁻¹ ALPHA MATRX	:M/D → G	ALPHA ÷ ÷ ALPHA x⁻¹
:Goto 5	PRGM 2 5		STO TAN
:Lbl 6	PRGM 1 6	:Disp D	PRGM 1 ALPHA x⁻¹
M+1 → M	ALPHA ÷ + 1 STO ÷	:Disp E	PRGM 1 ALPHA SIN
:abs Z/M → N	2nd x⁻¹ ALPHA 2 ÷ ALPHA	:Disp F	PRGM 1 ALPHA COS
	÷ STO LOG	:Disp G	PRGM 1 ALPHA TAN 2nd
:If M>N	PRGM 3 ALPHA ÷ 2nd		CLEAR
	MATH 3 ALPHA LOG		
:Goto 2	PRGM 2 2		
:Goto 3	PRGM 2 3		
:Lbl 2	PRGM 1 2		
:Disp "No"	PRGM 1 2nd ALPHA +		
	LOG 7 +		

To factor $10x^2 + x - 24$, do:

PRGM 5 **ENTER** 10 **ENTER** 1

ENTER **(-)** 24 **ENTER**, and the values 5, 8, 2, -3 appear, which means the expression is $(5x + 8)(2x - 3)$.

31. $\frac{4(a-2)}{15(a-4)}$ 33. $\frac{x^3 - x^2 - 5x - 3}{4}$

35. $\frac{6y^2 + 107y - 190}{10y^2 - 40}$

37. $\frac{3y^2 - y + 15}{y^3 + 3y^2 - 4y - 12}$

39. $\frac{3m^2 - 13m + 12}{m + 3}$ 41. $\frac{1}{2x - 10}$

43. $\frac{5}{3}$ 45. $\frac{ab + b}{2ab - 3}$ 47. $\frac{15x - 20}{8x + 4}$

49. $\frac{1}{7}$ 51. $m(m + n)$ 53. $-\frac{1}{3}$

55. $\frac{-x + 3}{3x - 9}$ 57. $\frac{2a^2}{3}$

59. $\frac{R_2R_3V_1 + R_1R_3V_2 + R_1R_2V_3}{R_2R_3 + R_1R_3 + R_1R_2}$

61. $\frac{2r_1r_2}{r_1 + r_2}$ 63. $\frac{3}{x^2 + 3x}$

65. $\frac{P}{Q} + \frac{R}{S} = \frac{PS}{QS} + \frac{RQ}{SQ} = \frac{PS + QR}{QS}$

$\frac{P}{Q} - \frac{R}{S} = \frac{PS}{QS} - \frac{RQ}{SQ} = \frac{PS - QR}{QS}$

67. 529 hours

69. An even number (integer) greater than two is not prime because all even numbers are divisible by two. A prime number must be divisible by only one and itself. Even integers must have 0, 2, 4, 6, or 8 for their last digit. The following is a programming solution for the TI-81. It also works for even integers.

```
Prgm6:PRIME
:Input N
:2 → D
:If FPart (N/D)=0
:Goto 5
:√N → L
:3 → D
:Lbl 1
:If FPart (N/D)=0
:Goto 5
:D+2 → D
:If D ≤ L
:Goto 1
:Disp "PRIME"
:Stop
:Lbl 5
:Disp D
:N/D → N
:Disp N
```

Solutions to skill and review problems

1. a. $\sqrt{5^2} = 5$ b. $\sqrt{10^2} = 10$
c. $\sqrt{20^2} = 20$
2. a. $\sqrt[3]{2^3} = 2$ b. $\sqrt[3]{4^3} = 4$
c. $\sqrt[6]{2^6} = 2$
3. a. $\sqrt{36} = 6$ b. $2 \cdot 3 = 6$
4. $2 \cdot 2^3 x^2 y^3 y$
 $2^4 x^4 y^4$
 $16x^4 y^4$
5. a. 8 b. 8 c. 16 d. 44 e. 56
6. Observe that $81 = 3^4$
 $3^4 a^4 b^8 = 3 \cdot 3^3 a^4 b^8$
 $3^4 a^4 b^8 = 3^{1+x} a^{2+y} b^{5+z}$
 $4 = 1 + x$ so $x = 3$
 $4 = 2 + y$ so $y = 2$
 $8 = 5 + z$ so $z = 3$

Solutions to trial exercise problems

$$11. \frac{8-2a}{a^3-64}$$

$$\frac{2(4-a)}{(a-4)(a^2+4a+16)}$$

$$\frac{-2(a-4)}{(a-4)(a^2+4a+16)}$$

$$\frac{-2}{a^2+4a+16}$$

$$19. \frac{3x-5}{x-4} + \frac{3x+2}{-(x-4)}$$

$$\frac{3x-5}{x-4} - \frac{3x+2}{x-4}$$

$$\frac{(3x-5) - (3x+2)}{x-4}$$

$$\frac{-7}{x-4}$$

$$24. \frac{a-6}{6a+18}(a+3)$$

$$\frac{a-6}{6(a+3)} \cdot \frac{a+3}{1}$$

$$\frac{a-6}{6}$$

$$27. \frac{-6a}{(a-3)(a+2)} - \frac{7a}{(a+5)(a+2)}$$

$$\frac{-6a(a+5)}{(a-3)(a+2)(a+5)}$$

$$- \frac{7a(a-3)}{(a+5)(a+2)(a-3)}$$

$$\frac{(-6a^2 - 30a) - (7a^2 - 21a)}{(a+5)(a+2)(a-3)}$$

$$\frac{-13a^2 - 9a}{(a+5)(a+2)(a-3)}$$

$$\frac{-a(13a+9)}{(a+5)(a+2)(a-3)}$$

$$43. 6\left(\frac{1}{3} + \frac{1}{2}\right)$$

$$6\left(\frac{2}{3} - \frac{1}{6}\right)$$

$$\frac{2+3}{2(2)-1}$$

$$\frac{5}{3}$$

$$54. \frac{6}{x(x-1)} - 2$$

$$\frac{3}{x-1} + 2$$

$$x(x-1)\left(\frac{6}{x(x-1)} - 2\right)$$

$$x(x-1)\left(\frac{3}{x-1} + 2\right)$$

$$\frac{6-2(x)(x-1)}{3x+2(x)(x-1)}$$

$$\frac{-2x^2+2x+6}{2x^2+x}$$

$$66. \text{MTBF}_S = \left(\frac{1}{\text{MTBF}_1} + \frac{1}{\text{MTBF}_2} + \frac{1}{\text{MTBF}_3} \right)^{-1}$$

$$= \left(\frac{1}{\text{MTBF}_1} \cdot \frac{\text{MTBF}_2 \cdot \text{MTBF}_3}{\text{MTBF}_2 \cdot \text{MTBF}_3} + \frac{1}{\text{MTBF}_2} \cdot \frac{\text{MTBF}_1 \cdot \text{MTBF}_3}{\text{MTBF}_1 \cdot \text{MTBF}_3} \right. \\ \left. + \frac{1}{\text{MTBF}_3} \cdot \frac{\text{MTBF}_1 \cdot \text{MTBF}_2}{\text{MTBF}_1 \cdot \text{MTBF}_2} \right)^{-1}$$

$$= \left(\frac{\text{MTBF}_2 \cdot \text{MTBF}_3}{\text{MTBF}_1 \cdot \text{MTBF}_2 \cdot \text{MTBF}_3} + \frac{\text{MTBF}_1 \cdot \text{MTBF}_3}{\text{MTBF}_2 \cdot \text{MTBF}_1 \cdot \text{MTBF}_3} \right. \\ \left. + \frac{\text{MTBF}_1 \cdot \text{MTBF}_2}{\text{MTBF}_3 \cdot \text{MTBF}_1 \cdot \text{MTBF}_2} \right)^{-1}$$

$$= \left(\frac{\text{MTBF}_2 \cdot \text{MTBF}_3 + \text{MTBF}_1 \cdot \text{MTBF}_3 + \text{MTBF}_1 \cdot \text{MTBF}_2}{\text{MTBF}_1 \cdot \text{MTBF}_2 \cdot \text{MTBF}_3} \right)^{-1}$$

$$= \frac{\text{MTBF}_1 \cdot \text{MTBF}_2 \cdot \text{MTBF}_3}{\text{MTBF}_2 \cdot \text{MTBF}_3 + \text{MTBF}_1 \cdot \text{MTBF}_3 + \text{MTBF}_1 \cdot \text{MTBF}_2}$$

Exercise 1-5

Answers to odd-numbered problems

1. 17 3. 2 5. -5 7. $2|x|$
9. $5y^4|x^3|$ 11. $\frac{4|x^3|}{3|y^5|}$ 13. $|x^2-3|$
15. $2\sqrt[3]{5}$ 17. $10\sqrt{2}$ 19. $2ab\sqrt[3]{2a^2}$

$$21. 10xy^4z^6\sqrt{2y} \quad 23. 2y\sqrt[4]{x^3y^2} \quad 25. a^4$$

$$27. 5ab^3c\sqrt[3]{5a} \quad 29. 4\sqrt{2} \quad 31. \frac{6\sqrt{2a}}{5}$$

$$33. \frac{2\sqrt{6}}{9} \quad 35. \frac{2\sqrt[3]{3}}{3} \quad 37. \frac{2x^2y^3\sqrt{15xz}}{5z}$$

$$39. \frac{2x^2y\sqrt[5]{2x^2w^2z}}{w^2z} \quad 41. \frac{2a\sqrt[3]{2a^2b^2c}}{b^3c}$$

$$\begin{array}{ll}
 43. \frac{2\sqrt[4]{3x^2y^2z^2}}{3z^2} & 45. \frac{\sqrt{10xy}}{5y} \\
 47. -2\sqrt{2} & 49. 14\sqrt{3} \quad 51. -\sqrt{3} \\
 53. 2\sqrt[3]{2} + 10\sqrt[3]{3} & 55. 8a\sqrt{b} \\
 57. -30a & 59. 2x + 2\sqrt[3]{2x^2} - x\sqrt[3]{4} \\
 61. 12 - 22\sqrt{x} + 8x & 63. 45 - 18\sqrt[3]{2} \\
 65. 8x^2 - 8x\sqrt{2x} + 4x & 67. \frac{5 + \sqrt{10}}{3}
 \end{array}$$

$$\begin{array}{ll}
 69. \frac{a^2 + 2a\sqrt{b} + b}{a^2 - b} & 71. \frac{x\sqrt{3} - \sqrt{x}}{3x - 1} \\
 73. \frac{\sqrt{7} - 4}{15} & 75. \frac{4\sqrt{15}}{15} \\
 77. \frac{\sqrt{6 - \sqrt{2}}}{2} & 79. \frac{\sqrt{32 - \sqrt{5}}}{4} \\
 81. a\sqrt{a} & 83. y\sqrt{2x}
 \end{array}$$

85. Recall, $a^3 - b^3$

$$= (a - b)(a^2 + ab + b^2) \text{ and } a^3 + b^3$$

$$= (a + b)(a^2 - ab + b^2).$$

a. Using $a^3 - b^3$

$$= (a - b)(a^2 + ab + b^2),$$

let $a = \sqrt[3]{x}$ and $b = \sqrt[3]{y}$ so that

$$x - y = (\sqrt[3]{x} - \sqrt[3]{y})[(\sqrt[3]{x})^2 + \sqrt[3]{x}\sqrt[3]{y} + (\sqrt[3]{y})^2]$$

$$= (\sqrt[3]{x} - \sqrt[3]{y})(\sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2})$$

$$\text{Thus, } Q(x, y) = \sqrt[3]{x^2} + \sqrt[3]{xy} + \sqrt[3]{y^2}.$$

b. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$;let $a = \sqrt[3]{x}$ and $b = \sqrt[3]{y}$:

$$x + y = (\sqrt[3]{x} + \sqrt[3]{y})(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2})$$

c. $8x - y = (2\sqrt[3]{x})^3 - (\sqrt[3]{y})^3$

$$= (2\sqrt[3]{x} - \sqrt[3]{y})[(2\sqrt[3]{x})^2 + 2\sqrt[3]{x}\sqrt[3]{y} + (\sqrt[3]{y})^2]$$

$$= (2\sqrt[3]{x} - \sqrt[3]{y})(4\sqrt[3]{x^2} + 2\sqrt[3]{xy} + \sqrt[3]{y^2})$$

$$8x - y = (\sqrt{8x} - \sqrt{y})(\sqrt{8x} + \sqrt{y})$$

$$= (2\sqrt{2x} - \sqrt{y})(2\sqrt{2x} + \sqrt{y})$$

$$\begin{array}{l}
 87. \frac{\sqrt[3]{x}}{\sqrt[3]{2x^2} - \sqrt[3]{3x}} \cdot \frac{(\sqrt[3]{2x^2})^2 + \sqrt[3]{2x^2}\sqrt[3]{3x} + (\sqrt[3]{3x})^2}{(\sqrt[3]{2x^2})^2 + \sqrt[3]{2x^2}\sqrt[3]{3x} + (\sqrt[3]{3x})^2} \\
 \frac{\sqrt[3]{x}(\sqrt[3]{4x^4} + \sqrt[3]{6x^3} + \sqrt[3]{9x^2})}{2x^2 - 3x} \\
 \frac{\sqrt[3]{4x^5} + \sqrt[3]{6x^4} + \sqrt[3]{9x^3}}{2x^2 - 3x} \\
 \frac{x\sqrt[3]{4x^2} + x\sqrt[3]{6x} + x\sqrt[3]{9}}{2x^2 - 3x} \\
 \frac{x(\sqrt[3]{4x^2} + \sqrt[3]{6x} + \sqrt[3]{9})}{x(2x - 3)} \\
 \frac{\sqrt[3]{4x^2} + \sqrt[3]{6x} + \sqrt[3]{9}}{2x - 3}
 \end{array}$$

$$\begin{array}{ll}
 89. \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}} & \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \\
 & \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
 \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} & \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\
 \frac{\sqrt{(2 - \sqrt{3})^2}}{1} & \frac{\sqrt{3} - 1}{\sqrt{3} + 1} \cdot \frac{\sqrt{3} - 1}{\sqrt{3} - 1} \\
 \sqrt{(2 - \sqrt{3})^2} & \frac{4 - 2\sqrt{3}}{2} \\
 2 - \sqrt{3} & 2 - \sqrt{3}
 \end{array}$$

$$\begin{array}{l}
 91. \text{ a. } \sqrt[3]{5} = \sqrt[6]{5}. \text{ This seems logical} \\
 \text{because } (\sqrt[6]{5})^6 = 5 \text{ and } (\sqrt[3]{5})^6 = 5. \\
 \text{ b. } \sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x}.
 \end{array}$$

Solutions to skill and review problems

$$\begin{array}{ll}
 1. \frac{4 + 9}{12} = \frac{13}{12} & 2. \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{1} \cdot \frac{1}{4} = \frac{1}{4} \\
 3. 3\left(\frac{1}{a^2}\right) = \frac{3}{a^2} & 4. 3a^2b^3 \\
 5. \frac{1}{\sqrt[3]{-8}} = \frac{1}{-2} = -\frac{1}{2} & 6. \frac{8a^8}{2a^2} = 4a^6 \\
 7. \frac{-2a^{-2}}{8a^8} = -\frac{1}{4a^{10}}
 \end{array}$$

Solutions to trial exercise problems

$$\begin{array}{ll}
 11. \frac{\sqrt{16x^6}}{\sqrt[3]{9y^{10}}} & 18. \frac{\sqrt[3]{8,000}}{\sqrt[3]{2^6 \cdot 5^3}} \\
 \frac{4|x^3|}{3|y^5|} & \frac{2^2 \cdot 5}{20}
 \end{array}$$

$$\begin{array}{l}
 27. \frac{\sqrt[3]{25a^2b^4c} \sqrt[3]{25a^2b^5c^2}}{\sqrt[3]{5^4a^4b^9c^3}} \\
 \frac{5ab^3c \sqrt[3]{5a}}{\sqrt[3]{5^4a^4b^9c^3}}
 \end{array}$$

$$\begin{array}{l}
 36. \sqrt[3]{\frac{3}{20}} \\
 \frac{\sqrt[3]{3}}{\sqrt[3]{2^2 \cdot 5}} \cdot \frac{\sqrt[3]{2 \cdot 5^2}}{\sqrt[3]{2 \cdot 5^2}} \\
 \frac{\sqrt[3]{2 \cdot 3 \cdot 5^2}}{\sqrt[3]{2^3 \cdot 5^3}} \\
 \frac{\sqrt[3]{150}}{10}
 \end{array}$$

$$\begin{array}{ll}
 41. \sqrt[3]{\frac{16a^5}{b^7c^2}} & 45. \frac{\sqrt[4]{8x^5y^7}}{\sqrt[4]{50x^3y^9}} \\
 \sqrt[3]{\frac{16a^5}{b^7c^2} \cdot \frac{b^2c}{b^2c}} & \sqrt[4]{\frac{8x^5y^7}{50x^3y^9}} \\
 \frac{\sqrt[3]{2^4a^5b^2c}}{\sqrt[3]{b^9c^3}} & \sqrt[4]{\frac{4x^2}{25y^2}} \\
 \frac{2a\sqrt[3]{2a^2b^2c}}{b^3c} & \sqrt[4]{\frac{4x^2}{5^2y^2} \cdot \frac{5^2y^2}{5^2y^2}} \\
 & \frac{\sqrt[4]{100x^2y^2}}{\sqrt[4]{5^4y^4}} \\
 & \frac{\sqrt[4]{100x^2y^2}}{5y} \\
 & \frac{\sqrt[4]{10^2x^2y^2}}{5y} \\
 & \frac{\sqrt{10xy}}{5y}
 \end{array}$$

$$\begin{array}{l}
 59. \sqrt[3]{4x}(\sqrt[3]{2x^2} + \sqrt[3]{4x} - \sqrt{x^2}) \\
 \sqrt[3]{8x^3} + \sqrt[3]{16x^2} - \sqrt[3]{4x^3} \\
 2x + 2\sqrt[3]{2x^2} - x\sqrt[3]{4}
 \end{array}$$

$$\begin{array}{l}
 63. (5\sqrt{3} - 2\sqrt{6})(\sqrt{3} + \sqrt{12}) \\
 5(3) + 5\sqrt{36} - 2\sqrt{18} - 2\sqrt{72} \\
 15 + 30 - 6\sqrt{2} - 12\sqrt{2} \\
 45 - 18\sqrt{2}
 \end{array}$$

$$\begin{array}{l}
 71. \frac{\sqrt{2x}}{\sqrt{6x} + \sqrt{2}} \cdot \frac{\sqrt{6x} - \sqrt{2}}{\sqrt{6x} - \sqrt{2}} \\
 \frac{\sqrt{12x^2} - \sqrt{4x}}{6x - \sqrt{12x} + \sqrt{12x} - 2} \\
 \frac{2x\sqrt{3} - 2\sqrt{x}}{6x - 2} \\
 \frac{2(x\sqrt{3} - \sqrt{x})}{2(3x - 1)} \\
 \frac{x\sqrt{3} - \sqrt{x}}{3x - 1}
 \end{array}$$

$$76. -\frac{5}{2\sqrt{2}}\left(-\frac{1}{\sqrt{2}}\right) - \frac{1}{2}\left(\frac{\sqrt{5}}{3}\right)$$

$$\frac{5}{4} - \frac{\sqrt{5}}{6}$$

$$\frac{6(5) - 4\sqrt{5}}{4(6)}$$

$$\frac{30 - 4\sqrt{5}}{24}$$

$$\frac{2(15 - 2\sqrt{5})}{24}$$

$$\frac{15 - 2\sqrt{5}}{12}$$

$$77. \sqrt{\frac{3 - \frac{1}{\sqrt{2}}}{2}}$$

$$\sqrt{\frac{1}{2}\left(3 - \frac{\sqrt{2}}{2}\right)}$$

$$\sqrt{\frac{1}{2} \cdot \frac{6 - \sqrt{2}}{2}}$$

$$\sqrt{\frac{6 - \sqrt{2}}{4}}$$

$$\frac{\sqrt{6 - \sqrt{2}}}{2}$$

$$84. \frac{\sqrt[8]{a^8 b^{12} c^{16}}}{\sqrt[8]{a^{8 \div 4} b^{12 \div 4} c^{16 \div 4}}}$$

$$\frac{\sqrt{a^2 b^3 c^4}}{abc^2 \sqrt{b}}$$

$$86. \frac{3xy \cdot \frac{\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2}}{\sqrt[3]{x} + \sqrt[3]{y}}}{3xy(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2})}$$

$$\frac{1}{x + y}$$

Exercise 1-6

Answers to odd-numbered problems

1. $2\sqrt{2}$ 3. $\frac{1}{4}$ 5. $\frac{1}{4}$ 7. $4\sqrt{x}$
 9. $3\sqrt[4]{x^3}$ 11. $2xy\sqrt[4]{2x^2y^3}$ 13. $2x\sqrt{2x}$
 15. 5 17. $8a^{\frac{3}{4}}$ 19. b^2
 21. $2x^{\frac{2}{7}}y^{\frac{3}{5}}z^{\frac{1}{2}}$ 23. $\frac{1}{8}x^{\frac{1}{2}}y^{\frac{3}{4}}$ 25. $\frac{1}{ab^{\frac{1}{2}}}$
 27. $a^{\frac{3}{4}}$ 29. $3a^{\frac{1}{4}}$ 31. $x^{\frac{1}{4}}y^{\frac{4}{3}}$
 33. $x^{\frac{1}{6}}y^{\frac{1}{5}}z$ 35. $a^{2m}b^4$ 37. $2^m y^n$
 39. 75.3760 41. -2.8854
 43. -9.6549 45. 15.1539
 47. 1.4238 49. 15.8322 51. 0.5
 53. 849.1202 55. 167.6478

57. To find the new value of D_l replace L_b by $4L_b$.

$$D_l = c(4L_b)^{1.5} = c(4^{1.5})L_b^{1.5}$$

Compare this new value to the original value of $cL_b^{1.5}$:

$$\frac{c(4^{1.5})L_b^{1.5}}{cL_b^{1.5}} = 4^{1.5} = 4^{3/2} = (\sqrt{4})^3 = 8$$

Thus the new leg diameter must be eight times the original diameter if the body length increases by a factor of 4.

59. \$394.91 61. 1 63. 1

Solutions to skill and review problems

1. a. not real
 2. $10i^2 + 35i - 6i - 21$
 $-10 + 29i - 21$
 $-31 + 29i$
 3. a. -1 b. 1 c. -1
 4. $3(2) + \sqrt{12} - 3\sqrt{6} - \sqrt{18}$
 $6 + 2\sqrt{3} - 3\sqrt{6} - 3\sqrt{2}$
 5. $\frac{2\sqrt{3}}{\sqrt{6} + \sqrt{2}} \cdot \frac{\sqrt{6} - \sqrt{2}}{\sqrt{6} - \sqrt{2}}$
 $\frac{2\sqrt{18} - 2\sqrt{6}}{6 + \sqrt{12} - \sqrt{12} - 2}$
 $\frac{6\sqrt{2} - 2\sqrt{6}}{4}$
 $\frac{2(3\sqrt{2} - \sqrt{6})}{4}$
 $\frac{3\sqrt{2} - \sqrt{6}}{2}$

Solutions to trial exercise problems

5. $(-8)^{-\frac{2}{3}} = \frac{1}{(-8)^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{-8})^2}$
 $= \frac{1}{(-2)^2} = \frac{1}{4}$
 23. $\left(\frac{1}{4}x^{\frac{1}{4}}y^{\frac{1}{2}}\right)\left(\frac{1}{2}x^{\frac{1}{4}}y^{\frac{1}{4}}\right)$
 $= \frac{1}{4}\left(\frac{1}{2}\right)x^{\frac{1}{4} + \frac{1}{4}}y^{\frac{1}{2} + \frac{1}{4}} = \frac{1}{8}x^{\frac{1}{2}}y^{\frac{3}{4}}$
 31. $\frac{x^{\frac{5}{8}}y^{\frac{2}{3}}}{x^{\frac{3}{8}}y^{\frac{-2}{3}}}$
 $= x^{\frac{5}{8} - \frac{3}{8}}y^{\frac{2}{3} - \frac{-2}{3}}$
 $= x^{\frac{1}{4}}y^{\frac{4}{3}}$
 32. $\left(\frac{-3}{x^{\frac{5}{3}}y^{\frac{3}{4}}}\right)^{20}$
 $= \frac{(-3)^{20}}{x^{\frac{100}{3}}y^{\frac{15}{2}}}$
 $= \frac{y^{15}}{x^{\frac{100}{3}}z^6}$

49. $(\sqrt[3]{-500})^{\frac{4}{3}}$

500 $\boxed{x^{1/y}}$ 3 $\boxed{=}$ $\boxed{y^x}$ $\boxed{()}$
 4 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{=}$
 TI-81: $\boxed{()}$ $\boxed{\text{MATH}}$ 4 500 $\boxed{)}$
 $\boxed{\wedge}$ $\boxed{()}$ 4 $\boxed{\div}$ 3 $\boxed{)}$ $\boxed{\text{ENTER}}$
 15.8322

$$60. B_n = 45,000 \left[\frac{0.10 \cdot \left(1 + \frac{0.10}{12}\right)^0}{\left(1 + \frac{0.10}{12}\right)^{360} - 1} \right]$$

$$= \frac{45,000 \cdot 0.10}{12} \cdot \frac{1}{\left(1 + \frac{0.10}{12}\right)^{360} - 1}$$

$$= 375 \cdot \frac{1}{\left(1 + \frac{0.10}{12}\right)^{360} - 1} \approx 19.91$$

1 $\boxed{+}$ 0.10 $\boxed{\div}$ 12 $\boxed{=}$ $\boxed{x^y}$ 360
 $\boxed{-}$ 1 $\boxed{=}$ $\boxed{1/x}$ $\boxed{\times}$ 375 $\boxed{=}$
 TI-81: 375 $\boxed{()}$ 1 $\boxed{\div}$ $\boxed{()}$ $\boxed{()}$ 1
 $\boxed{+}$.1 $\boxed{\div}$ 12 $\boxed{)}$ $\boxed{\wedge}$ 360 $\boxed{-}$ 1
 $\boxed{)}$ $\boxed{)}$ $\boxed{\text{ENTER}}$

$$I_n = 394.91 \left[1 - \left(1 + \frac{0.10}{12}\right)^{1-1-360} \right]$$

$$= 394.91 \left[1 - \left(1 + \frac{0.10}{12}\right)^{-360} \right]$$

$$\approx 375.00$$

1 $\boxed{-}$ $\boxed{()}$ 1 $\boxed{+}$ 0.10 $\boxed{\div}$ 12 $\boxed{)}$
 $\boxed{x^y}$ 360 $\boxed{+/-}$ $\boxed{=}$ $\boxed{\times}$
 394.91 $\boxed{=}$
 TI-81: 394.91 $\boxed{()}$ 1 $\boxed{-}$ $\boxed{()}$ 1
 $\boxed{+}$.1 $\boxed{\div}$ 12 $\boxed{)}$ $\boxed{\wedge}$ $\boxed{(-)}$
 360 $\boxed{)}$ $\boxed{\text{ENTER}}$

Exercise 1-7

Answers to odd-numbered problems

1. $-5 + 8i$ 3. $13 + 8i$
 5. $5 + 62i$ 7. 34 9. $21 - 20i$
 11. $-30 - i$ 13. $-\frac{14}{29} - \frac{23}{29}i$
 15. $\frac{5}{13} + \frac{12}{13}i$ 17. $-\frac{18}{13} + \frac{12}{13}i$
 19. $-6\sqrt{2}$ 21. $5\sqrt{2}i$
 23. $13 - 7\sqrt{2}i$ 25. $15 - \sqrt{3}i$
 27. $6 + 2\sqrt{3} + (-2\sqrt{2} + 3\sqrt{6})i$
 29. $\frac{4 - 3\sqrt{3}}{11} - \frac{6\sqrt{2} + \sqrt{6}}{11}i$
 31. $-\frac{7\sqrt{2}}{22} - \frac{27}{11}i$ 33. $-\sqrt{3}i$

35. -1 37. $-i$ 39. $-i$ 41. i

43. $-7 + 11i$ 45. $-\frac{125}{58} + \frac{95}{58}i$

47. -5 49. $T = \frac{6,190}{43,709} + \frac{1,094}{43,709}i$

51. $6 - 2i = X_c$

53. The value is complex for $x - 16 < 0$, so $x < 16$.

55. Subtraction:

$$\begin{aligned}(a + bi) - (c + di) \\&= (a - c) + (bi - di) \\&= (a - c) + (b - d)i \\ \text{Rule: } (a + bi) - (c + di) \\&= (a - c) + (b - d)i\end{aligned}$$

Multiplication:

$$\begin{aligned}(a + bi)(c + di) \\&= ac + bdi^2 + adi + bci \\&= (ac - bd) + (ad + bc)i \\ \text{Rule: } (a + bi)(c + di) \\&= (ac - bd) + (ad + bc)i\end{aligned}$$

Division:

$$\begin{aligned}\frac{a + bi}{c + di} &= \frac{a + bi}{c + di} \cdot \frac{c - di}{c - di} \\&= \frac{ac - bdi^2 - adi + bci}{c^2 - di^2 - cdi + cdi} \\&= \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2} \\&= \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i\end{aligned}$$

Rule: $\frac{a + bi}{c + di} = \frac{ac + bd}{c^2 + d^2} + \frac{bc - ad}{c^2 + d^2}i$

57. After approximately 18 iterations, the value of z repeats the value 0.1074991191 + 0.0636941246i. The following is a program for a TI-81:

```

Prgm2: JULIA
:5→A
:~.2→B
:Lbl 1
:A^2-B^2→C
:2AB→D
:C+.1→A
:D+.05→B
:Disp A
:Disp B
:Pause
:Goto 1

```

Solutions to skill and review problems

$$\begin{aligned}1. & -5[3x - 2(1 - 4x)] \\& -5[3x - 2 + 8x] \\& -5[11x - 2] \\& -55x + 10\end{aligned}$$

2. $x + 5 = 12$; 7, since $7 + 5 = 12$

3. $5x = 20$; 4, since $5 \cdot 4 = 20$

4. $\frac{x}{6} = 48$; 288, since $\frac{288}{6} = 48$

5. $3(2 - 3x) = 1 - 10x$

Replace x by -5 :

$3[2 - 3(-5)] = 1 - 10(-5)$

$3(2 + 15) = 1 + 50$

$3(17) = 51$

$51 = 51$

yes

6. any value, since $x + x$ combines into $2x$ regardless of considering the value of x

7. $C = \frac{5}{9}(72 - 32)$

$= \frac{5}{9}(40)$

$= \frac{200}{9} = 22\frac{2}{9} \text{ centigrade}$

8. $0.06(1,000 - 2x)$

$0.06(1,000) - 0.06(2)x$

$60 - 0.12x$

9. $8\% = \frac{8}{100}$, so d. 0.08

10. $0.08(12,000) = 960$

11. $0.06(4,000) + 0.1(12,000)$

$240 + 1,200$

$1,440$

Solutions to trial exercise problems

$$\begin{aligned}11. & i[(5 - 3i)(-2 + 4i) - (2 - i)^2] \\& i[(5 - 3i)(-2 + 4i) - (2 - i)(2 - i)] \\& i[(-10 + 20i + 6i - 12i^2) \\& \quad - (4 - 2i - 2i + i^2)] \\& i[(2 + 26i) - (3 - 4i)] \\& i[-1 + 30i] \\& -i + 30i^2 \\& -30 - i\end{aligned}$$

15. $\frac{6 + 4i}{6 - 4i} = \frac{3 + 2i}{3 - 2i}$

$\frac{3 + 2i}{3 - 2i} \cdot \frac{3 + 2i}{3 + 2i}$

$\frac{9 + 6i + 6i + 4i^2}{9 + 6i - 6i - 4i^2}$

$\frac{5 + 12i}{9 - 4}$

$\frac{5 + 12i}{5}$

$1 + \frac{12}{5}i$

$$\begin{aligned}27. & (2 + \sqrt{-6})(3 - \sqrt{-2}) \\& (2 + \sqrt{6}i)(3 - \sqrt{2}i) \\& 6 - 2\sqrt{2}i + 3\sqrt{6}i - \sqrt{12}i^2 \\& 6 + \sqrt{12} - 2\sqrt{2}i + 3\sqrt{6}i \\& 6 + 2\sqrt{3} + (-2\sqrt{2} + 3\sqrt{6})i\end{aligned}$$

33. $\frac{\sqrt{-6} + \sqrt{6}}{\sqrt{-2} - \sqrt{2}}$

$\frac{\sqrt{6}i + \sqrt{6}}{\sqrt{2}i - \sqrt{2}}$

$\frac{\sqrt{6} + \sqrt{6}i}{-\sqrt{2} + \sqrt{2}i}$

$\frac{\sqrt{6} + \sqrt{6}i}{-\sqrt{2} + \sqrt{2}i} \cdot \frac{-\sqrt{2} - \sqrt{2}i}{-\sqrt{2} - \sqrt{2}i}$

$\frac{-\sqrt{12} - \sqrt{12}i - \sqrt{12}i - \sqrt{12}i^2}{2 + 2\sqrt{2}i - 2\sqrt{2}i - 2i^2}$

$\frac{-2\sqrt{12}i}{4}$

$\frac{-4\sqrt{3}i}{4}$

$-\sqrt{3}i$

39. $i^{-5} = \frac{1}{i^5} = \frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{-1} = -i$

$$\begin{aligned}47. & (2 - i)^3 - 3(2 - i)^2 + (2 - i) \\& (2 - i)(2 - i)(2 - i) \\& \quad - 3(2 - i)(2 - i) + (2 - i) \\& (2 - i)(3 - 4i) - 3(3 - 4i) + 2 - i \\& 6 - 8i - 3i + 4i^2 - 9 + 12i + 2 - i \\& -5\end{aligned}$$

Chapter 1 review

1. $\{0, 1, 2, 3, 4, 5, 6, 7\}$

2. $\{1, 2, 3, 4, \dots\}$

3. $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{100}{101}\right\}$ 4. 0.41666

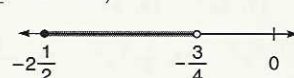
5. 0.230769230769 6. $-\frac{43}{4}$ 7. $-\frac{7}{24}$

8. $-\frac{29}{45}$ 9. $\frac{5}{112}$ 10. $\frac{12bx - 5ay}{20ab}$

11. $\frac{8a + 13ab + 3b}{4ab}$ 12. $\frac{3x^3}{25y^3}$

13. $\frac{6a^2 + b^2}{5a^2}$

14. $\left[-2\frac{1}{2}, -\frac{3}{4}\right)$

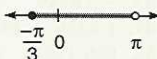


15. $\left(-\frac{1}{4}, \infty\right)$

16. $\{x | x \leq -1\}$



17. $\left\{x | -\frac{\pi}{3} \leq x < \pi\right\}$



18. $\left(-\frac{1}{2}, 3\frac{1}{4}\right]; \left\{x | -\frac{1}{2} < x \leq 3\frac{1}{4}\right\}$

19. $[\pi, \infty)$; $\{x \mid x \geq \pi\}$ 20. $-\frac{1}{2}$
 21. $\pi + 9$ 22. $\sqrt{2} - 5$ 23. $5x^2$
 24. $\frac{3x^2}{2|y^3|}$ 25. $5|2x - 1|$
 26. $-(x - 2)^2|x + 1|$ 27. $\frac{27x^6}{y^3}$
 28. $\frac{3x}{y^2}$ 29. $\frac{x^3y^2}{25}$ 30. $\frac{6x}{y}$
 31. $-\frac{36x^6}{y^6}$ 32. $\frac{3y^5}{x^5}$ 33. $-\frac{y^6}{64}$
 34. $\frac{36}{a^{20}}$ 35. $\frac{1}{x^{2n}}$ 36. $\frac{x^{8n}}{y^{4n-12}}$
 37. 4.2182×10^{16} 38. -4.605×10^{-11}
 39. 0.000 000 405 2
 40. $-340,900,000,000$ 41. 27 42. 21
 43. -515 44. $-15x^6 - 21x^4 + x^3 + 6$
 45. $5a^3 - 49a^2 + 115a + 25$
 46. $-3x^5 + 13x^4 - 5x^3 - 7x^2 - 15x + 15$
 47. $-3x^4 - x^2 + 1$
 48. $-10a^4 - 3 - 4b^8$
 49. $y^3 + y^2 + y + 1$
 50. $2x^3 - 3x^2 - x - 4\frac{1}{2} + \frac{-\frac{1}{2}}{2x + 1}$
 51. $x^3(5 - x)(5 + x)$
 52. $(x + 4)(x + 9)$
 53. $a(2a - 1)(4a - 5)$
 54. $(3ab - 4)(ab + 2)$
 55. $b(2a + 5b)(4a^2 - 10ab + 25b^2)$
 56. $5a(x - 1)(x + 1)(3x - 1)(3x + 1)$
 57. $(5x - y)(x - 10y)$
 58. $(a - b + 2x + y)(a - b - 2x - y)$
 59. $2(3x^2 - y)(9x^4 + 3x^2y + y^2)$
 60. $12(x - 2y)(x + 2y)$
 61. $(x + 2y)(3a - b)$
 62. $(2a^3 - bc)(4a^6 + 2a^3bc + b^2c^2)$
 63. $7a^2 - 32a - 21$ 64. $4(x^2 - 3a^2)^2$
 65. $(3x^3 + 1)(x^3 - 3)$
 66. $4b(x + 3y)(1 - 2a)(1 + 2a)$
 67. $(a - 2x - 10y)(a + 2x + 10y)$
 68. $(3x + 13)(x - 7)$
 69. $3x^2(xy^3 + 3z^2)(x^2y^6 - 3xy^3z^2 + 9z^4)$
 70. $(x - 1)(x + 1)(x - 2)^2$
 71. $\frac{a}{3a + 1}$ 72. $\frac{-y}{2x + 3}$
 73. $\frac{2x + 1}{4x^2 + 2x + 1}$ 74. $\frac{x - 2}{(x + 2)(2x - 1)}$
 75. $\frac{15x^2 - 2xy - 6y^2}{15xy}$ 76. $\frac{2x - 8}{x - 2}$
 77. $\frac{-8x^2 + 4x - 5}{4x(4x - 5)}$ 78. $\frac{12}{b}$

79. $\frac{2x(x - 4)}{4x + 3}$ 80. $\frac{1}{(x + 1)(x + 2)}$
 81. $\frac{-2x^2 + 12x - 10}{(x - 3)(x - 2)}$
 82. $\frac{-2x^2 - 13x + 13}{(x + 3)(x - 3)(x - 1)}$
 83. $\frac{-6x + 5y}{2(2x + y)}$ 84. $\frac{38a - 12b}{6a + 9b - 10}$
 85. $\frac{-2a + 2b + 3}{5a - 5b - 3}$ 86. -2 87. 4
 88. $6\sqrt[3]{2}$ 89. $3x^2y^3z\sqrt[3]{6y}$
 90. $2a^2bc^2\sqrt[3]{6b^2c^2}$ 91. $3a^2b\sqrt[3]{3a^2b}$
 92. $5ab^2\sqrt[4]{bc^3}$ 93. $b\sqrt[3]{a^2}$
 94. $8a^5b^5\sqrt{2a}$ 95. $\frac{\sqrt{3a}}{a}$
 96. $\frac{2xy^2\sqrt{10wxyz}}{5w^2z}$ 97. $9ab\sqrt{2b}$
 98. $-c\sqrt[4]{3b^3c^2}$
 99. $3xy^2 - 3xy\sqrt{y} + 3y^2\sqrt{2x}$
 100. $3x - 6x\sqrt[4]{27x} + 9x\sqrt[4]{9x^2}$
 101. $\frac{6x - 5\sqrt{6x} - 2\sqrt{3x} + 5\sqrt{2}}{2(3x - 1)}$
 102. $\frac{a\sqrt{a} - \sqrt{ab} + a - \sqrt{b}}{a^2 - b}$
 103. $\frac{3\sqrt{2} + 2\sqrt{6}}{16}$ 104. $\frac{\sqrt{9 - \sqrt{2}}}{3}$
 105. $5x\sqrt{x}$ 106. $8x^6$ 107. $\frac{\sqrt{x}}{3x^2}$
 108. $2|x^3|y^4\sqrt{2y}$ 109. $\frac{\sqrt{2}|x^3|}{4|y^5|}$
 110. $4|x^3(x - 3)|$ 111. $\left(x^{\frac{17}{12}}y^{\frac{1}{4}}\right)$
 112. $\frac{4x^{\frac{8}{15}}z^{\frac{2}{3}}}{\frac{1}{y^2}}$ 113. $\frac{x^{\frac{1}{4}}}{2y^{\frac{5}{4}}}$ 114. $\frac{x^{\frac{8}{5}}y^2z^2}{256}$
 115. $x^{\frac{c}{2}}y^{\frac{b}{3}}$ 116. $64^m x^{3m+2n} y^n$
 117. 50.2304 118. 625 119. 3.5342
 120. 1.5551 121. $-19\frac{1}{2} + 15i$
 122. $5 + 62i$ 123. $7\frac{8}{9} + 11\frac{2}{9}i$
 124. $-\frac{24}{29} - \frac{27}{29}i$ 125. $\frac{5}{17} + \frac{14}{17}i$
 126. $12 - 6\sqrt{2}i$ 127. $-\frac{3}{7} + \frac{9\sqrt{3}}{7}i$
 128. $-28 - 16\sqrt{2}i$ 129. $-\frac{1}{10} + \frac{3}{10}i$
 130. $-16 + 2i$ 131. $\frac{57}{29} - \frac{41}{29}i$
 132. $-2 - 10i$

Chapter 1 test

1. $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}$ 2. 14 3. $\frac{5}{24}$
 4. $\frac{a^2 - 5ab - 12b^2}{4ab}$ 5. $\frac{6a^2 + b^2}{5a^2}$
 6. $(-2\frac{1}{2}, -\frac{3}{4})$



7. $\{x \mid x \geq -3\}$
 8. $\{x \mid -1\frac{1}{2} \leq x < 2\}$, $[-1\frac{1}{2}, 2)$ 9. -4
 10. $\pi - 2$ 11. $3x^2|y|$ 12. $6x^3y^3$
 13. $-16x^4y^4$ 14. $-3x^3y^5$ 15. $\frac{36}{a^{20}}$
 16. $\frac{1}{x^n}$ 17. 2.05×10^{11} 18. 0.000213
 19. 3 20. $-\frac{7}{3}$
 21. $-2x^6 - x^4 + 6x^3 + 4$
 22. $a^4 - 50a^2 + 625$
 23. $2x^2 + 3x + 10 + \frac{15}{x - 2}$
 24. $4a(a - 2)(a + 2)$
 25. $(3x - 2)(3x + 1)$
 26. $(x - 2)(x + 2)(x^2 + 4)$
 27. $[(2x - 1)(4x^2 + 2x + 1)]$
 $[(2x + 1)(4x^2 - 2x + 1)]$
 28. $(x + 1)(x - 3)$
 29. $(3c + d)(a - 2b)$ 30. $\frac{x}{x + 1}$
 31. $\frac{x + 1}{2x + 1}$ 32. $\frac{2x - 3}{x - 2}$
 33. $\frac{x^2 + x + 1}{3x(2x + 1)}$ 34. $\frac{x^2 - 3x + 2}{x^2 + 3x + 2}$
 35. $\frac{2b - 9a}{2 + 6ab}$ 36. $\frac{a - b - 1}{a - b + 1}$ 37. $4\sqrt[3]{2}$
 38. $5x^2y\sqrt{2yz}$ 39. $4b^3\sqrt[3]{4a^2b}$
 40. $6b^2c\sqrt[4]{a^3b}$ 41. $\sqrt{6x}$ 42. $\frac{2x\sqrt[3]{xyz^2}}{yz}$
 43. $2a\sqrt{5ab}$
 44. $-2xy\sqrt{x} + 4x\sqrt{xy} - 2xy\sqrt{3}$
 45. $\frac{a\sqrt{6} - \sqrt{3a} - \sqrt{6a} + \sqrt{3}}{2a - 1}$
 46. $\frac{\sqrt{3} + 3\sqrt{5}}{6}$ 47. $2x\sqrt[3]{2}$ 48. $\frac{\sqrt[4]{x}}{2x^3y}$
 49. $2\sqrt{5}|x^3|y^4$ 50. $5x^2|x - 3|$
 51. $\frac{x}{y^{\frac{1}{4}}}$ 52. $\frac{81xz^{\frac{4}{3}}}{y^{\frac{1}{2}}}$ 53. $\frac{1}{4x^{\frac{1}{6}}y}$

54. $\frac{256}{x^4y^2z^2}$ 55. $a^{\frac{1}{2}}b^{\frac{1}{m}}$ 56. 0.0494
57. $-226 - 481i$ 58. $-\frac{24}{29} - \frac{27i}{29}$
59. $2\sqrt{3} - 3i$ 60. $\frac{10}{7} - \frac{2\sqrt{3}}{7}i$
61. $\frac{2}{3} + \frac{4}{3}i$

Chapter 2

Exercise 2-1

1. $\{\frac{5}{16}\}$ 3. $\{-8\}$ 5. $\{88\}$ 7. $\{-\frac{6}{5}\}$
 9. $\{\frac{7}{9}\}$ 11. $\{\frac{2}{5}\}$ 13. R 15. $\{-\frac{7}{3}\}$
 17. $\{-\frac{59}{16}\}$ 19. $\{0\}$ 21. $\{-\frac{1}{9}\}$
 23. $\{4\}$ 25. R 27. 1.2356
 29. 0.3571 31. $\frac{V-k}{g} = t$

33. $\frac{2S + gt^2}{2t} = V$

35. $\frac{2S - dn^2 + nd}{2n} = a$

37. $\frac{d - d_1 - jd_3}{k - 1 - d_3} = d_2$ 39. $y = \frac{3x}{10}$

41. $x = -\frac{7}{3}y$ 43. $\frac{V + br^2}{r^2} = a$

45. $8y = x$ 47. $\frac{3V + \pi h^3}{3\pi h^2} = R$

49. $\frac{by - 4b - 3a}{a} = x$ 51. $R_0 = \frac{RT}{a + T}$

53. $\frac{p-s}{s(p-1)} = f$

55. \$10,000 at 8% and \$5,000 at 6%

57. \$10,000 at 14% gain, \$8,000 at 9% loss

59. \$5,000 at 5% and \$6,000 at 8%

61. \$12,285.71 at 5% and \$5,714.29 at 9%

63. 40 gallons of 35% and 40 gallons of

65% **65.** 1,500 gallons of 4% solution

67. 128.6 liters of 20% and 171.4 liters of

55% 69. a. 1 hr 17 min b. 6 hr 3

min 71. 11 hours 31 minutes 73. 45

mph for the truck and 60 mph for the car

75. $56\frac{1}{4}$ mph 77. $\frac{7}{12}$ mile

79. assume $\frac{a}{b} = \frac{c}{d}$; multiply each

member by bd

$$bd\left(\frac{a}{b}\right) = bd\left(\frac{c}{d}\right)$$

$$d(a) = b(c)$$

$$ad = bc$$

1. $(x - 2)(x + 5) = 0$
 $x - 2 = 0$ or $x + 5 = 0$
 $x = 2$ or $x = -5$
2. $(2x - 3)(x + 2)$
 $2x^2 + 4x - 3x - 6$
 $2x^2 + x - 6$
3. $4x^2 - 16x$
 $4x(x - 4)$
4. $4x^2 - 1$
 $(2x - 1)(2x + 1)$
5. $6x^2 - 5x - 4$
 $(3x - 4)(2x + 1)$
6. $\sqrt{-20}$
 $\sqrt{4 \cdot 5i}$
 $2\sqrt{5i}$

$$\begin{aligned} 7. & \sqrt{8 - 4(3)(-2)} \\ & \sqrt{8 + 24} \\ & \sqrt{32} = \sqrt{2^5} = 2^2\sqrt{2} = 4\sqrt{2} \end{aligned}$$

$$8. \frac{8 - \sqrt{32}}{4}$$
$$\frac{8 - 4\sqrt{2}}{4}$$

$$\frac{4(2 - \sqrt{2})}{4}$$

9. Area = length \times width
 $= 8(6) = 48 \text{ in.}^2$
 Perimeter = $2(\text{length}) + 2(\text{width})$
 $= 2(8) + 2(6)$
 $= 28 \text{ inches}$

	1	2	3	...				
6								
						...	47	48

$$\text{time} = \frac{\text{distance}}{\text{rate}} = \frac{135}{45} = 3 \text{ hours}$$

$$\begin{aligned} 5. \quad \frac{1}{4}x + 3 &= \frac{3}{8}x - 8 \\ 8(\frac{1}{4}x + 3) &= 8(\frac{3}{8}x - 8) \\ 2x + 24 &= 3x - 64 \\ 88 &= x \\ \{88\} \end{aligned}$$

$$\begin{aligned} 11. \quad \frac{2-3x}{4} &= \frac{x}{2} \\ 2(2-3x) &= 4x \\ 4-6x &= 4x \\ 4 &= 10x \\ \frac{4}{10} &= x \\ \left\{ \frac{2}{5} \right\} \end{aligned}$$

$$\begin{aligned} 29. \quad & 150x - 13.8 = 0.04(1,500 - 1,417x) \\ & 150x - 13.8 = 60 - 56.68x \\ & 206.68x = 73.8 \\ & x = \frac{73.8}{206.68} \approx 0.3571 \end{aligned}$$

$$\begin{aligned} 35. S &= \frac{n}{2}[2a + (n-1)d]; \text{ for } a \\ 2S &= n[2a + (n-1)d] \\ 2S &= 2an + dn^2 - nd \\ 2S - dn^2 + nd &= 2an \\ \frac{2S - dn^2 + nd}{2n} &= a \end{aligned}$$

$$\begin{aligned} 39. \quad \frac{x+2y}{x-2y} &= 4; \text{ for } y \\ x+2y &= 4x-8y \\ 10y &= 3x \\ y &= \frac{3x}{10} \end{aligned}$$

$$\begin{aligned} 51. \quad T &= \frac{aR_0}{R - R_0}; \text{ for } R_0 \\ T(R - R_0) &= aR_0 \\ TR - TR_0 &= aR_0 \\ TR &= aR_0 + TR_0 \\ TR &= R_0(a + T) \\ \frac{TR}{a + T} &= R_0 \end{aligned}$$

59. Let x be the smaller amount, which was invested at 5%. Then the larger amount was $x + 1,000$, and was invested at 8%. The return from the larger investment minus the return from the smaller investment is \$230.

$$0.08(x + 1,000) - 0.05x = 230$$
$$0.08x + 80 - 0.05x = 230$$
$$0.03x = 150$$
$$x = \frac{150}{0.03} = 5,000; \text{ thus, } \$5,000 \text{ was}$$

invested at 5% and \$6,000 at 8%.

62. x = additional amount invested at 8%
 $\$6,000$ at 5% earns $\$300$
 x at 8% earns $0.08x$
 The total amount earned is 300
 $+ 0.08x$
 The total investment is $x + 6,000$
 We want $0.06(x + 6,000) = 300$
 $+ 0.08x$
 $0.06x + 360 = 300 + 0.08x$
 $60 = 0.02x$
 $\frac{60}{0.02} = x$
 $3,000 = x$
 Thus we want to invest $\$3,000$ at 8%.

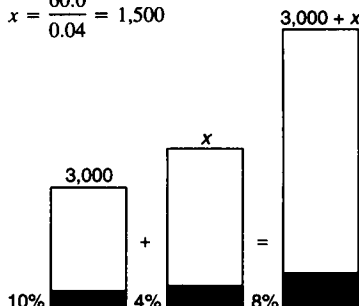
65. Let x = amount of 4% pesticide solution. The total amount of solution will be $3,000 + x$. The total amount of pesticide will be 10% of 3,000 (which is 300) and 4% of x , $(0.04x)$, or $300 + 0.04x$. We want this amount to be 8% of $3,000 + x$:

$$0.08(3,000 + x) = 300 + 0.04x$$

$$240 + 0.08x = 300 + 0.04x$$

$$0.04x = 60$$

$$x = \frac{60.0}{0.04} = 1,500$$



Thus, 1,500 gallons of the 4% solution should be added to the 3,000 gallons of 10% solution.

68. rate \times time = work, so rate = $\frac{\text{work}}{\text{time}}$;

$$\text{first rate is } \frac{5,000}{35} = \frac{1,000}{7} \text{ flyers/}$$

$$\text{minute; second rate is } \frac{5,000}{50} = 100$$

$$\text{flyers/minute. Combined rate is } \frac{1,000}{7}$$

$$+ 100.$$

$$\text{rate} \times \text{time} = \text{work}$$

$$\text{a. } \left(\frac{1,000}{7} + 100\right)t = 5,000$$

$$\frac{1,000}{7}t + 100t = 5,000$$

$$1,000t + 700t = 35,000;$$

$$t = \frac{35,000}{1,700} = 20.5882 \text{ minutes}$$

$$= 20 \text{ minutes } 35 \text{ seconds}$$

$$\text{b. } \left(\frac{1,000}{7} + 100\right)t = 8,000$$

$$\frac{1,000}{7}t + 100t = 8,000$$

$$1,000t + 700t = 56,000$$

$$t = \frac{56,000}{1,700} = 32 \text{ min } 56 \text{ sec}$$

74. x = speed of current; upstream the boat's rate is $16 - x$, and downstream it is $16 + x$; times are equal, and

$$t = \frac{d}{r}, \text{ so}$$

$$\frac{20}{16 - x} = \frac{14}{16 + x}$$

$$20(16 + x) = 14(16 - x)$$

$$320 - 20x = 224 + 14x$$

$$96 = 34x$$

$$x = 2\frac{14}{17} \text{ mph for the speed of the current}$$

Exercise 2-2

Answers to odd-numbered problems

1. $\{-1, 8\}$ 3. $\{0, 5\}$ 5. $\{-1, 3\}$

7. $\{-2, \frac{1}{3}\}$ 9. $\{-8, 1\}$ 11. $\{-3, 1\}$

13. $\{-\frac{3y}{5}, 2y\}$ 15. $\{\frac{3a}{2}, -\frac{5a}{6}\}$

17. $\{\pm 3\}$ 19. $\{\pm 2\sqrt{10}\}$

21. $\{\pm \frac{2\sqrt{10}}{3}\}$ 23. $\{\pm \frac{4\sqrt{10}}{5}\}$

25. $\{3 \pm \sqrt{10}\}$ 27. $\{-1 \pm \frac{2\sqrt{6}}{3}\}$

29. $\{-\frac{c}{b} \pm \frac{\sqrt{ad}}{ab}\}$ 31. $\frac{5 \pm \sqrt{97}}{6}$

33. $\frac{-2 \pm \sqrt{94}}{6}$ 35. $3 \pm \sqrt{14}$

37. $5\left(x - \frac{4 + 2\sqrt{19}}{5}\right)\left(x - \frac{4 - 2\sqrt{19}}{5}\right)$

39. $2\left(x - \frac{-3 + \sqrt{17}}{2}\right)\left(x - \frac{-3 - \sqrt{17}}{2}\right)$

41. 16 43. $w = \frac{\sqrt{3,999} + 3}{2}$
 $\approx 33.1 \text{ ft, length} = \frac{3\sqrt{3,999} - 1}{2} \approx 94.4 \text{ ft}$

45. length = 52 m; $w = 23 \text{ m}$ 47. \$3.33

49. $\frac{\sqrt{265} + 13}{2} \approx 14.6 \text{ hours and}$

$\frac{\sqrt{265} + 19}{2} \approx 17.6 \text{ hours}$

51. $\frac{\sqrt{321} - 1}{4} \approx 4.3 \text{ hours}$

53. $\{x | x \neq \frac{1}{2}\}$ 55. $\{z | z \neq -2\}$

57. $\{m | m \neq 0, 2\}$ 59. $\{x | x \neq 7, -3\}$

61. R 63. $\{\pm 2, \pm \sqrt{7}i\}$ 65. $\{5 \pm \sqrt{13}\}$

67. $\left\{\frac{73 + 3\sqrt{137}}{32}\right\}$ 69. $\{1, 729\}$

71. $\{\frac{1}{2}, 2\}$

$$\begin{aligned} 73. & a\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)^2 \\ & + b\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right) + c \\ & = a\left(\frac{b^2 - 2b\sqrt{b^2 - 4ac} + b^2 - 4ac}{4a^2}\right) \\ & + \frac{-b^2 + b\sqrt{b^2 - 4ac}}{2a} + c \\ & = \frac{b^2 - 2b\sqrt{b^2 - 4ac} + b^2 - 4ac}{4a} \\ & + \frac{-2b^2 + 2b\sqrt{b^2 - 4ac}}{4a} + \frac{4ac}{4a} = 0 \end{aligned}$$

The case for $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$ is

almost the same.

75. $2 - 3i$ or $-2 + 3i$

77. $a = \sqrt{\frac{c + \sqrt{c^2 + d^2}}{2}}$,

$$b = \frac{d}{\sqrt{2}(\sqrt{c + \sqrt{c^2 + d^2}})}$$
 when c and

d are not both 0. When c and d are zero, let $b = 0$.

Solutions to skill and review problems

1. $(3\sqrt{2x})^2 = 3^2(\sqrt{2x})^2 = 9(2x) = 18x$

2. $(3 + \sqrt{2x})(3 + \sqrt{2x}) = 9 + 3\sqrt{2x} + 3\sqrt{2x} + 2x = 9 + 6\sqrt{2x} + 2x$

3. $\sqrt{x} = 4$ 4. $\sqrt[3]{x} = 4$
 $(\sqrt{x})^2 = 4^2$ $(\sqrt[3]{x})^3 = 4^3$

$x = 16$ $x = 64$

5. $\sqrt{\frac{2}{3}} + 2 = \sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{\sqrt{3}}$

$= \frac{2\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{6}}{3}; d$

Solutions to trial exercise problems

9. $\frac{x}{2} + \frac{7}{2} = \frac{4}{x}$

$2x\left(\frac{x}{2}\right) + 2x\left(\frac{7}{2}\right) = 2x\left(\frac{4}{x}\right)$

$x^2 + 7x = 8$

$x^2 + 7x - 8 = 0$

$(x + 8)(x - 1) = 0$

$\{-8, 1\}$

10. $(p + 4)(p - 6) = -16$

$p^2 - 2p - 24 = -16$

$p^2 - 2p - 8 = 0$

$(p - 4)(p + 2) = 0$

$\{-2, 4\}$

$$\begin{aligned}
 12. \quad & x^2 - 4ax + 3a^2 = 0 \\
 & (x - 3a)(x - a) = 0 \\
 & x - 3a = 0 \text{ or } x - a = 0 \\
 & x = 3a \text{ or } x = a \\
 & \{a, 3a\}
 \end{aligned}$$

$$27. \quad 3(x + 1)^2 = 8$$

$$(x + 1)^2 = \frac{8}{3}$$

$$x + 1 = \pm \sqrt{\frac{8}{3}}$$

$$x = -1 \pm \frac{2\sqrt{6}}{3}$$

$$\left\{-1 \pm \frac{2\sqrt{6}}{3}\right\}$$

$$\begin{aligned}
 36. \quad & \frac{1}{x+2} - x = 5 \\
 & 1 - x(x+2) = 5(x+2) \\
 & 0 = x^2 + 7x + 9 \\
 & a = 1, b = 7, c = 9; \\
 & x = \frac{-7 \pm \sqrt{7^2 - 4(1)(9)}}{2(1)} \\
 & x = \frac{-7 \pm \sqrt{13}}{2}
 \end{aligned}$$

$$\begin{aligned}
 39. \quad & 2x^2 + 6x - 4 \\
 & 2(x^2 + 3x - 2) \\
 & 2\left(x - \frac{-3 + \sqrt{17}}{2}\right)\left(x - \frac{-3 - \sqrt{17}}{2}\right)
 \end{aligned}$$

$$\begin{aligned}
 43. \quad & w = \text{width, so length} = 3w - 5; \\
 & 100^2 = w^2 + (3w - 5)^2 \\
 & w = \frac{\sqrt{3,999} + 3}{2} \approx 33.1 \text{ ft,} \\
 & \text{length} = 3w - 5 \\
 & = \frac{3\sqrt{3,999} - 1}{2} \approx 94.4 \text{ ft}
 \end{aligned}$$

$$\begin{aligned}
 49. \quad & x = \text{time for one press, so } x + 3 \text{ is the} \\
 & \text{time for the other press; rate} \times \text{time} \\
 & = \text{work, so rate} = \frac{\text{work}}{\text{time}}. \text{ One rate is} \\
 & \frac{10,000}{x}, \text{ second is } \frac{10,000}{x+3}; \text{ combined} \\
 & \text{rate is } \frac{10,000}{x} + \frac{10,000}{x+3}, \text{ so using rate} \\
 & \times \text{time} = \text{work we obtain} \\
 & \left(\frac{10,000}{x} + \frac{10,000}{x+3}\right)8 = 10,000 \\
 & \text{Multiply each member by } \frac{1}{10,000} \\
 & \left(\frac{1}{x} + \frac{1}{x+3}\right)8 = 1 \\
 & \frac{(x+3) + x}{x(x+3)} \cdot 8 = 1 \\
 & \frac{8(2x+3)}{x^2+3x} = 1
 \end{aligned}$$

$$\begin{aligned}
 16x + 24 &= x^2 + 3x \\
 0 &= x^2 - 13x - 24 \\
 x &= \frac{\sqrt{265} + 13}{2}
 \end{aligned}$$

$$\text{So the rates are } \frac{\sqrt{265} + 13}{2} \approx 14.6$$

$$\text{hours and } \frac{\sqrt{265} + 19}{2} \approx 17.6 \text{ hours.}$$

$$\begin{aligned}
 51. \quad & x = \text{time in no wind condition; } x + \frac{1}{2} \\
 & = \text{time into wind. Rate} = \frac{\text{distance}}{\text{time}}; \text{ so} \\
 & \text{rate in no wind} = \frac{600}{x}, \text{ rate in wind is} \\
 & \frac{600}{x + \frac{1}{2}} = \frac{1,200}{2x + 1}, \text{ and the difference of} \\
 & \text{the rates is 15 mph, so} \\
 & \frac{600}{x} - \frac{1,200}{2x + 1} = 15 \\
 & \frac{600(2x + 1) - 1,200x}{x(2x + 1)} = 15 \\
 & 600 = 15(2x^2 + x) \\
 & 40 = 2x^2 + x \\
 & 0 = 2x^2 + x - 40, x = \frac{\sqrt{321} - 1}{4} \\
 & \approx 4.3 \text{ hours}
 \end{aligned}$$

$$\begin{aligned}
 59. \quad & \frac{2 - 3x}{x^2 - 4x - 21} \\
 & x^2 - 4x - 21 = 0 \\
 & (x - 7)(x + 3) = 0 \\
 & x - 7 = 0 \text{ or } x + 3 = 0 \\
 & x = 7 \text{ or } x = -3 \\
 & \{x \mid x \neq 7, -3\}
 \end{aligned}$$

$$\begin{aligned}
 71. \quad & 4y^{-4} + 4 = 17y^{-2} \\
 & u = y^{-2}, \text{ so } u^2 = y^{-4} \\
 & 4u^2 - 17u + 4 = 0 \\
 & (4u - 1)(u - 4) = 0 \\
 & u = y^{-2} = \frac{1}{4} \text{ or } 4; \\
 & (y^{-2})^{-1/2} = \left(\frac{1}{4}\right)^{-1/2} \text{ or} \\
 & (y^{-2})^{-1/2} = 4^{-1/2} \text{ or} \\
 & y = 2 \text{ or } y = \frac{1}{2} \\
 & \left\{\frac{1}{2}, 2\right\}
 \end{aligned}$$

$$\begin{aligned}
 75. \quad & (a + bi)^2 = -5 - 12i; a^2 - b^2 + 2abi \\
 & = -5 - 12i, \text{ so } a^2 - b^2 = -5 \text{ and } 2ab \\
 & = -12, \text{ or } ab = -6, \text{ or } b = -\frac{6}{a} \text{ thus,}
 \end{aligned}$$

$$a^2 - \left(-\frac{6}{a}\right)^2 = -5$$

$$\begin{aligned}
 & a^4 + 5a^2 - 36 = 0, a^2 = 4, a = \pm 2; \\
 & \text{choose } a = 2; \text{ then } b = -\frac{6}{2} = -3; \\
 & \text{thus one value for } a + bi \text{ is } 2 - 3i. \text{ If} \\
 & a = -2, b \text{ is } 3, \text{ giving } -2 + 3i.
 \end{aligned}$$

$$\begin{aligned}
 77. \quad & (a + bi)^2 = c + di \\
 & a^2 - b^2 + 2abi = c + di \\
 & a^2 - b^2 = c, 2ab = d; b = \frac{d}{2a}
 \end{aligned}$$

$$a^2 - \left(\frac{d}{2a}\right)^2 = c$$

$$4a^4 - 4a^2c - d^2 = 0; \text{ this is quadratic in } a^2, \text{ so}$$

$$a^2 = \frac{-(-4c) \pm \sqrt{(-4c)^2 - 4(4)(-d^2)}}{2(4)}$$

$$a = \pm \sqrt{\frac{4c \pm \sqrt{16c^2 + 16d^2}}{8}}$$

$$= \pm \sqrt{\frac{4c \pm 4\sqrt{c^2 + d^2}}{8}}$$

$$= \pm \sqrt{\frac{c \pm \sqrt{c^2 + d^2}}{2}}. \text{ Because } a \text{ must be}$$

$$\text{real, we require that } c \pm \sqrt{c^2 + d^2} \geq 0; \text{ since}$$

$$\sqrt{c^2 + d^2} \geq c \text{ we choose } c + \sqrt{c^2 + d^2}, \text{ and choose } a \geq 0,$$

$$\text{obtaining } a = \sqrt{\frac{c + \sqrt{c^2 + d^2}}{2}}.$$

$$\text{Using } b = \frac{d}{2a} \text{ it can be shown that}$$

$$b = \frac{d}{\sqrt{2}(\sqrt{c + \sqrt{c^2 + d^2}})} \text{ when } c \text{ and}$$

$$d \text{ are not both 0. When } c \text{ and } d \text{ are zero, let } b = 0.$$

78. a.

$$\begin{aligned}
 A &= h\left(\frac{a+b}{2}\right) \\
 &= 36p\left(\frac{24p+30p}{2}\right) = 36p(27p) = 972p^2
 \end{aligned}$$

$$b. \quad 972p^2 = 972 + m$$

$$p^2 = \frac{972 + m}{972} = 1 + \frac{m}{972}$$

$$p = \sqrt{1 + \frac{m}{972}}$$

$$\text{If } m = 100, \text{ then } p = \sqrt{1 + \frac{100}{972}}$$

$$\approx 1.050$$

$$\begin{aligned}
 & \text{New dimensions: } a = 24p \approx 25.2 \text{ units,} \\
 & b = 30p \approx 31.5 \text{ units, and } h = 36p \\
 & \approx 37.8 \text{ units.}
 \end{aligned}$$

Exercise 2-3

Answers to odd-numbered problems

1. $2x + 3$ 3. $4x + 3$
5. $x - 10\sqrt{x} + 25$ 7. $9x + 18$
9. $2x - 4\sqrt{2x} + 4$
11. $x + 3 - 4\sqrt{x-1}$ 13. $4x - 4$
15. $x + 12 + 6\sqrt{x+3}$
17. $-x + 2 - 2\sqrt{1-x}$ 19. $\{32\}$

21. $\{54\}$ 23. Φ 25. $\{-4\}$
 27. $\{-7\frac{2}{3}\}$ 29. $\{-64\}$ 31. $\{22\}$
 33. $\{-3, 27\}$ 35. $\{-2, 8\}$ 37. $\{-2, 7\}$
 39. $\{2\}$ 41. $\{1\}$ 43. $\{9\}$ 45. $\{9\}$
 47. $\{5\}$ 49. $\{2, 3\}$ 51. $\{3, 11\}$
 53. $\{0, 8\}$ 55. $\{2\}$ 57. Φ
 59. $x = \pm\sqrt{-2y^2 + 18y - 27}$

61. $\frac{\pi}{6}D^3 = A$ 63. $s = t^2 + 7t + 9$

65. 300 feet = S 67. $\frac{4}{3}\pi r^3 = V$

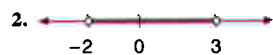
69. $i2\sqrt{\frac{A}{P}} - 2$
 $i + 2 = 2\sqrt{\frac{A}{P}}$
 $(i + 2)^2 = 2^2\left(\sqrt{\frac{A}{P}}\right)^2$
 $i^2 + 4i + 4 = 4\left(\frac{A}{P}\right)$
 $P\left(1 + i + \frac{i^2}{4}\right) = A$
 $P\left(1 + \frac{i}{2}\right)^2 = A$

Solutions to skill and review problems

1. $3[2(\frac{1}{2}) + 1] > \frac{1}{2} + 6$

$3[2] > 6\frac{1}{2}$

$6 > 6\frac{1}{2}$; no



3. no since it is equivalent to $\frac{3}{6} < \frac{2}{6}$

4. yes since $-\frac{3}{6} < -\frac{2}{6}$ is true

5. $\frac{-2}{-2-3} > -2$

$\frac{2}{5} > -2$ is true

6. only f, or $-5 > -2$ is false

Solutions to trial exercise problems

12. $(x + \sqrt{x+1})^2$
 $(x + \sqrt{x+1})(x + \sqrt{x+1})$
 $x^2 + x\sqrt{x+1} + x\sqrt{x+1} + (x+1)$
 $x^2 + x + 1 + 2x\sqrt{x+1}$
 33. $\sqrt[4]{x^2 - 24x} = 3$
 $(\sqrt[4]{x^2 - 24x})^4 = (3)^4$
 $x^2 - 24x = 81$
 $x^2 - 24x - 81 = 0$
 $x = -3$ or 27
 $\{-3, 27\}$

43. $\sqrt{m}\sqrt{m-8} = 3$
 $(\sqrt{m}\sqrt{m-8})^2 = 3^2$
 $m(m-8) = 9$
 $m^2 - 8m - 9 = 0$
 $m = -1$ or 9
 -1 does not check.
 $\{9\}$

51. $\sqrt{2n+3} - \sqrt{n-2} = 2$
 $(\sqrt{2n+3})^2 = (\sqrt{n-2} + 2)^2$
 $2n+3 = (n-2) + 4\sqrt{n-2} + 4$
 $(n+1)^2 = (4\sqrt{n-2})^2$
 $n^2 + 2n + 1 = 16n - 32$
 $n^2 - 14n + 33 = 0$
 $n = 3$ or 11
 $\{3, 11\}$

55. $(2y+3)^{1/2} - (4y-1)^{1/2} = 0$
 $[(2y+3)^{1/2}]^2 = [(4y-1)^{1/2}]^2$
 $2y+3 = 4y-1$
 $2 = y$
 $\{2\}$

63. $\sqrt{s-t} = t+3$; for s
 $(\sqrt{s-t})^2 = (t+3)^2$
 $s-t = t^2 + 6t + 9$
 $s = t^2 + 7t + 9$

Exercise 2-4

Answers to odd-numbered problems

1. $\{x | x > -5\}$

3. $\{x | x \leq 1\frac{1}{2}\}$

5. $\{x | x \leq -2\frac{1}{3}\}$

7. $\{x | x < 2\frac{2}{3}\}$

9. $\{x | x \leq 3\frac{5}{6}\}$

11. $\{x | x < 2\frac{7}{10}\}$

13. $\{x | x \geq -3\frac{3}{5}\}$

15. $\{x | x \leq 0\}$

17. $\{x | x \geq \frac{3}{5}\}$

19. $\{x | x \leq \frac{3}{4}\}$

21. $\{x | x < -\frac{3}{10}\}$

23. $\{x | x \geq 4\frac{7}{8}\}$

25. $\{x | x < 4\frac{1}{5}\}$

27. $\{x | -8 \leq x \leq 3\}$

29. $\{x | x \leq -1 \text{ or } 1 \leq x \leq 3\}$

31. $\{q | 0 < q < 1\frac{2}{3}\}$

33. $\{r | r \leq -1 \text{ or } 0 \leq r \leq 1\frac{1}{2}\}$

35. $\{x | x < -2 \text{ or } 1 < x < 2\}$

37. $\{x | -2 \leq x \leq 2\}$

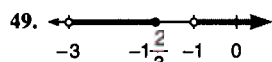
39. $\{x | x = -1 \text{ or } x > 1\frac{1}{2}\}$

41. $\{x | x > 0\}$

43. $\{x | x \leq -1 \text{ or } 1 \leq x < 3\}$

45. $\{x | -2 < x < 3 \text{ or } x = 5\}$

47. $\{x | -6 \leq x < -5 \text{ or } x \geq 3\}$



$$\{x \mid -3 < x \leq -1\frac{2}{3} \text{ or } x > -1\}$$

51. $x \geq \frac{5}{2}$ 53. $x \leq \frac{9}{2}$ 55. $x \leq -1$ or

$$x \geq 6 \quad 57. x \leq -\frac{1}{2} \text{ or } x \geq 1\frac{1}{2}$$

59. a. $x \geq 79$ b. $80\frac{1}{4}$ = average

61. $0 < W < 20$

63. The side must be between 4 and 21 inches long. 65. $0 < x \leq 35 + 5\sqrt{65}$

67. $3 < t \leq \frac{4 + \sqrt{10}}{2}$

69. a. conforms d. does not conform

b. conforms e. does not conform

c. conforms f. does not conform

71. between 3 hours 49 minutes and 10 hours

Solutions to skill and review problems

1. $\frac{2x-3}{4} = 2$

$$2x - 3 = 8$$

$$2x = 11$$

$$x = \frac{11}{2}$$

2. $\frac{2x^2-4}{7} = x$

$$2x^2 - 4 = 7x$$

$$2x^2 - 7x - 4 = 0$$

$$(2x+1)(x-4) = 0$$

$$2x+1=0 \text{ or } x-4=0$$

$$2x=-1 \text{ or } x=4$$

$$x = -\frac{1}{2} \text{ or } x=4$$

3. If $|x| = 8$, then $x = 8$ or $x = -8$ (c).

4. If $|x| < 8$, then $-8 < x < 8$ (b). (Try some values for x .)

5. If $|x| > 8$, then $x < -8$ or $x > 8$ (c). (Try some values for x .)

6. $\left| \frac{1-x}{2} \right| < x$ 7. R .

$$\left| \frac{1-x}{2} \right| < 3$$

$$|-1| < 3$$

$$1 < 3$$

$$\text{True (Yes)}$$

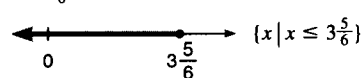
Solutions to trial exercise problems

9. $-3(x-2) + 2(x+1) \geq 5(x-3)$

$$-3x + 6 + 2x + 2 \geq 5x - 15$$

$$-6x \geq -23$$

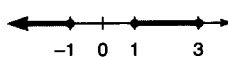
$$x \leq \frac{23}{6}$$



29. $(x-3)(x+1)(x-1) \leq 0$

critical points: -1 (true), 1 (true), 3 (true)

test points: -2 (true), 0 (false), 2 (true), 4 (false)



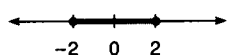
$$\{x \mid x \leq -1 \text{ or } 1 \leq x \leq 3\}$$

37. $(x^2-4x+4)(x^2-4) \leq 0$

$$(x-2)^3(x+2) \leq 0$$

critical points: -2 (true), 2 (true)

test points: -3 (false), 0 (true), 3 (false)

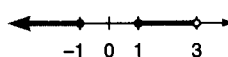


$$\{x \mid -2 \leq x \leq 2\}$$

43. $\frac{x^2-1}{x-3} \leq 0$

critical points: -1 (true), 1 (true), 3

test points: -2 (true), 0 (false), 2 (true), 4 (false)



$$\{x \mid x \leq -1 \text{ or } 1 \leq x < 3\}$$

49. $\frac{x}{x+1} - \frac{2}{x+3} \leq 1$

$$\frac{x(x+3) - 2(x+1)}{(x+1)(x+3)} \leq 0$$

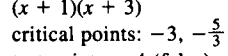
$$\frac{(x+1)(x+3) - 2(x+1)}{(x+1)(x+3)} \leq 0$$

$$\frac{-3x-5}{(x+1)(x+3)} \leq 0$$

critical points: -3, $-\frac{5}{3}$ (true), -1

test points: -4 (false), -2 (true),

$-1\frac{1}{10}$ (false), 0 (true)



$$\{x \mid -3 < x \leq -1\frac{2}{3} \text{ or } x > -1\}$$

61. $P = 2\ell + 2w$ (perimeter is twice the

length plus twice the width)

$$P = 2(30) + 2w$$

$$P = 60 + 2w$$

$$\text{We want } 60 + 2w < 100$$

$$2w < 40$$

$$w < 20$$

We also want $w > 0$, so we require

$$0 < w < 20$$

67. $\frac{1}{1,500} \cdot \frac{1,500}{t} + \frac{1}{1,500} \cdot \frac{1,500}{t-3}$

$$\geq \frac{1}{1,500} \cdot 3,000$$

$$\frac{1}{t} + \frac{1}{t-3} \geq 2$$

$$\frac{2t-3}{t(t-3)} - 2 \geq 0$$

$$\frac{2t-3}{t(t-3)} - \frac{2t(t-3)}{t(t-3)} \geq 0$$

$$\frac{-2t^2 + 8t - 3}{t(t-3)} \geq 0$$

critical points are 0, 3, and $\frac{4 \pm \sqrt{10}}{2}$

($\approx 3.6, 0.4$) (true); using test points of

-1, 0.2, 1, 3.5, and 4 we find

$0 < t \leq \frac{4 - \sqrt{10}}{2}$ or

$3 < t \leq \frac{4 + \sqrt{10}}{2}$. If $t - 3 > 0$ then

$t > 3$, so the solution is

$$3 < t \leq \frac{4 + \sqrt{10}}{2}$$

72. critical points: $3x - 5 = 6x$

$$-5 = 3x$$

$$-1\frac{2}{3} = x$$

test points: -2, -1

$x = -2$: $3(-2) - 5 \leq 6(-2)$

$$-11 \leq -12 \text{ False}$$

$x = -1$: $3(-1) - 5 \leq 6(-1)$

$$-8 \leq -6 \text{ True}$$

Solution: $x \geq -1\frac{2}{3}$

Exercise 2-5

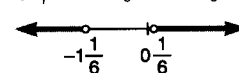
Answers to odd-numbered problems

1. $\{\pm\frac{8}{5}\}$ 3. $\{-5\frac{1}{2}, \frac{1}{2}\}$ 5. $\{-1, 4\}$

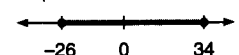
7. $\{-4\}$ 9. $\{\frac{1}{3}, 3\}$

11. $\left\{ \frac{5 \pm \sqrt{97}}{4}, \frac{5 \pm \sqrt{47}i}{4} \right\}$

13. $\{x \mid x < -1\frac{1}{6} \text{ or } x > \frac{1}{6}\}$

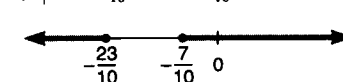


15. $\{x \mid -26 \leq x \leq 34\}$

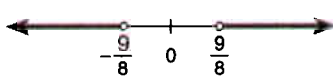


17. Φ

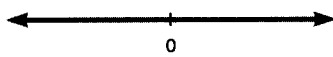
19. $\{x \mid x \leq -\frac{23}{10} \text{ or } x \geq -\frac{7}{10}\}$



21. $\{x \mid x > \frac{9}{8} \text{ or } x < -\frac{9}{8}\}$



23. R



25. $\{x \mid x > 7\frac{1}{3} \text{ or } x < -7\frac{1}{3}\}$

27. $\{x \mid x < -10 \text{ or } x > 15\}$

29. $\{x \mid x < -34 \text{ or } x > 38\}$

31. $\{-7\frac{1}{3}, 7\frac{1}{3}\}$

33. $\{-10, 15\}$ 35. $\{-34, 38\}$

37. $x \mid -7\frac{1}{3} < x < 7\frac{1}{3}\}$

39. $\{x \mid -10 < x < 15\}$

41. $\{x \mid -34 < x < 38\}$

43. $\{-1\frac{1}{4}, 1\frac{1}{4}\}$

45. $\{x \mid -\frac{1}{3} < x < 1\frac{1}{3}\}$

47. $\{x \mid x \leq -2\frac{2}{3} \text{ or } x \geq 2\frac{2}{3}\}$

49. $\{x \mid -32\frac{1}{2} \leq x \leq 35\frac{1}{2}\}$

51. $\{x \mid x \leq -2\frac{23}{36} \text{ or } x \geq -\frac{31}{36}\}$

53. $\{x \mid x < -2 \text{ or } x > 2\}$

55. $\{x \mid x \geq 4 \text{ or } x \leq -8\}$

57. The dimension y is $5\frac{5}{8}$ and the dimension x is $7\frac{3}{8}$ inches.

59. $0 \leq x \leq 18$

Solutions to skill and review problems

	x	y	$2x - y = ?$	$= 5?$
1.	1	-3	$2(1) - (-3) = 5$	True
2.	-3	-11	$2(-3) - (-11) = 5$	True
3.	3	1	$2(3) - 1 = 5$	True
4.	0	-5	$2(0) - (-5) = 5$	True

 5. If $x = 2$ and $y = -3$, which of the statements is true?

a. $3(2) + (-3) = 3$

b. $-2 + 5(-3) = -17$

c. $-3 + 9 = 3(2)$

d. $2 = -3 + 5$ All are true.

 6. If $x = -2$ and $y = 4$, which of the statements is true?

a. $3(-2) + 4 = -2$

b. $-(-2) + 5(4) \neq 18$

c. $4 + 10 \neq 3(-2)$

d. $-2 \neq 4 + 6$ Only a is true.

 7. Solve $2x + y = 8$ for y .

$$y = -2x + 8$$

 8. Solve $x - 2y = 4$ for y .

$$x - 4 = 2y$$

$$\frac{x - 4}{2} = y$$

Solutions to trial exercise problems

9. $\left| \frac{3x - 5}{4} \right| = 1$

$$\frac{3x - 5}{4} = 1 \text{ or } \frac{3x - 5}{4} = -1$$

$$3x - 5 = 4 \text{ or } 3x - 5 = -4$$

$$3x = 9 \text{ or } 3x = 1$$

$$x = 3 \text{ or } x = \frac{1}{3}$$

$$\left\{ \frac{1}{3}, 3 \right\}$$

10. $|x^2 - 2x| = 3$

$$x^2 - 2x = 3 \text{ or } x^2 - 2x = -3$$

$$x^2 - 2x - 3 = 0 \text{ or }$$

$$x^2 - 2x + 3 = 0$$

$$x = -1 \text{ or } 3 \text{ or } x = 1 \pm \sqrt{2}i$$

$$\{-1, 3, 1 \pm \sqrt{2}i\}$$

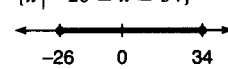
15. $\left| \frac{x - 4}{3} \right| \leq 10$

$$-10 \leq \frac{x - 4}{3} \leq 10$$

$$-30 \leq x - 4 \leq 30$$

$$-26 \leq x \leq 34$$

$$\{x \mid -26 \leq x \leq 34\}$$



21. $\left| 3x - \frac{x}{3} \right| > 3$

$$\left| \frac{9x}{3} - \frac{x}{3} \right| > 3$$

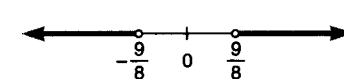
$$\left| \frac{8x}{3} \right| > 3$$

$$\frac{8x}{3} > 3 \text{ or } \frac{8x}{3} < -3$$

$$8x > 9 \text{ or } 8x < -9$$

$$x > \frac{9}{8} \text{ or } x < -\frac{9}{8}$$

$$\{x \mid x > \frac{9}{8} \text{ or } x < -\frac{9}{8}\}$$



27. $25 < |5 - 2x|$

$$|5 - 2x| > 25$$

$$5 - 2x > 25 \text{ or } 5 - 2x < -25$$

$$-20 > 2x \text{ or } 30 < 2x$$

$$-10 > x \text{ or } 15 < x$$

$$\{x \mid x < -10 \text{ or } x > 15\}$$

42. $8 > \left| \frac{3 - 2x}{5} \right|$

$$8 > \frac{3 - 2x}{5} > -8$$

$$40 > 3 - 2x > -40$$

$$37 > -2x > -43$$

$$-\frac{37}{2} < x < \frac{43}{2}$$

$$\{x \mid -18\frac{1}{2} < x < 21\frac{1}{2}\}$$

56. $\frac{3}{4} \leq \left| \frac{3x + 1}{8} \right|$

$$\left| \frac{3x + 1}{8} \right| \geq \frac{3}{4}$$

$$\frac{3x + 1}{8} \geq \frac{3}{4} \text{ or } \frac{3x + 1}{8} \leq -\frac{3}{4}$$

$$3x + 1 \geq 6 \text{ or } 3x + 1 \leq -6$$

$$3x \geq 5 \text{ or } 3x \leq -7$$

$$x \geq \frac{5}{3} \text{ or } x \leq -\frac{7}{3}$$

$$\{x \mid x \geq \frac{5}{3} \text{ or } x \leq -\frac{7}{3}\}$$

60. $(x - y)^2 \geq 0$

$$x^2 - 2xy + y^2 \geq 0$$

$$x^2 + y^2 \geq 2xy$$

$$\frac{x^2 + y^2}{2} \geq xy$$

Chapter 2 review

1. $\{4\frac{4}{9}\}$ 2. $\{-22\frac{1}{3}\}$ 3. $\{2\frac{8}{11}\}$ 4. R

5. $\{0\}$ 6. 0.6595 7. -0.9619

8. $Q = \frac{px - m}{p}$ 9. $b = \frac{W - 2kc - R}{k}$

10. $P_1 = \frac{nP_2 - 5c - 5P}{n}$ 11. $x = \frac{y}{1 - y}$

12. $x = \frac{y}{y - 1}$ 13. \$3,550 at 7% and \$4,450 at 5%

14. \$3,545.45 at 12%, \$11,454.55 at 10%

15. \$2,900 at 5% and \$2,100 at 9%

16. 1,333.3 pounds of the 10% mixture, 666.7 pounds of the 25% mixture

17. 7.5 tons of 55% copper

18. $23\frac{1}{3}$ minutes 19. 3 mph

20. $21\frac{2}{3}$ mph 21. $-\frac{5}{2}$ or 6

22. $-\frac{1}{2}$ or 3 23. $\frac{4}{3}$ or $-\frac{3}{2}$

24. $-\frac{5b}{3a}$ or $\frac{b}{2a}$ 25. $\pm \frac{2\sqrt{30}}{9}$

26. $\frac{3}{2} \pm \sqrt{3}$ 27. $-1 \pm \sqrt{2}$

$$28. -\frac{c}{b} \pm \frac{\sqrt{ad}}{ab} \quad 29. \frac{15 \pm \sqrt{345}}{10}$$

$$30. \frac{1 \pm \sqrt{17}i}{6} \quad 31. -2 \text{ or } 4$$

$$32. \frac{-37 \pm \sqrt{193}}{12}$$

$$33. 3 \left(x - \frac{4 + 2\sqrt{13}}{3} \right) \left(x - \frac{4 - 2\sqrt{13}}{3} \right)$$

$$34. 5 \left(x - \frac{-3 + \sqrt{29}}{5} \right) \left(x - \frac{-3 - \sqrt{29}}{5} \right)$$

$$35. 24 \quad 36. 12 \text{ and } 7.5 \quad 37. 10 \text{ units}$$

$$38. 74 \text{ hours} \quad 39. 3 \text{ hours} \quad 40. x \neq 3$$

$$41. x \neq -2 \text{ and } x \neq -6$$

$$42. x = \pm 3 \text{ or } x = \pm \frac{3}{2}\sqrt{2} \quad 43. 1 \text{ or } 13$$

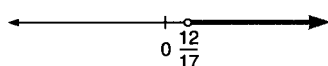
$$44. \frac{9}{4} \quad 45. 1 \text{ or } 16 \quad 46. \pm \frac{\sqrt{2}}{4} \text{ or } \pm \frac{1}{2}i$$

$$47. \frac{1}{6} \text{ or } -1 \quad 48. \Phi \text{ (the empty set)}$$

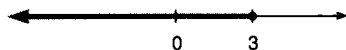
$$49. 4\frac{4}{5} \quad 50. 8 \quad 51. \frac{19}{2}$$

$$52. k + 1 \quad 53. \frac{\pi r^2}{1 - \pi R^2} = A$$

$$54. x > \frac{12}{17}$$



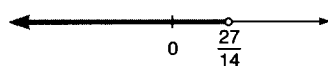
$$55. x \leq 3$$



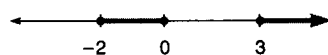
$$56. x \geq -30$$



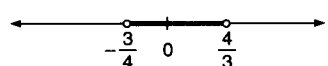
$$57. x < \frac{27}{14}$$



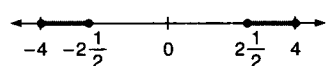
$$58. -2 \leq r \leq 0 \text{ or } r \geq 3$$



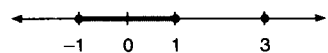
$$59. -\frac{3}{4} < w < \frac{4}{3}$$



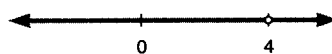
$$60. -4 \leq x \leq -2\frac{1}{2} \text{ or } 2\frac{1}{2} \leq x \leq 4$$



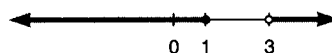
$$61. -1 \leq x \leq 1 \text{ or } x = 3$$



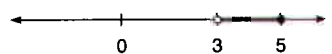
$$62. x \neq 4$$



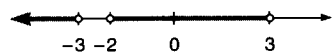
$$63. x \leq 1 \text{ or } x > 3$$



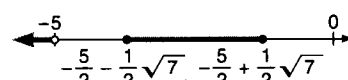
$$64. 3 < x \leq 5$$



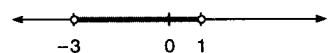
$$65. x < -3 \text{ or } -2 < x < 3$$



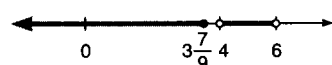
$$66. x < -5 \text{ or } -\frac{5}{2} - \frac{1}{2}\sqrt{7} \leq x \leq -\frac{5}{2} + \frac{1}{2}\sqrt{7}$$



$$67. -3 < x < 1$$



$$68. x \leq 3\frac{7}{9} \text{ or } 4 < x < 6$$



$$69. -\frac{3}{16} \text{ or } \frac{9}{16}$$

$$70. -5, 10, \frac{1}{2}(1 \pm \sqrt{7}i)$$

$$71. 0, \pm\sqrt{2}i \quad 72. -16 \leq x \leq 24$$

$$73. -1, 2, \frac{1}{2}(1 \pm \sqrt{7}i)$$

$$74. -24\frac{1}{2} < x < 25\frac{1}{2}$$

$$75. x > 38 \text{ or } x < -34 \quad 76. -34, 38$$

$$77. -4 < x < 1 \quad 78. x \geq -\frac{13}{3} \text{ or}$$

$$x \leq -\frac{29}{3} \quad 79. x > \frac{14}{15} \text{ or } x < -\frac{2}{5}$$

Chapter 2 test

$$1. \{\frac{16}{7}\} \quad 2. \{\frac{7}{8}\} \quad 3. \{1\} \quad 4. -2.4407$$

$$5. x = \frac{m + pQ}{p} \quad 6. P_2 = \frac{5P + nP_1 + 5c}{n}$$

$$7. y = \frac{x}{x+1} \quad 8. \$3,000 \text{ at } 9\% \text{ and}$$

$$\$9,000 \text{ at } 5\% \quad 9. 18.67 \text{ tons}$$

$$10. 14.3 \text{ minutes} \quad 11. 1\frac{3}{7} \text{ miles per hour}$$

$$12. -3 \text{ or } \frac{5}{3} \quad 13. -\frac{3}{2} \text{ or } \frac{1}{5}$$

$$14. \pm \frac{5\sqrt{2}}{2} \quad 15. 1 \pm \frac{2}{3}\sqrt{6}$$

$$16. \frac{3 \pm \sqrt{33}}{2} \quad 17. -1 \pm \frac{\sqrt{3}}{3}i$$

$$18. 2 \left[x - \left(\frac{1}{4} + \frac{\sqrt{97}}{4} \right) \right] \left[x - \left(\frac{1}{4} - \frac{\sqrt{97}}{4} \right) \right]$$

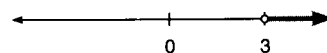
$$19. 24 \text{ units} \quad 20. 2 \text{ units}$$

$$21. 23.5 \text{ hours} \quad 22. 100 \text{ mph}$$

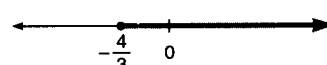
$$23. x \neq \pm 9 \quad 24. \pm \frac{1}{2}, \pm 3 \quad 25. 4$$

$$26. \frac{1}{2}, -1 \quad 27. 4 \quad 28. b = \frac{1}{\pi} - \frac{r^2}{A}$$

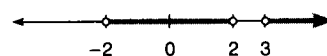
$$29. x > 3$$



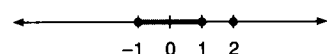
$$30. x \geq -\frac{4}{3}$$



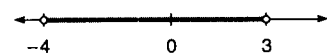
$$31. -2 < x < 2 \text{ or } x > 3$$



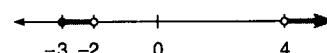
$$32. -1 \leq x \leq 1 \text{ or } x = 2$$



$$33. -4 < x < 3$$



$$34. -3 \leq x < -2 \text{ or } x > 4$$



$$35. -\frac{3}{2}, \frac{13}{2} \quad 36. -\frac{8}{3} \leq x \leq 4$$

$$37. x < -2 \text{ or } x > \frac{10}{3}$$

$$38. x \geq \frac{7}{5} \text{ or } x \leq -1 \quad 39. 0 < x < 3$$

$$40. -1 \pm \sqrt{11}, -1 \pm 3i \quad 41. \frac{1}{3}$$

$$42. -\frac{3}{2} \pm \frac{\sqrt{7}}{2}i \quad 43. \frac{5}{2} \pm \frac{\sqrt{41}}{2}$$

$$44. \pm\sqrt{2} \quad 45. \frac{17}{3} \quad 46. \frac{17}{16} \pm \frac{\sqrt{449}}{16}$$

$$47. -11 \quad 48. -\frac{2}{3} \pm \frac{\sqrt{11}}{3}i$$

Chapter 3

Exercise 3–1

Answers to odd-numbered problems

Answers to problems 1–15 will vary.

$$1. (0, -8), (1, -5), (2, -2) \quad 3. (-4, -6),$$

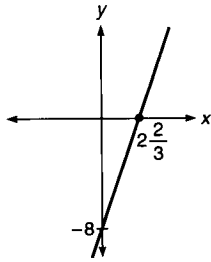
$$(0, -3), (4, 0) \quad 5. (0, -2), (1, -1), (2, 0)$$

$$7. (0, -2), (1, 1), (2, 4) \quad 9. (1, 1), (2, 2),$$

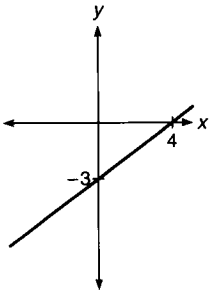
$$(3, 3)$$

11. $(-1, 7), (0, 7), (1, 7)$
 13. $(-2, -6), (0, -3), (2, 0)$
 15. $(0, -\frac{10}{3}), (1, -\frac{8}{3}), (2, -2)$

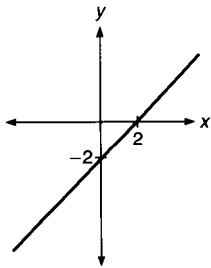
17.



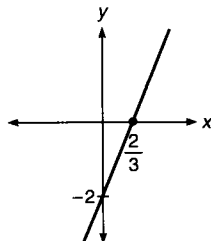
19.



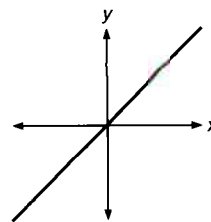
21.



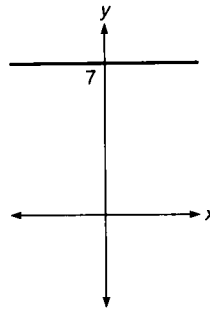
23.



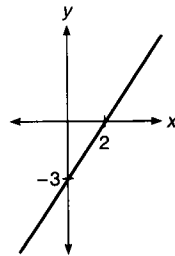
25.



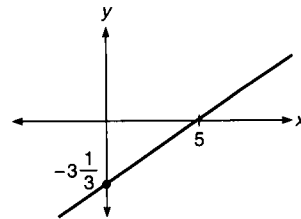
27.



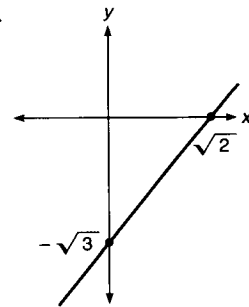
29.



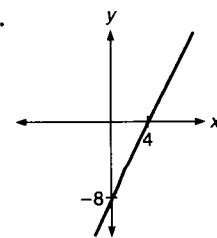
31.



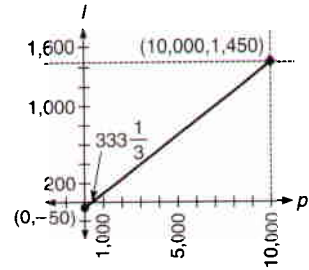
33.



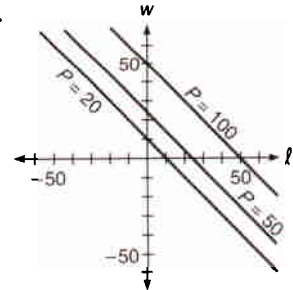
35.



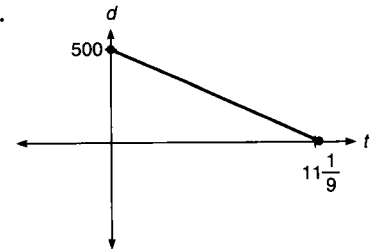
37.



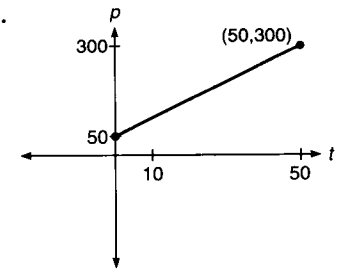
39.



41.



43.

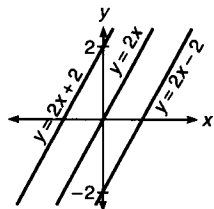


45. $(-1, 7)$ 47. $(4, 5)$ 49. $(2, -1\frac{1}{2})$
 51. $(2\frac{7}{12}, 1\frac{2}{3})$ 53. $(-2\frac{1}{2}, 6)$
 55. $(1, \frac{7\sqrt{2}}{2})$ 57. $5\sqrt{2}$ 59. $2\sqrt{10}$
 61. 1 63. $6\sqrt{5}$ 65. $3\sqrt{2}$ 67. $2\frac{1}{2}$
 69. $\frac{2\sqrt{101}}{5}$ 71. $2\sqrt{73}$
 73. $3\sqrt{a^2 + 4b^2}$ 75. 4
 77. $y = -\frac{4}{3}x + \frac{35}{3}$ 79. $y = \frac{3}{7}x - \frac{10}{7}$
 81. $a = \frac{5}{3}, b = -\frac{34}{15}$
 83. 6

85. a. $d = |x_2 - x_1| + |y_2 - y_1|$
 b. Taxicab distance is always longer unless the two points lie on the same horizontal or vertical line, in which case they are the same.
87. $a = 10$
89. The second line is $a_2x + b_2y + c_2 = 0$; $a_2 = ka_1$, $b_2 = kb_1$, $c_2 = kc_1$, so the second line is also $ka_1x + kb_1y + kc_1 = 0$, and since $k \neq 0$ we can divide each term by it obtaining $a_1x + b_1y + c_1 = 0$, which is the first line.
91. 44.5 square units

Solutions to skill and review problems

1. Compute $\frac{a-b}{c-d}$ if $a = 9$, $b = -3$,
 $c = -5$, $d = -1$.
 $\frac{9 - (-3)}{-5 - (-1)} = \frac{12}{-4} = -3$
2. Solve $3y - 2x = 5$ for y .
 $3y = 2x + 5$
 $y = \frac{2x + 5}{3}$
3. Solve $ax + by + c = 0$ for y .
 $by = -ax - c$
 $y = \frac{-(ax + c)}{b}$
 $y = -\frac{ax + c}{b}$
4. If $y = 3x - b$ contains the point $(-2, 4)$, find b .
 $4 = 3(-2) + b$
 $4 = -6 + b$
 $10 = b$



6. Solve the equation $2x^2 - x = 3$.
 $2x^2 - x - 3 = 0$
 $(2x - 3)(x + 1) = 0$
 $2x - 3 = 0$ or $x + 1 = 0$
 $2x = 3$ or $x = -1$
 $x = \frac{3}{2}$ or -1 ; $\{-1, \frac{3}{2}\}$
7. Solve the equation $|2x - 3| = 5$.
 $2x - 3 = 5$ or $2x - 3 = -5$
 $2x = 8$ or $2x = -2$
 $x = 4$ or $x = -1$
 $\{-1, 4\}$

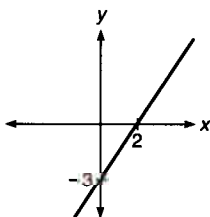
8. Simplify $\sqrt{48x^4}$.
 $\sqrt{2^4 \cdot 3 \cdot x^4} = 2^2 x^2 \sqrt{3} = 4x^2 \sqrt{3}$
9. Calculate $\frac{5}{8} - \frac{1}{4} + \frac{2}{3}$.
 $\frac{5}{8} - \frac{2}{8} + \frac{2}{3}$
 $\frac{3}{8} + \frac{2}{3} = \frac{9 + 16}{24} = \frac{25}{24}$

Solutions to trial exercise problems

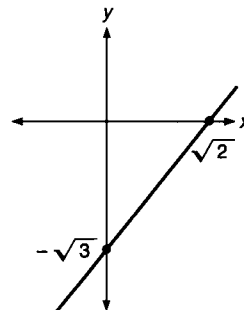
13. Answers to problem 13 will vary. We solve for y and select values of x that produce integer values of y (for convenience).

$$\begin{aligned} \frac{1}{2}x - \frac{1}{3}y &= 1 \\ -\frac{1}{3}y &= -\frac{1}{2}x + 1 \\ y &= \frac{3}{2}x - 3 \\ (-2, -6), (0, -3), (2, 0) \end{aligned}$$

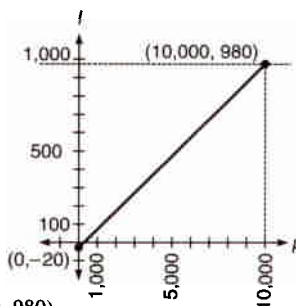
29. $\frac{1}{2}x - \frac{1}{3}y = 1$
 $y = \frac{3x - 6}{2}$
 $(1, -1\frac{1}{2}), (2, 0), (3, 1\frac{1}{2})$
 x -intercept ($y = 0$):
 $\frac{1}{2}x - 0 = 1$
 $x = 2; (2, 0)$
 y -intercept ($x = 0$):
 $y = \frac{0 - 6}{2} = -3; (0, -3)$



33. $\sqrt{3}x - \sqrt{2}y = \sqrt{6}$
 $y = \frac{\sqrt{6}x - 2\sqrt{3}}{2}$
 $(1, \frac{\sqrt{6} - 2\sqrt{3}}{2})$,
 $(2, \sqrt{6} - \sqrt{3})$,
 $(3, \frac{3\sqrt{6} - 2\sqrt{3}}{2})$
 x -intercept ($y = 0$):
 $\sqrt{3}x - 0 = \sqrt{6}$
 $x = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}; (\sqrt{2}, 0)$
 y -intercept ($x = 0$):
 $y = \frac{0 - 2\sqrt{3}}{2}; (0, -\sqrt{3})$



38. $I = 0.10p - 20$
 $0 \leq p \leq 10,000$
 I -intercept ($p = 0$):
 $I = 0 - 20$
 $(0, -20)$
 p -intercept ($I = 0$):
 $0 = 0.10p - 20$
 $p = \frac{20}{0.10} = 200$
 $(200, 0)$
 At $p = 0$, plot $(0, -20)$.
 At $p = 10,000$ plot $(10,000, 980)$.



51. $(\frac{2}{3}, 3), (4\frac{1}{2}, \frac{1}{3})$

$$\left(\frac{\frac{2}{3} + 4\frac{1}{2}}{2}, \frac{3 + \frac{1}{3}}{2}\right) = (2\frac{7}{12}, 1\frac{2}{3})$$

69. $(3, \frac{1}{5}), (-1, \frac{3}{5})$

$$\begin{aligned} d &= \sqrt{(3 - (-1))^2 + (\frac{1}{5} - \frac{3}{5})^2} \\ &= \sqrt{4^2 + (\frac{2}{5})^2} = \sqrt{16 + \frac{4}{25}} \\ &= \sqrt{\frac{16(25)}{25} + \frac{4}{25}} = \sqrt{\frac{404}{25}} \\ &= \frac{\sqrt{4(101)}}{\sqrt{25}} = \frac{2\sqrt{101}}{5} \end{aligned}$$

86. Let $M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ be the

midpoint; we need to show that the distance from M to P_1 equals the distance from M to P_2 .

$$\sqrt{\left(x_1 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_1 - \frac{y_1 + y_2}{2}\right)^2}$$

$$= \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2}$$

Squaring both sides:

$$\left(x_1 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_1 - \frac{y_1 + y_2}{2}\right)^2$$

$$= \left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2$$

$$\left(\frac{2x_1 - x_1 - x_2}{2}\right)^2 + \left(\frac{2y_1 - y_1 - y_2}{2}\right)^2$$

$$= \left(\frac{2x_2 - x_1 - x_2}{2}\right)^2 + \left(\frac{2y_2 - y_1 - y_2}{2}\right)^2$$

$$\left(\frac{x_1 - x_2}{2}\right)^2 + \left(\frac{y_1 - y_2}{2}\right)^2 = \left(\frac{x_2 - x_1}{2}\right)^2$$

$$+ \left(\frac{y_2 - y_1}{2}\right)^2$$

$$\frac{1}{4}(x_1 - x_2)^2 + \frac{1}{4}(y_1 - y_2)^2$$

$$= \frac{1}{4}(x_2 - x_1)^2 + \frac{1}{4}(y_2 - y_1)^2$$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2$$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2.$$

It is not too difficult to show that each side is the same by performing the indicated squaring operations.

Exercise 3-2

Answers to odd-numbered problems

1. $-\frac{1}{8}$ 3. $\frac{13}{3}$ 5. $\frac{5}{4}$ 7. $\frac{43}{144}$

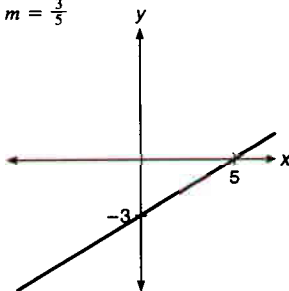
9. m is not defined 11. 0 13. 10

15. $-\frac{5\sqrt{2}}{2}$ 17. $-2\sqrt{3}$ 19. $-\frac{p-q}{2q}$

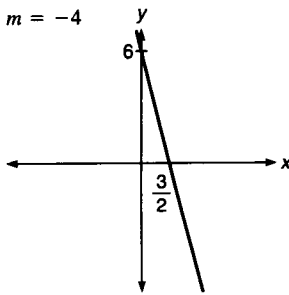
21. Use (1,1) and (2,-1):

$$m = \frac{-1 - 1}{2 - 1} = \frac{-2}{1} = -2$$

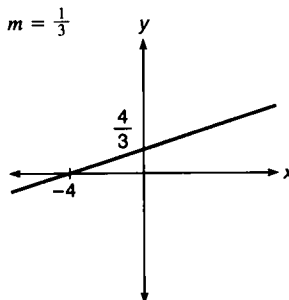
23. $m = \frac{3}{5}$



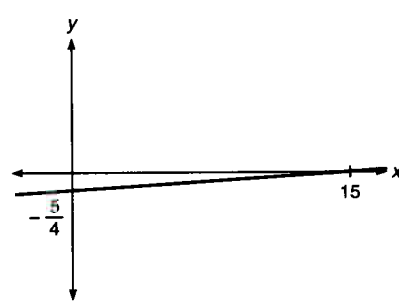
25. $m = -4$



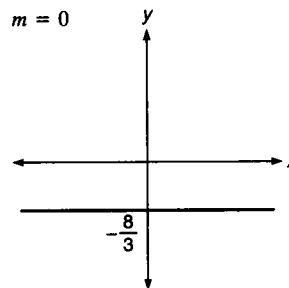
27. $m = \frac{1}{3}$



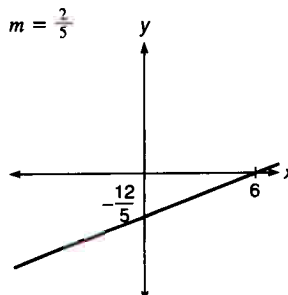
29. $m = \frac{1}{12}$



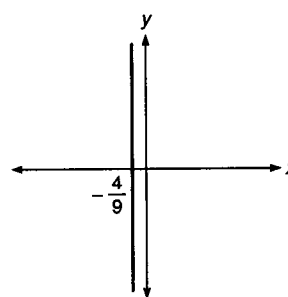
31. $m = 0$



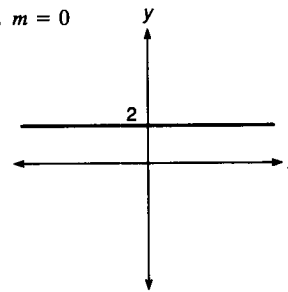
33. $m = \frac{2}{5}$



35. m is undefined



37. $m = 0$



39. $y = -2x - 1$ 41. $y = -4x + 9\frac{3}{4}$

43. $y = \frac{1}{a}x + (b - 1)$ 45. $y = \frac{3}{8}x + \frac{17}{8}$

47. $y = \frac{7}{8}x - \frac{1}{4}$ 49. $y = \frac{22}{3}x - 120$

51. $y = -4x - \frac{6}{5}$ 53. $y = -\frac{5}{64}x + \frac{11}{16}$

55. $y = -\frac{5\sqrt{2}}{2}x$

57. $y = \frac{4n}{m-n}x + \frac{m^2 - 3mn - 2n^2}{m-n}$

59. $y = 5x - 3$ 61. $y = -2x - 2$

63. $y = -5$ 65. $y = \frac{3}{5}x + 5$

67. $y = -\frac{4}{3}x + 1\frac{4}{3}$ 69. $y = 5x + 15$

71. $y = -x$ 73. $x = -1$

75. $y = -2x - 2$ 77. $x = 2$

79. a. -11.4° ; b. -35.8° 81. -52.1°

83. a. \$16,751; b. \$21,112

85. $(1\frac{3}{5}, -3\frac{1}{5})$

87. Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two different points on the line $y = 3x - 4$. Then $y_1 = 3x_1 - 4$, and $y_2 = 3x_2 - 4$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(3x_2 - 4) - (3x_1 - 4)}{x_2 - x_1} = \frac{3x_2 - 3x_1}{x_2 - x_1} = \frac{3(x_2 - x_1)}{x_2 - x_1} = 3$$

89. Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be two different points on the line $y = \frac{1}{3}x + 2$. Then $y_1 = \frac{1}{3}x_1 + 2$, and $y_2 = \frac{1}{3}x_2 + 2$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(\frac{1}{3}x_2 + 2) - (\frac{1}{3}x_1 + 2)}{x_2 - x_1} = \frac{\frac{1}{3}x_2 - \frac{1}{3}x_1}{x_2 - x_1} = \frac{\frac{1}{3}(x_2 - x_1)}{x_2 - x_1} = \frac{1}{3}$$

91. $y - y_1 = m(x - x_1)$ is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$, or $(y - y_1)(x_2 - x_1) = (x - x_1)(y_2 - y_1)$. Now plug the point $P_1 = (x_1, y_1)$ in for (x, y) , obtaining $(y_1 - y_1)(x_2 - x_1) = (x_1 - x_1)(y_2 - y_1)$, or $0 = 0$, which makes P_1 a solution. Now plug in $P_2 = (x_2, y_2)$ and observe that both sides are the same, making this point also a solution.

93. $(3\frac{1}{3}, -2\frac{2}{3})$ 95. $(-3\frac{1}{4}, 2\frac{3}{4})$

Solutions to skill and review problems

- Evaluate $3x^2 + 2x - 10$ for $x = -5$.
 $3(-5)^2 + 2(-5) - 10$
 $75 - 10 - 10$
 55
- Evaluate $3x^2 + 2x - 10$ for $x = c + 1$.
 $3(c + 1)^2 + 2(c + 1) - 10$
 $3(c^2 + 2c + 1) + 2c + 2 - 10$
 $3c^2 + 8c - 5$

97. $(-4\frac{3}{4}, -1\frac{1}{8})$

99. The point of intersection is found by substitution:
 $y = -3x + 15$ $y = \frac{1}{3}x + 2$
 $-3x + 15 = \frac{1}{3}x + 2$
 $-9x + 45 = x + 6$
 $39 = 10x$
 $x = \frac{39}{10}$

$$y = -3(\frac{39}{10}) + 15$$

$$y = \frac{33}{10}$$

$$\text{Thus the point } (h, k) = (\frac{39}{10}, \frac{33}{10}).$$

Using the distance formula we find a and b . a is the distance from the point (h, k) to $(0, 15)$:

$$a = \sqrt{(\frac{39}{10} - 0)^2 + (\frac{33}{10} - 15)^2}$$

$$a = \sqrt{(\frac{39}{10})^2 + (-\frac{117}{10})^2}$$

$$a = \sqrt{\frac{15,210}{100}}, \text{ so } a^2 = \frac{1,521}{10}$$

b is the distance from the point (h, k) to $(0, 2)$:

$$b = \sqrt{(\frac{39}{10} - 0)^2 + (\frac{33}{10} - 2)^2}$$

$$b = \sqrt{(\frac{39}{10})^2 + (\frac{13}{10})^2}$$

$$b = \sqrt{\frac{1,690}{100}}, \text{ so } b^2 = \frac{169}{10}$$

$$\text{Thus, } a^2 + b^2 = \frac{1,521}{10} + \frac{169}{10} = \frac{1,690}{10} = 169. c = 13, \text{ so } c^2 = 169.$$

$$\text{Thus, } a^2 + b^2 = c^2.$$

101. $(-3, 7)$ 103. $(\frac{7}{8}, -\frac{5}{8})$
 105. $(3, 33)$, $(7, 73)$ 107. $(\frac{1}{2}, 0)$, $(3, 2\frac{1}{2})$
 109. $(-1, -1)$, $(6, 13)$
 111. $(-4, -9)$, $(2, 3)$ 113. $(17, 270)$
 115. $(-1, -8)$, $(2\frac{1}{2}, -2\frac{3}{4})$

4. Simplify $\sqrt{\frac{2x}{5y^3}}$.

$$\frac{\sqrt{2x}}{\sqrt{5y^3}} = \frac{\sqrt{2x}}{y\sqrt{5y}} \cdot \frac{\sqrt{5y}}{\sqrt{5y}} = \frac{\sqrt{10xy}}{y(5y)} = \frac{\sqrt{10xy}}{5y^2}$$

5. Solve $|2x - 6| < 8$.
 $-8 < 2x - 6 < 8$
 $-2 < 2x < 14$
 $-1 < x < 7$

6. Solve $\frac{x-2}{4} = \frac{2x+1}{3}$.

$$3(x-2) = 4(2x+1)$$

$$3x-6 = 8x+4$$

$$-10 = 5x$$

$$-2 = x$$

7. Compute $(\frac{2}{3} - \frac{1}{4}) \div 5$.

$$\frac{2(4) - 1(3)}{3(4)} \div 5 = \frac{5}{12} \cdot \frac{1}{5} = \frac{1}{12}$$

Solutions to trial exercise problems

7. $(7\frac{7}{12}, -5, -3)$

$$(x_1, y_1) = (7\frac{7}{12}, -5), (x_2, y_2) = (-5, -3)$$

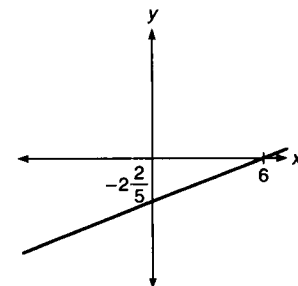
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-5)}{-5 - 7\frac{7}{12}} = \frac{-3 - (-5)}{-5 - 7\frac{7}{12}}$$

$$= \frac{-\frac{36}{12} - \frac{7}{12}}{-12} = \frac{-\frac{43}{12}}{-12} = \frac{43}{12} \cdot \frac{1}{12} = \frac{43}{144}$$

17. $(\sqrt{27}, 3)$, $(\sqrt{12}, 9)$

$$m = \frac{9 - 3}{\sqrt{12} - \sqrt{27}} = \frac{6}{2\sqrt{3} - 3\sqrt{3}} = \frac{6}{-\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{6\sqrt{3}}{3} = -2\sqrt{3}$$

33. $\frac{1}{3}x - \frac{5}{6}y = 2$
 $y = \frac{2}{5}x - \frac{12}{5}; m = \frac{2}{5}$
 intercepts: $(0, -2\frac{2}{5})$, $(6, 0)$



$$43. (a, b), m = \frac{1}{a}$$

$$y - b = \frac{1}{a}(x - a)$$

$$y - b = \frac{1}{a}x - 1$$

$$y = \frac{1}{a}x - 1 + b$$

$$y = \frac{1}{a}x + (b - 1)$$

$$53. (4, \frac{3}{8}), (12, -\frac{1}{4})$$

$$m = \frac{-\frac{1}{4} - \frac{3}{8}}{12 - 4} = \frac{-\frac{5}{8}}{8} = -\frac{5}{8} \cdot \frac{1}{8} = -\frac{5}{64}$$

$$y - (-\frac{1}{4}) = -\frac{5}{64}(x - 12)$$

$$y + \frac{1}{4} = -\frac{5}{64}x + \frac{15}{16}$$

$$y = -\frac{5}{64}x + \frac{11}{16}$$

67. A line that is perpendicular to the line $4y + 5 = 3x$ and passes through the point $(\frac{3}{2}, -\frac{1}{5})$.

$$4y + 5 = 3x$$

$$4y = 3x - 5$$

$$y = \frac{3}{4}x - \frac{5}{4} \text{ Solved for } y; m = \frac{3}{4}.$$

Use $m = -\frac{4}{3}$, the negative reciprocal of $\frac{3}{4}$, since we want a line perpendicular to the original line.

$$y - (-\frac{1}{5}) = -\frac{4}{3}(x - \frac{3}{2})$$

$$y + \frac{1}{5} = -\frac{4}{3}x + 2$$

$$y = -\frac{4}{3}x + 1\frac{4}{5}$$

81. First find the wind chill factor, wcf, for a -11.5° temperature for 15 mph and 20 mph winds. At 15 mph we use the (temperature, wcf) points $(-10, -45)$ and $(-15, -51)$. We compute y in the ordered pair $(-11.5, y)$. The wcf is -46.8° . At 20 mph we use the (temperature, wcf) points $(-10, -52)$ and $(-15, -60)$. We compute y in the ordered pair $(-11.5, y)$ and obtain the wcf -54.4° .

Now we have the ordered pairs (mph, wcf) of $(15, -46.8^\circ)$ and $(20, -54.4^\circ)$. We use these to compute y in the ordered pair $(18.5, y)$. The value of y is -52.12 , so the required wind chill factor for -11.5° and 18.5 mph is -52.1° .

86. Let $P_1(a, b)$ and $P_2(c, d)$ be any two points on the line $y = 5x - 2$. Then $b = 5a - 2$, and $d = 5c - 2$. Now we put P_1 and P_2 into the definition of m :

$$m = \frac{d - b}{c - a}$$

$$= \frac{(5c - 2) - (5a - 2)}{c - a}$$

Replace d by $5c - 2$ and b by $5a - 2$

$$= \frac{5c - 5a}{c - a}$$

Remove parentheses and combine like terms

$$= \frac{5(c - a)}{c - a}$$

Factor 5 from the numerator

$$= 5$$

Reduce by $c - a$

Thus, no matter what two points we choose on this line we will obtain the slope 5.

$$95. y = x + 6; 3y + x = 5$$

$$3(x + 6) + x = 5$$

Replace y in second equation by

$$x + 6$$

$$x = -\frac{13}{4}$$

Solve for x

$$y = x + 6$$

First equation

$$y = -\frac{13}{4} + \frac{24}{4}$$

Replace x by $-\frac{13}{4}$

$$= \frac{11}{4}$$

Solve for y

The point is $(-\frac{13}{4}, \frac{11}{4})$.

$$108. y = 4x^2 + 6x - 1; y = -2x + 4$$

$$-2x + 4 = 4x^2 + 6x - 1$$

$$0 = 4x^2 + 8x - 5$$

$$0 = (2x + 5)(2x - 1)$$

$$x = -\frac{5}{2}, \frac{1}{2}$$

$$y = -2(-\frac{5}{2}) + 4 = 9$$

$$y = -2(\frac{1}{2}) + 4 = 3$$

$$(-2\frac{1}{2}, 9), (\frac{1}{2}, 3)$$

$$116. y = x^2 + 3x + 13; y = -x^2 - 6x + 9$$

$$-x^2 - 6x + 9 = x^2 + 3x + 13$$

$$0 = 2x^2 + 9x + 4$$

$$0 = (2x + 1)(x + 4)$$

$$x = -4, -\frac{1}{2}$$

$$y = -(-4)^2 - 6(-4) + 9 = 17$$

$$y = -(-\frac{1}{2})^2 - 6(-\frac{1}{2}) + 9 = \frac{47}{4}$$

$$(-4, 17), (-\frac{1}{2}, 11\frac{3}{4})$$

5. not a function; the first element 2 repeats; domain $\{-10, 2, 4\}$, range $\{-5, 9, 12, 13\}$

7. $\{(-2, 10), (3, 5), (5, 3), (\frac{3}{4}, 7\frac{1}{4}), (7, 1)\}$;

function, one to one;

domain: $\{-2, 3, 5, 7\frac{1}{4}\}$,

range: $\{10, 5, 3, 7\frac{1}{4}, 1\}$

9. $\{(1, 1), (8, 2), (27, 3), (-1, -1), (-8, -2), (-27, -3)\}$; function, one to one;

domain: $\{\pm 1, \pm 8, \pm 27\}$, range: $\{\pm 1, \pm 2, \pm 3\}$

11. $D = R$; $f(-4) = -23$; $f(0) = -3$;

$$f(\frac{1}{2}) = -\frac{1}{2}; f(7) = 32; f(3\sqrt{2})$$

$$= 15\sqrt{2} - 3; f(c - 1) = 5c - 8$$

13. $D = \{x | x \geq \frac{1}{2}\}$; $g(-4)$, $g(0)$ are not defined since $-4, 0$ are not in the domain

of g ; $g(\frac{1}{2}) = 0$; $g(7) = \sqrt{13}$; $g(3\sqrt{2}) =$

$$\sqrt{6\sqrt{2} - 1}; g(c - 1) = \sqrt{2c - 3}$$

15. $D = \{x | x \neq -3\}$; $f(-4) = 9$;

$$f(0) = -\frac{1}{3}; f(\frac{1}{2}) = 0; f(7) = \frac{13}{10};$$

$$f(3\sqrt{2}) = \frac{13 - 7\sqrt{2}}{3}; f(c - 1) = \frac{2c - 3}{c + 2}$$

17. $D = R$; $m(-4) = 41$; $m(0) = -11$;

$$m(\frac{1}{2}) = -\frac{43}{4}; m(7) = 129; m(3\sqrt{2})$$

$$= 43 - 3\sqrt{2}; m(c - 1) = 3c^2 - 7c - 7$$

19. $D = \{x | x \geq 1\}$; $f(-4)$, $f(0)$, $f(\frac{1}{2})$ are not defined since $-4, 0, \frac{1}{2}$ are not in the

implied domain; $f(7) = \frac{\sqrt{3}}{2}$;

$$f(3\sqrt{2}) = \frac{\sqrt{3\sqrt{2} - 1}}{\sqrt{3\sqrt{2} + 1}} \text{ or } \frac{(3\sqrt{2} - 1)\sqrt{17}}{17};$$

$$f(c - 1) = \frac{\sqrt{c - 2}}{\sqrt{c}}$$

21. $D = R$; $h(-4) = -68$; $h(0) = -4$;

$$h(\frac{1}{2}) = -3\frac{7}{8}; h(7) = 339;$$

$$h(3\sqrt{2}) = 54\sqrt{2} - 4; f(c - 1) = c^3 - 3c^2$$

$$+ 3c - 5 \quad 23. \text{ domain: } x \neq -3; g(-4)$$

$$= 4; g(0) = 0; g(\frac{1}{2}) = \frac{1}{7}; g(7) = \frac{7}{10};$$

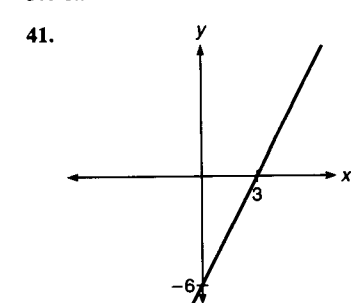
$$g(3\sqrt{2}) = 2 - \sqrt{2}; g(c - 1) = \frac{c - 1}{c + 2}$$

25. $2x - 3 + h$ 27. a. 105 b. 0

29. a. -3 b. -2 c. $\frac{5}{2}$ d. $\frac{8}{3}$

31. -3 33. $-\frac{1}{5}$ 35. 208

37. $5x + 2$ 39. $2x - 2$

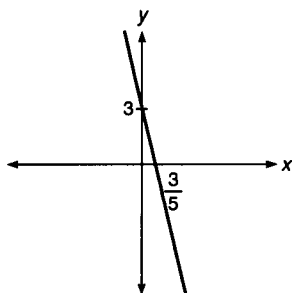


Exercise 3-3

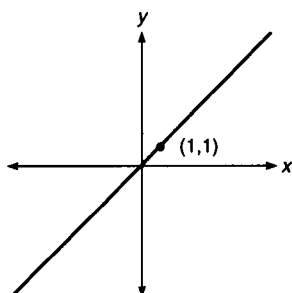
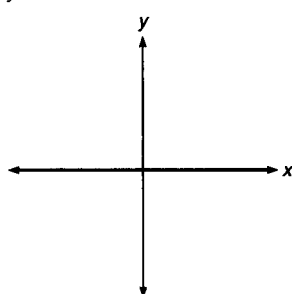
Answers to odd-numbered problems

1. A function is a relation in which no first element repeats. 3. function, one to one; domain $\{-3, 1, 4, 5\}$, range $\{1, 2, 5, 8\}$

43.



45.

47. $y = 0$ is the x -axis49. $C(m) = 0.34m + 500$ 51. $8,333\frac{1}{3}$ miles 53. $v = \frac{1}{20}t + 8$ 55. $W = \frac{44}{25}a + \frac{574}{5}$; using $a = 40$ wepredict $185\frac{1}{5}$ pounds57. $A = 800\pi - 40\pi h$ 59. $V = 4x^3 - 140x^2 + 1,200x$ **Solutions to skill and review problems**

1. $m = \frac{4-3}{-2-1} = -\frac{1}{3}$

$y - 3 = -\frac{1}{3}(x - 1)$

$y - 3 = -\frac{1}{3}x + \frac{1}{3}$

$y = -\frac{1}{3}x + 3\frac{1}{3}$

2. $2y - x = 4$

$2y = x + 4$

$y = \frac{1}{2}x + 2; m = \frac{1}{2}$

The y -intercept of 3 is the point $(0, 3)$:

$y - 3 = \frac{1}{2}(x - 0)$

$y = \frac{1}{2}x + 3$

3. $8x^3 - 1$

$(2x - 1)((2x)^2 + (1)(2x) + 1^2)$

$(2x - 1)(4x^2 + 2x + 1)$

4. $(3x - 2)^3$

$(3x - 2)(3x - 2)(3x - 2)$

$(9x^2 - 12x + 4)(3x - 2)$

$27x^3 - 36x^2 + 12x - 18x^2 + 24x - 8$

$27x^3 - 54x^2 + 36x - 8$

5. $\frac{2x-1}{3} - \frac{x-1}{2} = 6$

$6\left(\frac{2x-1}{3}\right) - 6\left(\frac{x-1}{2}\right) = 6(6)$

$2(2x - 1) - 3(x - 1) = 36$

$4x - 2 - 3x + 3 = 36$

$x + 1 = 36$

$x = 35$

6. $|x - 3| > 1$

$x - 3 > 1$ or $x - 3 < -1$

$x > 4$ or $x < 2$

Solutions to trial exercise problems

19. $f(x) = \frac{\sqrt{x-1}}{\sqrt{x+1}}$; domain: $x + 1 > 0$

and $x - 1 \geq 0$, so, $x > -1$ and $x \geq 1$.Both conditions are satisfied if $x \geq 1$.

$D = \{x | x \geq 1\}$

 $f(-4), f(0), f(\frac{1}{2})$ are not defined since $-4, 0, \frac{1}{2}$ are not in the implied

domain.

$f(7) = \frac{\sqrt{7-1}}{\sqrt{7+1}} = \frac{\sqrt{6}}{\sqrt{8}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$$f(3\sqrt{2}) = \frac{\sqrt{3\sqrt{2}-1}}{\sqrt{3\sqrt{2}+1}} \cdot \frac{\sqrt{3\sqrt{2}-1}}{\sqrt{3\sqrt{2}-1}}$$

$$= \frac{3\sqrt{2}-1}{\sqrt{18-1}} = \frac{3\sqrt{2}-1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}}$$

$$= \frac{(3\sqrt{2}-1)\sqrt{17}}{17}$$

$f(c-1) = \frac{\sqrt{(c-1)-1}}{\sqrt{(c-1)+1}} = \frac{\sqrt{c-2}}{\sqrt{c}}$

27. a. $g(3) = 2\sqrt{2}$ so $f(g(3)) = f(2\sqrt{2})$

$$= 2(2\sqrt{2})^4 - 3(2\sqrt{2})^2 + 1$$

$$= 2(2^4)(\sqrt{2})^4 - 3(2^2)(\sqrt{2})^2 + 1$$

$$= 2(16)(4) - 3(8) + 1 = 105$$

b. $g(\frac{2}{3}) = 1$, so $f(g(\frac{2}{3})) = f(1) = 0$

35. $(f(-3))^2 - 3(g(1))^2$

$$[(5(-3) - 1)]^2 - 3[(2(1) + 2)]^2$$

$$(-16)^2 - 3(4)^2$$

$$208$$

7. $\frac{x+3}{x-1} < 1$

Critical points:

a. Solve corresponding equality

$\frac{x+3}{x-1} = 1$

$x + 3 = x - 1$

$0 = -4$ (no solution)

b. Zeros of denominators

$x - 1 = 0$

$x = 1$

CP: 1

TP: 0, 2

$x = 0: \frac{0+3}{0-1} < 1$

$-3 < 1$, True

$x = 2: \frac{2+3}{2-1} < 1$

$5 < 1$, False

solution: $x < 1$ 56. We have two points (temperature, time) (t, T) : $(74^\circ, 3:05)$ and $(625^\circ, 4:15)$.

If we put the times in minutes these

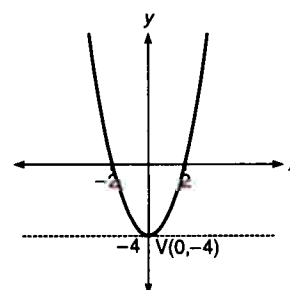
points are $(74, 0)$ and $(625, 70)$. If T $= mt + b$, then $74 = m(0) + b$, so b $= 74$. Using the second point 625 $= m(70) + 74$, $551 = 70m$,

$m = \frac{551}{70}$, so $T = \frac{551}{70}t + 74$, where t is

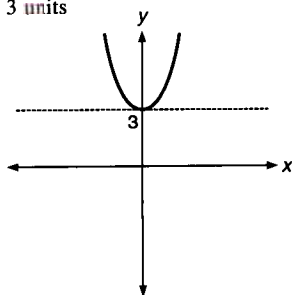
in minutes after 3:05 P.M. 4:00

corresponds to $t = 55$, so at this time

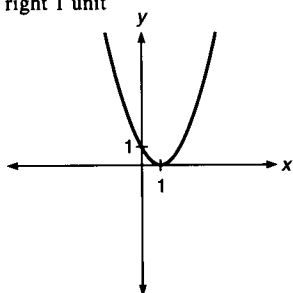
$T = \frac{551}{70}(55) + 74 \approx 507^\circ$

Exercise 3-4**Answers to odd-numbered problems**1. $y = x^2 - 4$; graph of $y = x^2$ shifted down 4 units

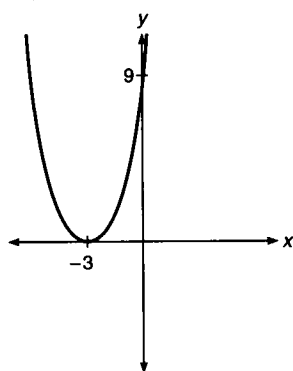
3. $y = x^2 + 3$; graph of $y = x^2$ shifted up 3 units



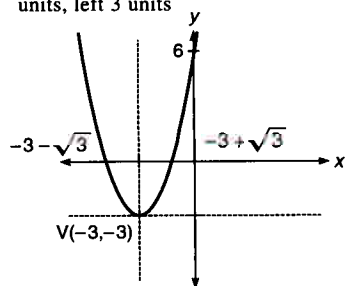
5. $y = (x - 1)^2$; graph of $y = x^2$ shifted right 1 unit



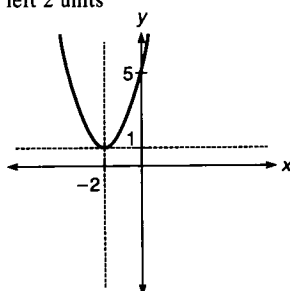
7. $y = (x + 3)^2$; $y = (x - (-3))^2$; graph of $y = x^2$ shifted left 3 units



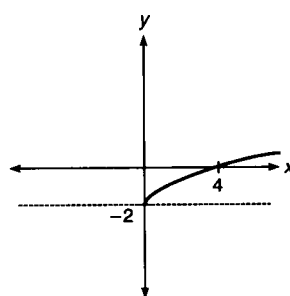
9. $y = (x + 3)^2 - 3$; $y = (x - (-3))^2 - 3$; graph of $y = x^2$ shifted down 3 units, left 3 units



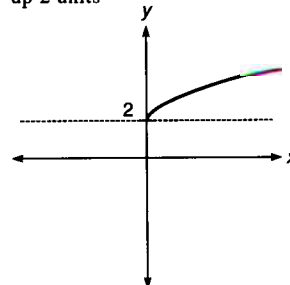
11. $y = (x + 2)^2 + 1$; $y = (x - (-2))^2 + 1$; graph of $y = x^2$ shifted up 1 unit, left 2 units



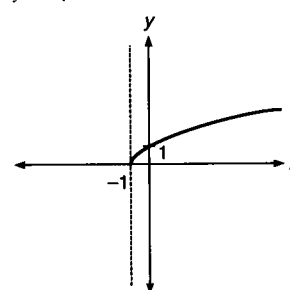
13. $y = \sqrt{x} - 2$; graph of $y = \sqrt{x}$ shifted down 2 units



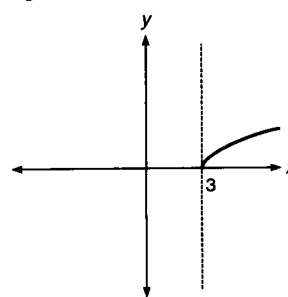
15. $y = \sqrt{x} + 2$; graph of $y = \sqrt{x}$ shifted up 2 units



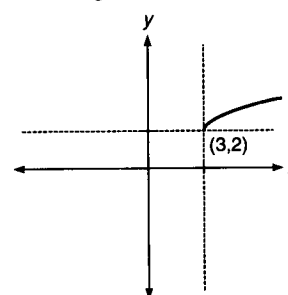
17. $y = \sqrt{x + 1}$; $y = \sqrt{x - (-1)}$; graph of $y = \sqrt{x}$ shifted left 1 unit



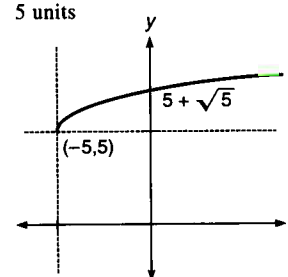
19. $y = \sqrt{x - 3}$; graph of $y = \sqrt{x}$ shifted right 3 units



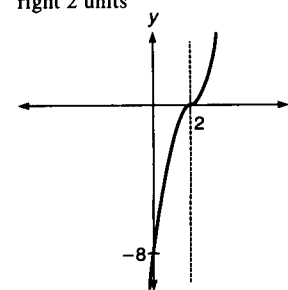
21. $y = \sqrt{x - 3} + 2$; graph of $y = \sqrt{x}$ shifted right 3 units, up 2 units



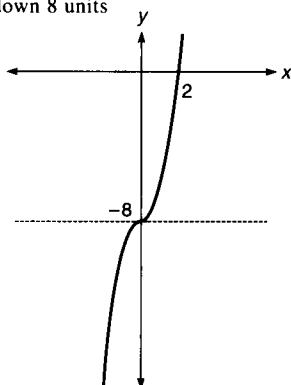
23. $y = \sqrt{x + 5} + 5$; $y = \sqrt{x - (-5)} + 5$; graph of $y = \sqrt{x}$ shifted left 5 units, up 5 units



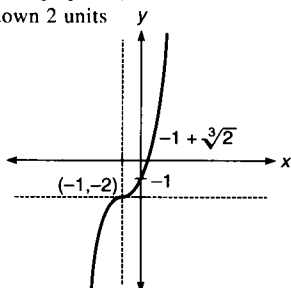
25. $y = (x - 2)^3$; graph of $y = x^3$ shifted right 2 units



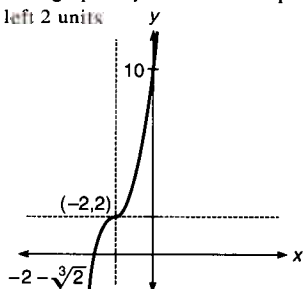
27. $y = x^3 - 8$; graph of $y = x^3$ shifted down 8 units



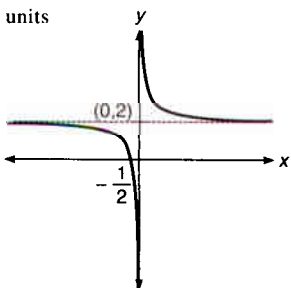
29. $y = (x + 1)^3 - 2$; $y = (x - (-1))^3 - 2$; graph of $y = x^3$ shifted left 1 unit, down 2 units



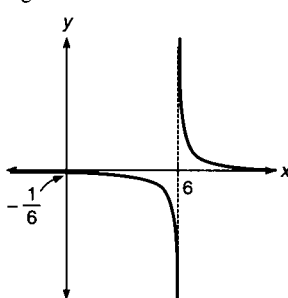
31. $y = (x + 2)^3 + 2$; $y = (x - (-2))^3 + 2$; graph of $y = x^3$ shifted up 2 units, left 2 units



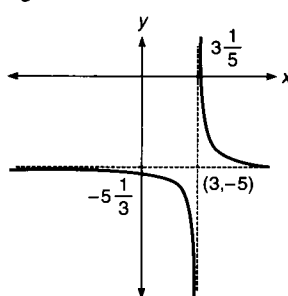
33. $y = \frac{1}{x} + 2$; graph of $y = \frac{1}{x}$ shifted up 2 units



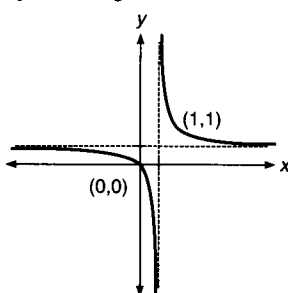
35. $y = \frac{1}{x - 6}$; graph of $y = \frac{1}{x}$ shifted right 6 units



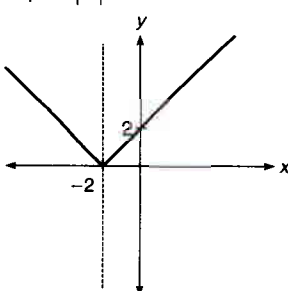
37. $y = \frac{1}{x - 3} - 5$; graph of $y = \frac{1}{x}$ shifted right 3 units, down 5 units



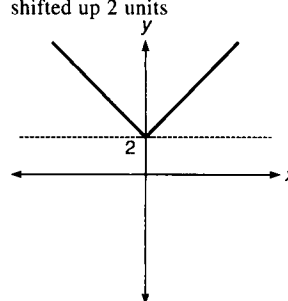
39. $y = \frac{1}{x - 1} + 1$; graph of $y = \frac{1}{x}$ shifted up 1 unit, right 1 unit



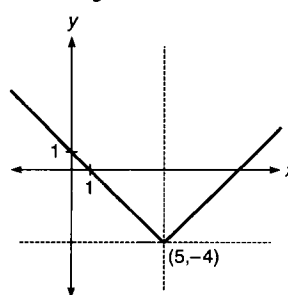
41. $y = |x + 2|$; $y = |x - (-2)|$; graph of $y = |x|$ shifted left 2 units



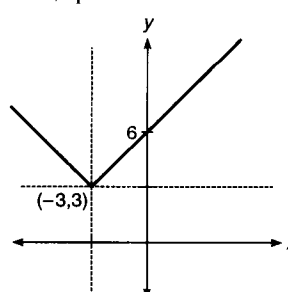
43. $y = |x| + 2$; graph of $y = |x|$ shifted up 2 units



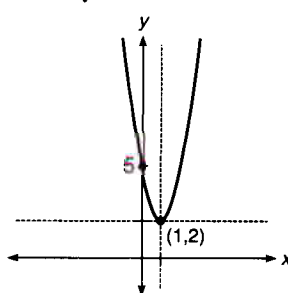
45. $y = |x - 5| - 4$; graph of $y = |x|$ shifted right 5 units, down 4 units



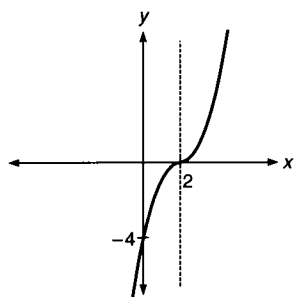
47. $y = |x + 3| + 3$; $y = |x - (-3)| + 3$; graph of $y = |x|$ shifted left 3 units, up 3 units



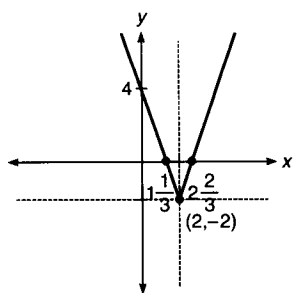
49. $y = 3(x - 1)^2 + 2$; graph of $y = x^2$ shifted up 2 units, right 1 unit, vertically scaled 3 units



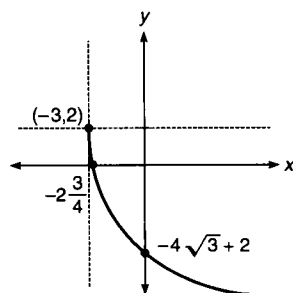
51. $y = \frac{1}{2}(x-2)^3$; graph of $y = x^3$ shifted right 2 units, vertically scaled $\frac{1}{2}$ units



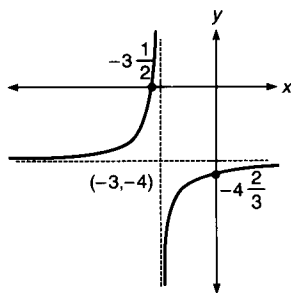
53. $y = |3x-6| - 2$; $y = 3|x-2| - 2$; graph of $y = |x|$ shifted down 2 units, right 2 units, vertically scaled 3 units



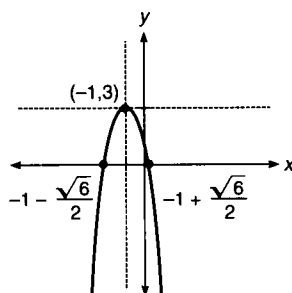
55. $y = -4\sqrt{x+3} + 2$;
 $y = -4\sqrt{x-(-3)} + 2$; graph of
 $y = \sqrt{x}$ shifted up 2 units, left 3 units,
 vertically scaled -4 units



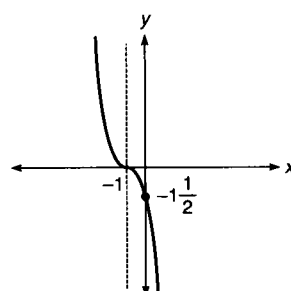
57. $y = \frac{-2}{x+3} - 4$; $y = \frac{-2}{x-(-3)} - 4$;
 graph of $y = \frac{1}{x}$ shifted down 4 units,
 left 3 units, vertically scaled -2 units



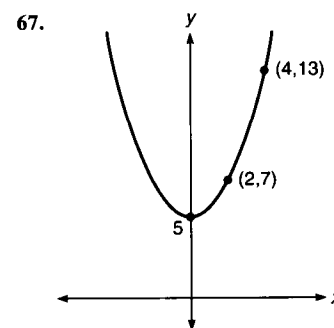
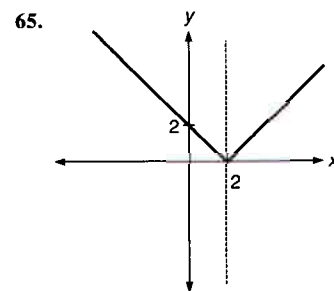
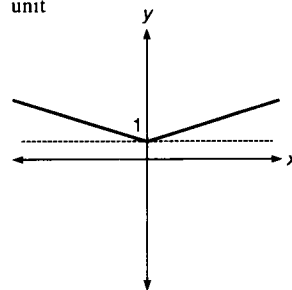
59. $y = -2(x+1)^2 + 3$;
 $y = -2(x-(-1))^2 + 3$; graph of
 $y = x^2$ shifted up 3 units, left 1 unit,
 vertically scaled -2 units



61. $y = -\frac{3}{2}(x+1)^3$; $y = -\frac{3}{2}(x-(-1))^3$;
 graph of $y = x^3$ shifted left 1 unit,
 vertically scaled $-1\frac{1}{2}$ units



63. $y = \left|\frac{x}{3}\right| + 1$; graph of $y = |x|$
 shifted up 1 unit, vertically scaled $\frac{1}{3}$
 unit



Solutions to skill and review problems

1. Find the distance between the points
 $(1,2)$ and $(6,8)$.
 $(x_1, y_1) = (1, 2)$; $(x_2, y_2) = (6, 8)$
 $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(6 - 1)^2 + (8 - 2)^2}$
 $= \sqrt{5^2 + 6^2}$
 $= \sqrt{61}$

2. Find the midpoint of the line segment which joins the points (1,2) and (6,8).

$$\begin{aligned}(x_1, y_1) &= (1, 2); (x_2, y_2) = (6, 8) \\ \text{midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1 + 6}{2}, \frac{2 + 8}{2} \right) \\ &= \left(3\frac{1}{2}, 5 \right).\end{aligned}$$

3. Find the equation that describes all points equidistant from the two points (1,2) and (6,8). Let (x,y) be a point which is equidistant from these two points. Then, using the distance formula (answer 1 above):

$$\begin{aligned}\sqrt{(x-1)^2 + (y-2)^2} &= \sqrt{(x-6)^2 + (y-8)^2} \\ (x^2 - 2x + 1) + (y^2 - 4y + 4) &= (x^2 - 12x + 36) + (y^2 - 16y + 64) \\ -2x + 1 - 4y + 4 &= -12x + 36 - 16y + 64 \\ 10x + 12y - 95 &= 0\end{aligned}$$

4. Find where the lines [1] $2y - 3x = 5$ and [2] $x + y = 3$ intersect.

$$\begin{aligned}y &= -x + 3 \\ \text{Solve [2] for } y & \\ 2(-x + 3) - 3x &= 5 \\ \text{Replace } y \text{ by } -x + 3 \text{ in [1]} & \\ -2x + 6 - 3x &= 5 \\ 1 &= 5x \\ \frac{1}{5} &= x \\ y &= -\frac{1}{5} + 3 \\ \text{Replace } x \text{ by } \frac{1}{5} \text{ in } y &= -x + 3 \\ y &= 2\frac{4}{5} \\ \text{The point is } &\left(\frac{1}{5}, 2\frac{4}{5} \right).\end{aligned}$$

5. Find the equation of a line that is perpendicular to the line $y = -2x + 3$ and passes through the point (1,-2). The slope of $y = -2x + 3$ is -2. We want a slope of $m = \frac{1}{2}$, since the slopes of perpendicular lines are negative reciprocals of each other.

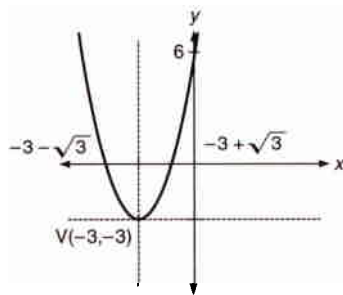
$$\begin{aligned}y - y_1 &= m(x - x_1) \\ \text{Point-slope formula} & \\ y - (-2) &= \frac{1}{2}(x - 1) \\ y + 2 &= \frac{1}{2}x - \frac{1}{2} \\ y &= \frac{1}{2}x - 2\frac{1}{2}\end{aligned}$$

6. Solve $x^2 - 4x = 32$.
 $x^2 - 4x - 32 = 0$
 $(x - 8)(x + 4) = 0$
 $x - 8 = 0$ or $x + 4 = 0$
 $x = 8$ or $x = -4$

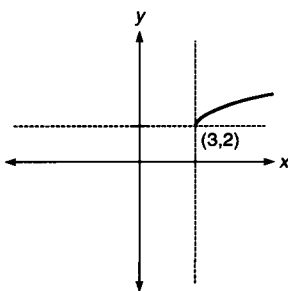
7. Solve $\frac{x}{x+y} = 3$ for x.
 $x = 3(x+y)$
 $x = 3x + 3y$
 $-3y = 2x$
 $-\frac{3}{2}y = x$

Solutions to trial exercise problems

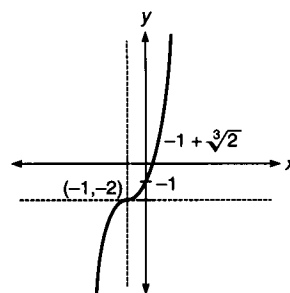
9. $y = (x+3)^2 - 3$
 $y = (x - (-3))^2 - 3$
 Graph of $y = x^2$ shifted down 3 units, left 3 units.
 Vertex at $(-3, -3)$.
 Intercepts:
 $x = 0: y + 3 = (0 + 3)^2$
 $y = 6$
 $y = 0: 3 = (x + 3)^2$
 $\pm\sqrt{3} = x + 3$
 $-3 \pm \sqrt{3} = x$



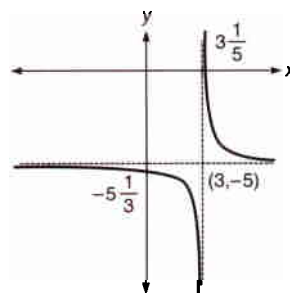
21. $y = \sqrt{x-3} + 2$
 Graph of $y = \sqrt{x}$ shifted left 3 units, up 2 units.
 Vertex at (3,2).
 Intercepts:
 $x = 0: y = \sqrt{-3} + 2$, not real so no y-intercept
 $y = 0: 0 = \sqrt{x-3} + 2$
 $-2 = \sqrt{x-3}$; a square root is nonnegative, so no x-intercept.



29. $y = (x+1)^3 - 2$
 $y = (x - (-1))^3 - 2$
 Graph of $y = x^3$ shifted left 1 unit, down 2 units; origin at $(-1, -2)$.
 Intercepts:
 $x = 0: y + 2 = 1^3, y = -1$
 $y = 0: 2 = (x + 1)^3$
 $\sqrt[3]{2} = x + 1$
 $-1 + \sqrt[3]{2} = x \approx 0.3$



37. $y = \frac{1}{x-3} - 5$
 Graph of $y = \frac{1}{x}$ shifted right 3 units, down 5 units; origin at (3,-5).
 Intercepts:
 $x = 0: y = -\frac{1}{3} - 5$
 $y = -5\frac{1}{3}$
 $y = 0: 5 = \frac{1}{x-3}$
 $5(x-3) = 1$
 $x-3 = \frac{1}{5}$
 $x = 3\frac{1}{5}$



45. $y = |x - 5| - 4$

Graph of $y = |x|$ shifted right 5 units, down 4 units; origin at $(5, -4)$.

Intercepts:

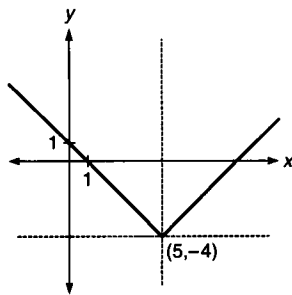
$$x = 0: y = |-5| - 4$$

$$= 1$$

$$y = 0: 4 = |x - 5|$$

$$x - 5 = 4 \text{ or } x - 5 = -4$$

$$x = 9 \text{ or } x = 1$$



53. $y = |3x - 6| - 2$

$$y = 3|x - 2| - 2$$

Graph of $y = |x|$ shifted down 2 units, right 2 units, vertically scaled 3 units; origin at $(2, -2)$.

Intercepts:

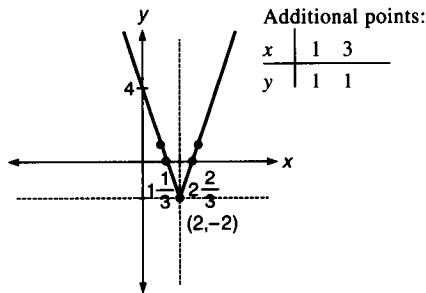
$$x = 0: y = |-6| - 2 = 4$$

$$y = 0: 2 = 3|x - 2|$$

$$\frac{2}{3} = |x - 2|, \text{ so}$$

$$x - 2 = \frac{2}{3} \text{ or } x - 2 = -\frac{2}{3}$$

$$x = 2\frac{2}{3} \text{ or } x = 1\frac{1}{3}$$



57. $y = \frac{-2}{x+3} - 4$

$$y = \frac{-2}{x - (-3)} - 4$$

Graph of $y = \frac{1}{x}$ shifted down 4 units, left 3 units, vertically scaled -2 units;

origin at $(-3, -4)$.

Intercepts:

$$x = 0: y = \frac{-2}{-3} - 4 = -4\frac{2}{3}$$

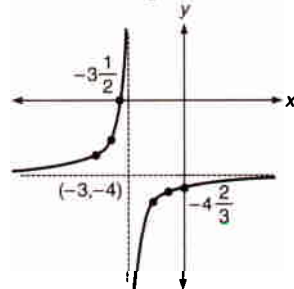
$$y = 0: 0 = \frac{-2}{x+3} - 4$$

$$4 = \frac{-2}{x+3}$$

$$4x + 12 = -2$$

$$4x = -14$$

$$x = -3\frac{1}{2}$$



Additional points:

x	-5	-4	-2	-1
y	-3	-2	-6	-5

60. $y = \sqrt{4x - 8} - 3$

$$y = \sqrt{4(x - 2)} - 3$$

$$y = 2\sqrt{x - 2} - 3$$

Graph of $y = \sqrt{x}$ shifted down 3 units, right 2 units, vertically scaled 2 units; origin at $(2, -3)$.

Intercepts:

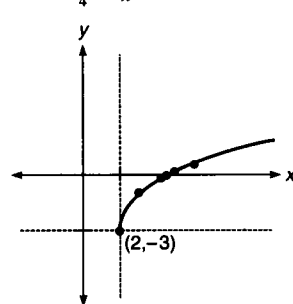
$$x = 0: y = \sqrt{-8} - 3, \text{ so no } y\text{-intercept}$$

$$y = 0: 3 = 2\sqrt{x - 2}$$

$$\frac{3}{2} = \sqrt{x - 2}$$

$$\frac{9}{4} = x - 2$$

$$\frac{17}{4} = x$$



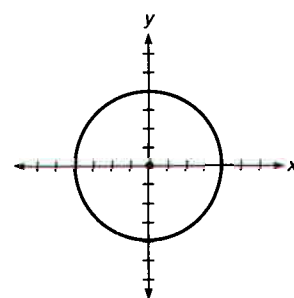
Additional points:

x	3	4	5	6
y	-1	-0.2	0.46	1

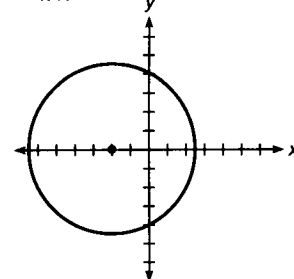
Exercise 3-5

Answers to odd-numbered problems

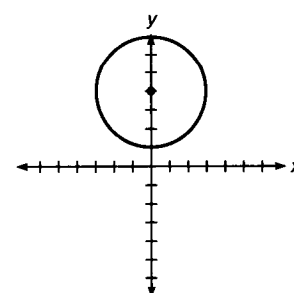
1. $C(0,0), r = \sqrt{16} = 4$



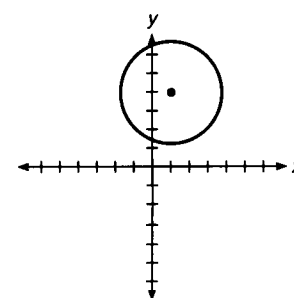
3. $C(-2,0); r = \sqrt{20} = 2\sqrt{5} \approx 4.47$



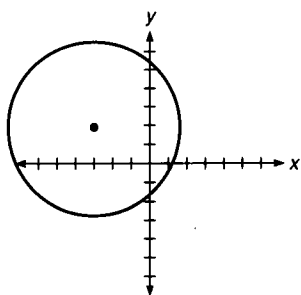
5. $C(0,4), r = 3$



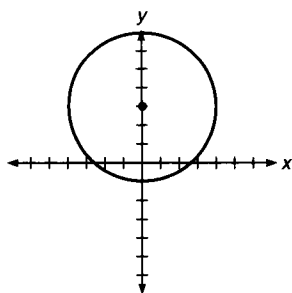
7. $C(1,4), r = \sqrt{8} = 2\sqrt{2} \approx 2.8$



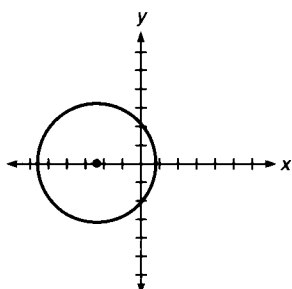
9. $C(-3, 2)$, $r = \sqrt{20} = 2\sqrt{5} \approx 4.47$



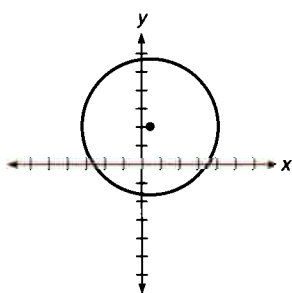
11. $x^2 + (y - 3)^2 = 15$
 $C(0, 3)$, $r = \sqrt{15} \approx 3.9$



13. $(x + \frac{5}{2})^2 + y^2 = \frac{41}{4}$
 $C(-\frac{5}{2}, 0)$, $r = \sqrt{\frac{41}{4}} = \frac{\sqrt{41}}{2} \approx 3.2$



15. $(x - \frac{1}{2})^2 + (y - 2)^2 = \frac{53}{4}$
 $C(\frac{1}{2}, 2)$, $r = \sqrt{\frac{53}{4}} \approx 3.6$



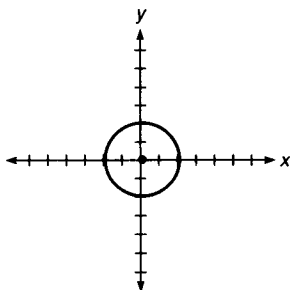
17. $(x - 1)^2 + (y + 2)^2 = 0$

$C(1, -2)$, $r = 0$

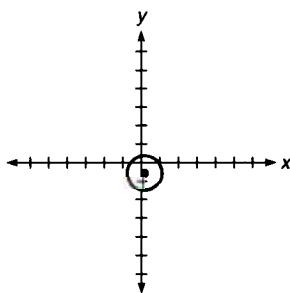
With $r = 0$ this “circle” is just the point $(1, -2)$.

19. $(x + 2)^2 + y^2 = -2$ Since the left side of the equation is nonnegative there are no real solutions to the equation, and so there is no graph.

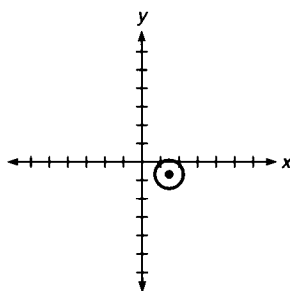
21. $x^2 + (y - \frac{1}{6})^2 = \frac{121}{36}$
 $C(0, \frac{1}{6})$, $r = \frac{11}{6}$



23. $(x - \frac{1}{5})^2 + (y + \frac{1}{2})^2 = \frac{89}{100}$
 $C(\frac{1}{5}, -\frac{1}{2})$, $r = \frac{\sqrt{89}}{10} \approx 0.9$



25. $(x - \frac{3}{2})^2 + (y + \frac{1}{2})^2 = \frac{1}{2}$
 $C(\frac{3}{2}, -\frac{1}{2})$, $r = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2} \approx 0.7$



27. $x^2 + 6x + y^2 - 4y + 9 = 0$

29. $x^2 - 4x + y^2 - (6 - 2\sqrt{2})y + 10$

$- 6\sqrt{2} = 0$ 31. $x^2 + y^2 - 6y - 55 = 0$

33. $x^2 - 2x + y^2 + 6y - 63 = 0$

35. $x^2 - 6x + y^2 - 10y - 24 = 0$

37. function (passes vertical line test); not one to one (fails horizontal line test)

39. not a function (fails vertical line test)

41. 40% 43. 0.35 45. a. 0.7;

b. -0.6 47. -2.2, -0.4, 1.3, 1.8

49. -1.25 to 0.5, 1.5 to 2.7

51. even, y-axis symmetry

53. odd, origin symmetry

55. even, y-axis symmetry

57. odd, origin symmetry

59. neither even nor odd

61. neither even nor odd

63. odd, origin symmetry

65. even, y-axis symmetry

67. odd, origin symmetry

69. $y = \frac{2}{3}x - 7$

71. $(x - 3)^2 + (y - 2)^2 = \frac{9}{16}$

Solutions to skill and review problems

1. Graph $f(x) = x^2 - 4$.

$y = x^2 - 4$

Graph of $y = x^2$ shifted down 4 units.

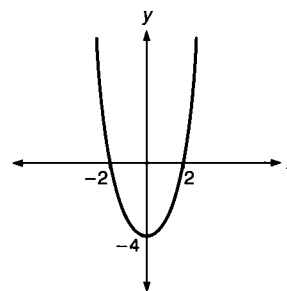
Vertex at $(0, -4)$.

Intercepts:

$x = 0$: $y = 0^2 - 4 = -4$

$y = 0$: $0 = x^2 - 4$

$4 = x^2$; $\pm 2 = x$



2. Graph $f(x) = (x - 4)^2$.

$$y = (x - 4)^2$$

Graph of $y = x^2$ shifted right 4 units.

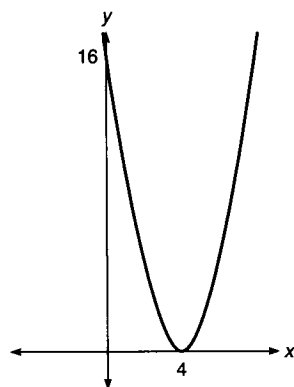
Vertex at (4, 0).

Intercepts:

$$x = 0: y = (0 - 4)^2 = 16$$

$$y = 0: 0 = (x - 4)^2$$

$$0 = x - 4; 4 = x$$



3. Graph $f(x) = (x - 4)^2 - 4$.

$$y = (x - 4)^2 - 4$$

Graph of $y = x^2$ shifted right 4 units, down 4 units.

Vertex at (4, -4).

Intercepts:

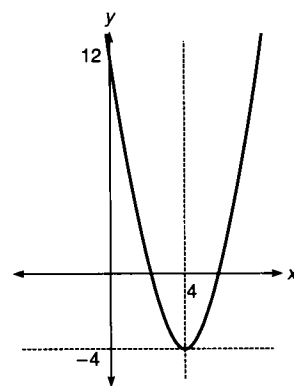
$$x = 0: y = (0 - 4)^2 - 4 = 12$$

$$y = 0: 0 = (x - 4)^2 - 4$$

$$4 = (x - 4)^2$$

$$\pm 2 = x - 4$$

$$4 \pm 2 = x; x = 2 \text{ or } 6$$



4. Solve $|2x - 3| = 8$.

$$2x - 3 = 8 \text{ or } 2x - 3 = -8$$

$$2x = 11 \text{ or } 2x = -5$$

$$x = 5\frac{1}{2} \text{ or } x = -2\frac{1}{2}$$

$$\{-2\frac{1}{2}, 5\frac{1}{2}\}$$

5. Factor $x^6 - 64$.

$$(x^3 - 8)(x^3 + 8)$$

Difference of two squares

$$(x - 2)(x^2 + 2x + 4)(x + 2)(x^2 - 2x + 4)$$

Difference of two cubes

6. Find the equation of the line that passes through the points (-4, 1) and (3, -5).

$$(x_1, y_1) = (-4, 1); (x_2, y_2) = (3, -5)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 1}{3 - (-4)} = \frac{-6}{7}$$

$$= -\frac{6}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{6}{7}(x - (-4))$$

$$y - 1 = -\frac{6}{7}x - \frac{24}{7}$$

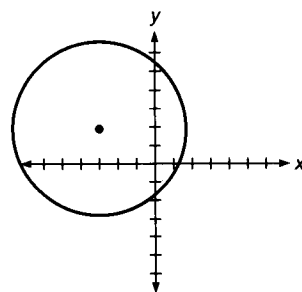
$$y = -\frac{6}{7}x - \frac{24}{7} + \frac{7}{7}$$

$$y = -\frac{6}{7}x - \frac{17}{7}$$

Solutions to trial exercise problems

9. $(x + 3)^2 + (y - 2)^2 = 20$

$$C(-3, 2), r = \sqrt{20} = 2\sqrt{5} \approx 4.47$$



21. $3x^2 + 3y^2 - y - 10 = 0$

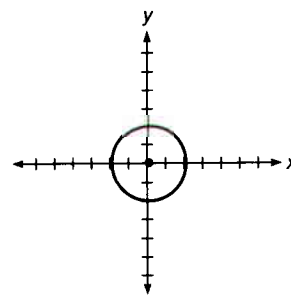
$$x^2 + y^2 - \frac{1}{3}y = \frac{10}{3}$$

$$\frac{1}{2}(-\frac{1}{3}) = -\frac{1}{6}, (-\frac{1}{6})^2 = \frac{1}{36}$$

$$x^2 + y^2 - \frac{1}{3}y + \frac{1}{36} = \frac{10}{3} + \frac{1}{36}$$

$$x^2 + (y - \frac{1}{6})^2 = \frac{121}{36}$$

$$C(0, \frac{1}{6}), r = \frac{11}{6}$$



29. $(h, k) = (2, 3 - \sqrt{2}), r = \sqrt{5}$

$$(x - 2)^2 + (y - (3 - \sqrt{2}))^2 = (\sqrt{5})^2$$

$$(x - 2)^2 + (y - (3 - \sqrt{2}))^2 = 5$$

$$x^2 - 4x + 4 + y^2 - 2(3 - \sqrt{2})y$$

$$+ (3 - \sqrt{2})^2 = 5$$

$$x^2 - 4x + 4 + y^2 - 2(3 - \sqrt{2})y$$

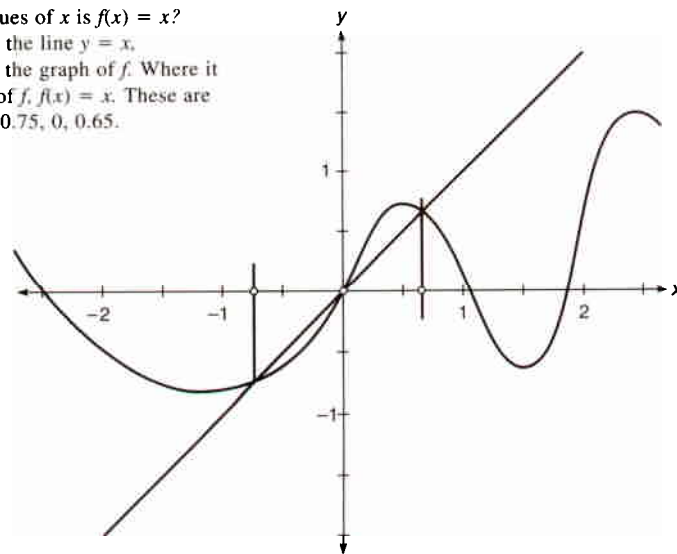
$$+ 9 - 6\sqrt{2} + 2 = 5$$

$$x^2 - 4x + y^2 - (6 - 2\sqrt{2})y + 10$$

$$- 6\sqrt{2} = 0$$

50. For what values of x is $f(x) = x$?

The graph shows the line $y = x$, superimposed on the graph of f . Where it meets the graph of f , $f(x) = x$. These are approximately -0.75, 0, 0.65.



54. $f(x) = x^5 - 4x^3 - x$

$$\begin{aligned} f(-x) &= (-x)^5 - 4(-x)^3 - (-x) \\ &= -x^5 + x^4 + x - f(x) \\ &= -(x^5 - 4x^3 - x) = -x^5 + 4x^3 + x \end{aligned}$$

$$f(-x) = -f(x): \text{ odd, origin symmetry}$$

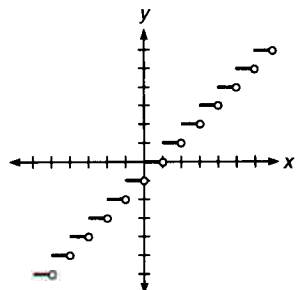
58. $f(x) = x^4 - x - 2$

$$f(-x) = x^4 + x - 2$$

$$-f(x) = -x^4 + x + 2$$

thus $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$,
so f is neither even nor odd

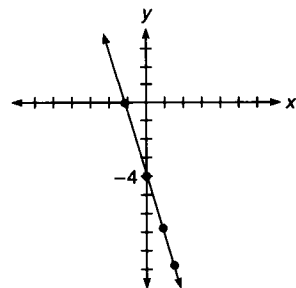
70.



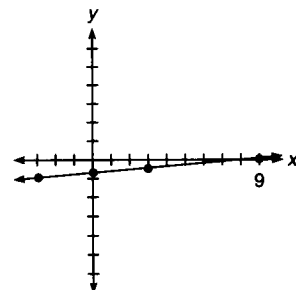
Chapter 3 review

In problems 1 through 6 the three points you use may differ from those shown, but the x - and y -intercepts, and the graph, should be the same.

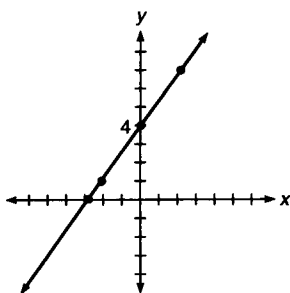
1. $(0, -4)$, $(1, -\frac{13}{2})$, $(2, -9)$, $(-1\frac{3}{8}, 0)$



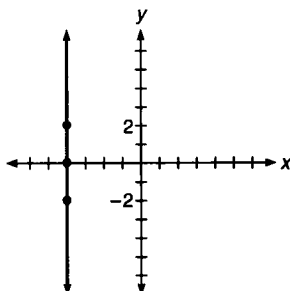
2. $(-3, -1)$, $(0, -\frac{3}{4})$, $(3, -\frac{1}{2})$, $(9, 0)$



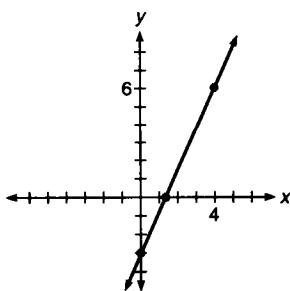
3. $(-2, 1)$, $(0, 4)$, $(2, 7)$, $(-2\frac{2}{3}, 0)$



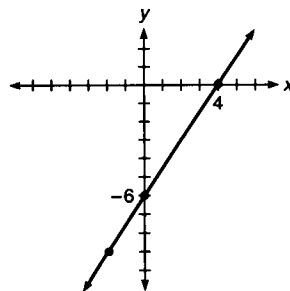
4. $(-4, -2)$, $(-4, 0)$, $(-4, 2)$



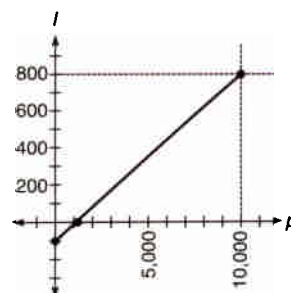
5. $(-4, -12)$, $(0, -3)$, $(4, 6)$, $(1\frac{1}{3}, 0)$



6. $(-2, -9)$, $(0, -6)$, $(4, 0)$



7. $(0, -100)$ I -intercept
 $(1, 111\frac{1}{9}, 0)$ p -intercept

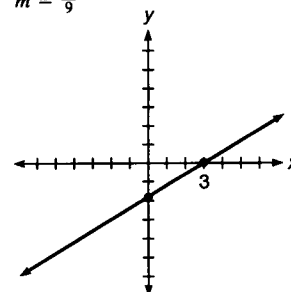


8. $(2, -2\frac{1}{2})$ 9. $(-4, \frac{3}{2}\sqrt{2})$

10. $\sqrt{65}$ 11. $3\sqrt{3}$

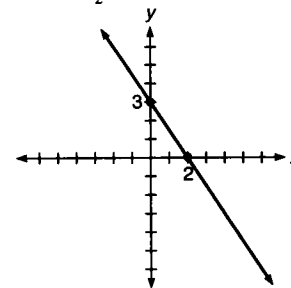
12. $(3, 0)$, $(0, -\frac{5}{3})$

$$m = \frac{5}{9}$$



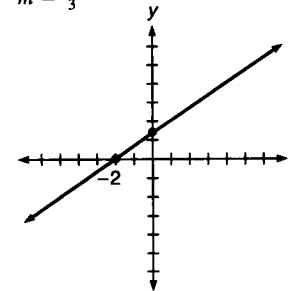
13. $(2, 0)$, $(0, 3)$

$$m = -1\frac{1}{2}$$

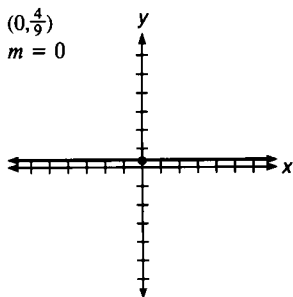


14. $(-2, 0)$, $(0, \frac{4}{3})$

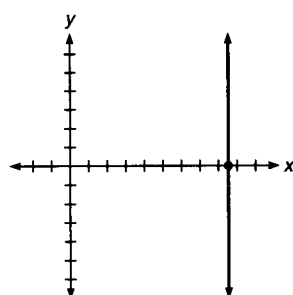
$$m = \frac{2}{3}$$



15. $(0, \frac{4}{9})$
 $m = 0$



16. $(8.5, 0)$
 m is undefined



17. $\frac{3}{8}$ 18. $-\frac{4}{3}$ 19. $\frac{5\sqrt{3}}{3}$

20. $y = -\frac{4}{3}x + \frac{11}{3}$ 21. $y = 4x - 11$

22. $y = -\frac{2}{3}x - 3$ 23. $y = -6x - 1$

24. $y = \frac{3}{2}x - \frac{5}{2}$ 25. $y = \frac{2}{5}x - \frac{9}{5}$

26. $(2\frac{2}{7}, \frac{6}{7})$ 27. $(1\frac{1}{2}, 6\frac{1}{2})$

28. $16x + 6y - 27 = 0$

 29. Let (a, b) and (c, d) be two points on the line $y = 3x - 4$ so that $a \neq c$. Then $b = 3a - 4$ and $d = 3c - 4$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{d - b}{c - a} \text{ if } c \neq a$$

$$m = \frac{(3c - 4) - (3a - 4)}{c - a}$$

$$m = \frac{3c - 3a}{c - a}$$

$$m = \frac{3(c - a)}{c - a}$$

$$m = 3$$

 30. 125 seconds 31. It is not a function since it is not true that all the first elements are different. Domain $\{-3, 1, 4\}$; range $\{2, 4, 5, 8\}$.

 32. Function since all first elements are different. Not one to one since it is not true that all the second elements are different. Domain $\{-3, -2, -1, 0, 1\}$; range $\{1, 3, 4, 5\}$.

 33. Function since all first elements are different. One to one since all the second elements are different. Domain $\{-10, 2, 3, 4\}$; range $\{-5, 2, 12, 13\}$.

 34. Not a function since it is not true that all the first elements are different. Domain $\{3, \pi, 17\}$; range

 $\{-\sqrt{2}, \frac{8}{13}, \sqrt{2}, \pi\}$.

 35. $\{(-3, 3), (9, -1), (\sqrt{18}, 2 - \sqrt{2}), (\frac{3}{4}, 1\frac{3}{4}), (\pi, 2 - \frac{\pi}{3})\}$;

one-to-one function

 36. $\{(-3, \sqrt{13}), (-3, -\sqrt{13}), (-2, \sqrt{10}), (-2, -\sqrt{10}), (-1, \sqrt{7}), (-1, -\sqrt{7}), (0, 2), (0, -2), (1, 1), (1, -1)\}$; not a function

 37. Implied domain: $x \neq \frac{3}{4}$; $f(-4) = \frac{15}{19}$,

 $f(0) = -\frac{1}{3}$, $f(\frac{1}{2}) = -\frac{3}{4}$,

 $f(3\sqrt{5}) = \frac{-44}{12\sqrt{5} - 3}$,

 $f(c - 2) = \frac{-c^2 + 4c - 3}{4c - 11}$

 38. Implied domain: $x \leq \frac{1}{2}$; $g(-4)$
 $= 6\sqrt{3}$, $g(0) = 2\sqrt{3}$, $g(\frac{1}{2}) = 0$,

 $3\sqrt{5} > \frac{1}{2}$, so it is not in the domain of g ,

 $g(c - 2) = 2\sqrt{15 - 6c}$

 39. Implied domain: R ; $v(-4) = -5$,

 $v(0) = 3$, $v(\frac{1}{2}) = 1\frac{3}{4}$, $v(3\sqrt{5}) = -42$
 $-6\sqrt{5}$, $v(c - 2) = -c^2 + 2c + 3$

 40. Implied domain: $x < -3$ or $x > 2$;

 $g(-4) = -\frac{2}{3}\sqrt{6}$. 0 is not in the domain of

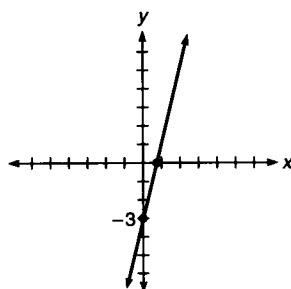
 g , $\frac{1}{2}$ is not in the domain of g , $g(3\sqrt{5})$
 $= \frac{-4}{\sqrt{3\sqrt{5} + 39}}$, $g(c - 2) = \frac{-4}{\sqrt{c^2 - 3c - 4}}$

 41. $2x - 5 + h$

 42. a. 89, b. -1, c. $a^6 - 3a^3 + 1$

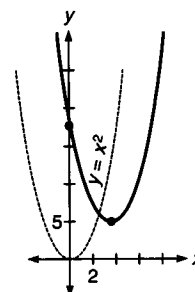
 43. a. 29, b. 1, c. $\frac{-19a - 11b}{a + b}$

 44. a. $\frac{9}{10}$, b. $-\frac{27}{34}$

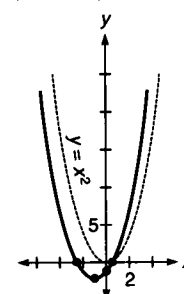
 45. $(\frac{3}{5}, 0)$, $(0, -3)$


46. $f(x) = 1.25x - 21.5, -4^\circ$

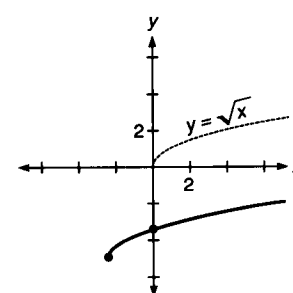
47. vertex: $(3\frac{1}{2}, 5)$; y-intercept: $(0, 17.25)$



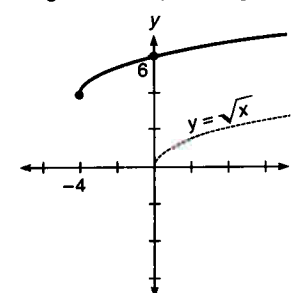
48. vertex: $(-1, -2)$; x-intercept: $(-1 + \sqrt{2}, 0)$, $(-1 - \sqrt{2}, 0)$; y-intercept: $(0, -1)$



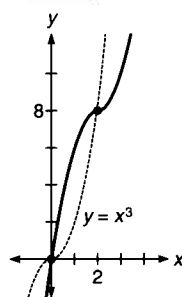
49. origin shifted to $(-2\frac{1}{2}, -5)$; x-intercept: $(22.5, 0)$; y-intercept: $\frac{\sqrt{10} - 10}{2} = -3.4$



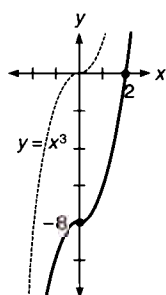
50. origin: $(-4, 4)$; y-intercept: $(0, 6)$



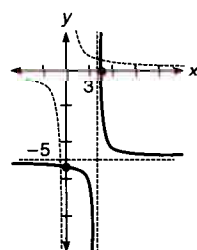
51. origin: (2,8); y-intercept: (0,0);
x-intercept: (0,0)



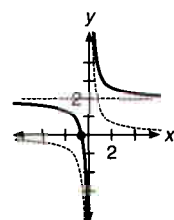
52. x-intercept: (2,0); y-intercept: (0,-8)



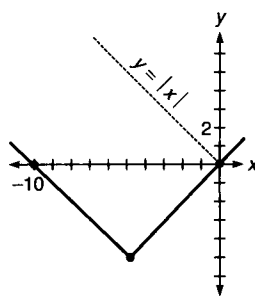
53. origin: (3,-5); x-intercept: $(3\frac{1}{3}, 0)$;
y-intercept: $(0, -5\frac{1}{3})$



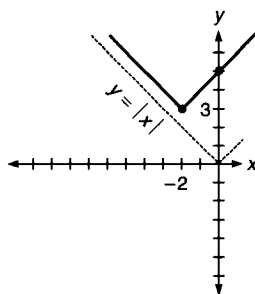
54. graph of $y = \frac{1}{x}$ raised up 2 units;
x-intercept: $(-\frac{1}{2}, 0)$



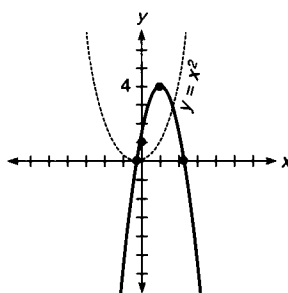
55. origin: (-5,-5); x-intercept: (0,0),
(-10,0); y-intercept: (0,0)



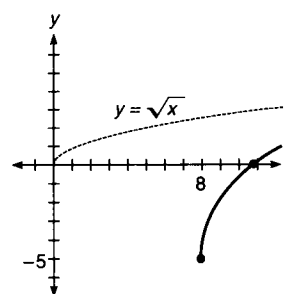
56. graph of $y = |x|$ shifted left 2 units,
up 3 units; y-intercept: (0,5)



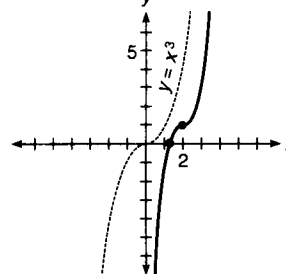
57. graph of $y = x^2$ but flipped over,
vertically scaled by 3, and origin
shifted to (1,4); x-intercept: (-0.2,0),
(2.2,0); y-intercept: (0,1)



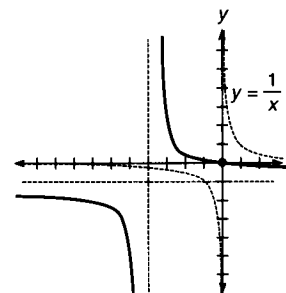
58. graph of $y = \sqrt{x}$ shifted to a new
origin of (8,-5), vertically scaled by 3
units; x-intercept: $(10\frac{7}{9}, 0)$



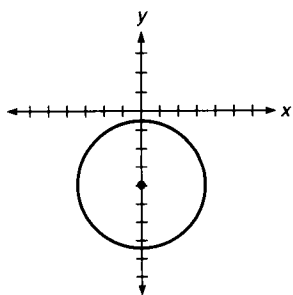
59. graph of $y = x^3$ shifted to a new origin
of (2,1), vertically scaled by 2 units;
x-intercept: $(2 - \frac{\sqrt[3]{4}}{2}, 0)$; y-intercept:
(0,-15)



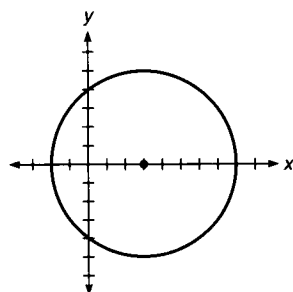
60. graph of $y = \frac{1}{x}$ shifted to a new origin
of (-4,-1), vertically scaled by 4; all
intercepts at the origin



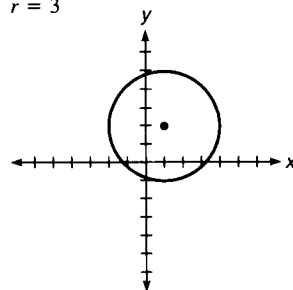
61. center: $(0, -4)$; $r = \sqrt{12} = 2\sqrt{3} \approx 3.46$



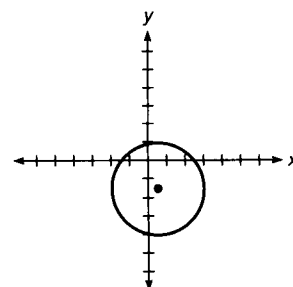
62. $(x - 3)^2 + (y - 0)^2 = 25$; center: $(3, 0)$; $r = 5$



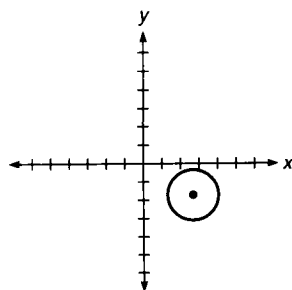
63. $(x - 1)^2 + (y - 2)^2 = 9$; center: $(1, 2)$; $r = 3$



64. $(x - \frac{1}{2})^2 + (y - (-\frac{3}{2}))^2 = \frac{15}{2}$; center: $(\frac{1}{2}, -1\frac{1}{2})$; $r = \frac{\sqrt{30}}{2} \approx 2.7$



65. $(x - \frac{5}{2})^2 + (y - (-\frac{3}{2}))^2 = 2$; center: $(2\frac{1}{2}, -1\frac{1}{2})$; $r = \sqrt{2} \approx 1.4$



66. $(x + \frac{7}{2})^2 + (y - 2)^2 = 80$

67. $(x - 1)^2 + (y + 3)^2 = 9$

68. $(x - 1)^2 + (y - 3)^2 = 73$

69. function, not one to one

70. function, one to one

71. function, one to one

72. even; y-axis symmetry

73. odd; origin symmetry

74. odd; origin symmetry

75. odd; origin symmetry

76. neither odd nor even

77. neither odd nor even

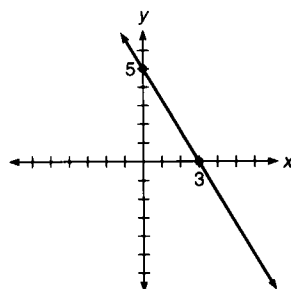
78. $y = \frac{5}{3}x - \frac{35}{3}$ 79. 40 and 55

80. 9% 81. 70

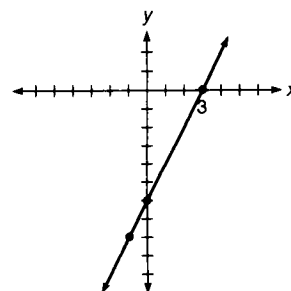
82. approximately generations 15–20, 30–45, 55–60, and 70–75

Chapter 3 test

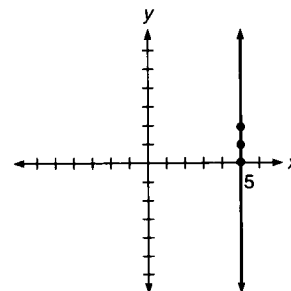
1. $(-3, 10)$, $(0, 5)$, $(3, 0)$



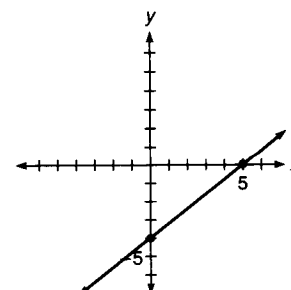
2. $(-1, -8)$, $(0, -6)$, $(3, 0)$



3. $(5, 0)$, $(5, 1)$, $(5, 2)$



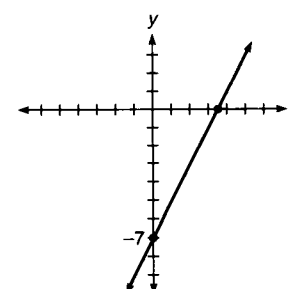
4. $(-5, -8)$, $(0, -4)$, $(5, 0)$



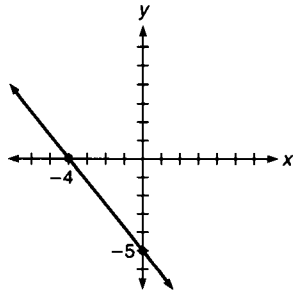
5. $(\frac{1}{2}, 4)$

6. $\sqrt{41}$

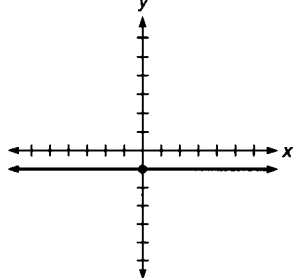
7. $m = 2$; $(3\frac{1}{2}, 0)$; $(0, -7)$



8. $m = -\frac{5}{4}$; $(-4, 0)$, $(0, -5)$



9. y-intercept is -1 ; no x-intercept;
 $m = 0$



10. $-\frac{1}{4}$ 11. $y = -\frac{1}{2}x + 2$

12. $y = -5x - 15$ 13. $y = -5x + 7$

14. $(x, y) = (2, 0)$ 15. $4x + 6y - 39 = 0$

16. $\{(1, 1), (2, \sqrt{3}), (3, \sqrt{5}), (4, \sqrt{7})\}$; a one-to-one function

17. domain: $x \neq 3$; $f(-2) = \frac{2}{5}$; $f(0) = 0$; 3 is not in the domain of f .

$$f(c-3) = \frac{c-3}{c-6}$$

18. $g(x) = \sqrt{6-2x}$; domain: $x \leq 3$;

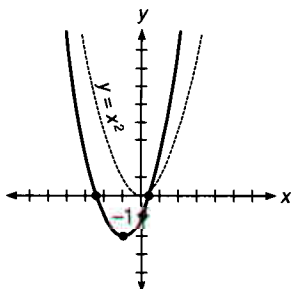
$$g(-2) = \sqrt{10}; g(0) = \sqrt{6}$$

$$g(3) = 0; g(c-3) = \sqrt{12-2c}$$

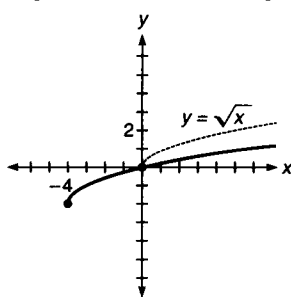
19. domain: R ; $v(-2) = 3$; $v(0) = 3$; $v(3) = -12$; $v(c-3) = 4c - c^2$

20. $4x + 2h$

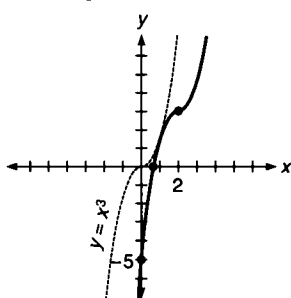
21. vertex: $(-1, -2)$; x-intercept: $(-1 \pm \sqrt{2}, 0)$; y-intercept: $(0, -1)$



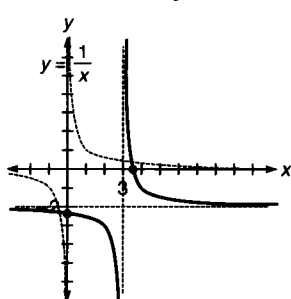
22. graph of $y = \sqrt{x}$ but shifted so new origin is at $(-4, -2)$; intercepts at $(0, 0)$



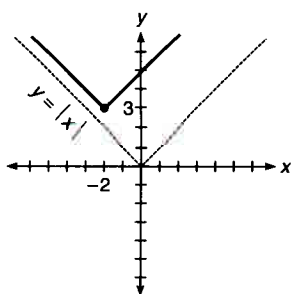
23. graph of $y = x^3$ with origin shifted to $(2, 3)$; x-intercept: $(2 + \sqrt[3]{-3}, 0)$; y-intercept: $(0, -5)$



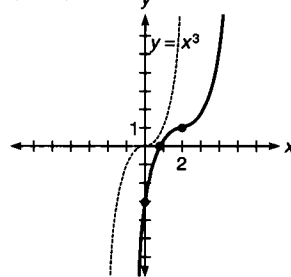
24. graph of $y = \frac{1}{x}$ with origin shifted to $(3, -2)$; x-intercept: $(3\frac{1}{2}, 0)$; y-intercept: $(0, -2\frac{1}{3})$



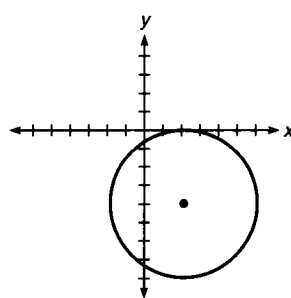
25. graph of $y = |x|$ with origin shifted to $(-2, 3)$; y-intercept: $(0, 5)$



26. graph of $y = x^3$ but with origin moved to $(2, 1)$, vertically scaled by $\frac{1}{2}$; x-intercept: $(2 + \sqrt[3]{-2}, 0)$; y-intercept: $(0, -3)$

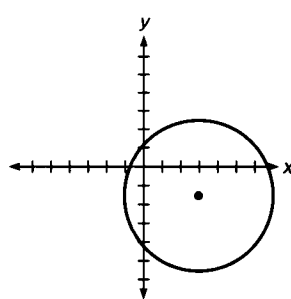


27. center: $(2, -4)$; $r = \sqrt{16} = 4$



28. $(x-3)^2 + (y - (-1\frac{1}{2}))^2 = \frac{65}{4}$; center:

$$(3, -1\frac{1}{2}), r = \frac{\sqrt{65}}{2} \approx 4$$



29. $(x + \frac{7}{2})^2 + (y - 2)^2 = 80$

30. $(x - 1)^2 + (y - 3)^2 = 73$

31. a. -2 ; b. 4 ; c. -3 ; d. -3

32. $-9, -7, -2, 5, 7.5$

33. $-8, -1, 4, 9$

34. -8 to -4 , 1.5 to 6

35. -9 to -8 , -4 to 1.5 , 6 to 10

36. -9 to 10 37. -3.5 to 4

38. even; y-axis symmetry

39. neither even nor odd

40. even; y-axis symmetry

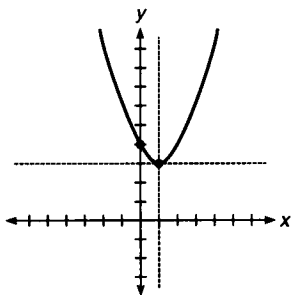
41. 94.7° Celsius; 88.2° Celsius

Chapter 4

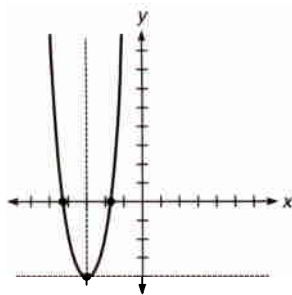
Exercise 4-1

Answers to odd-numbered problems

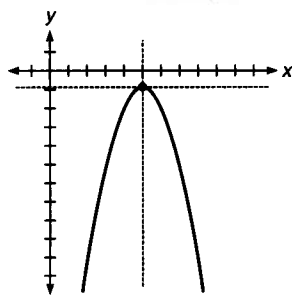
1. vertex: (1,3); intercepts: (0,4)



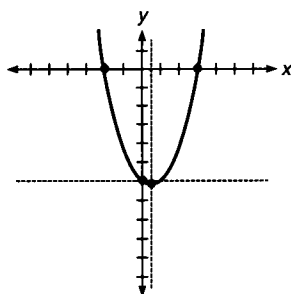
3. vertex: $(-3, -4)$; intercepts: (0,14), $(-3 - \sqrt{2}, 0)$, $(-3 + \sqrt{2}, 0)$



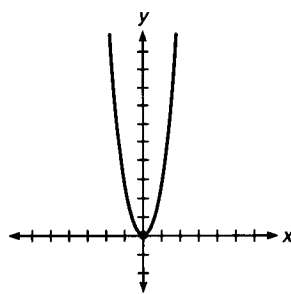
5. vertex: (5, -1); intercepts: (0, -26)



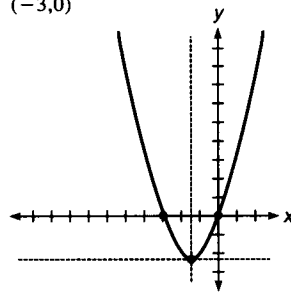
7. $y = (x - \frac{1}{2})^2 - 6\frac{1}{4}$
vertex: $(\frac{1}{2}, -6\frac{1}{4})$; intercepts: (0, -6), (-2, 0), (3, 0)



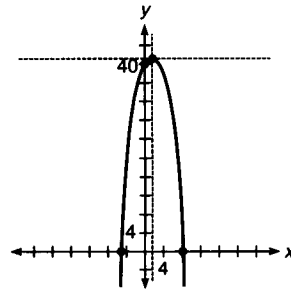
9. vertex: (0,0); intercepts: (0,0)



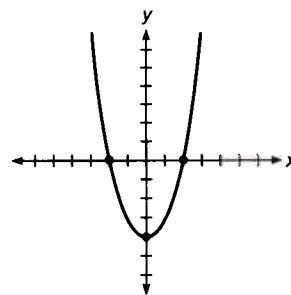
11. $y = (x + \frac{3}{2})^2 - \frac{9}{4}$
vertex: $(-1\frac{1}{2}, -\frac{9}{4})$; intercepts: (0,0), (-3,0)



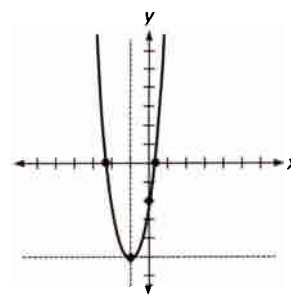
13. $y = -(x - \frac{3}{2})^2 + 42\frac{1}{4}$
vertex: $(1\frac{1}{2}, 42\frac{1}{4})$; intercepts: (0,40), (-5,0), (8,0)



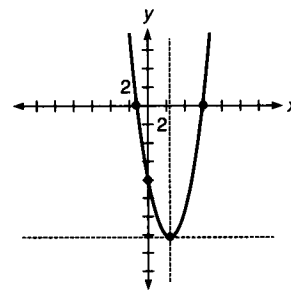
15. $y = x^2 - 4$
vertex: (0, -4); intercepts: (0, -4), (-2, 0), (2, 0)



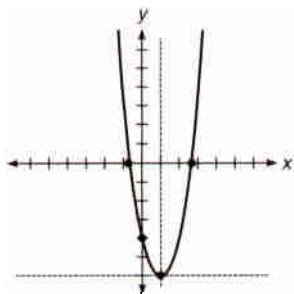
17. $y = 3(x + 1)^2 - 5$
vertex: $(-1, -5)$; intercepts: (0, -2), $(-1 \pm \frac{\sqrt{15}}{3}, 0) \approx (-2.3, 0)$, (0.3, 0)



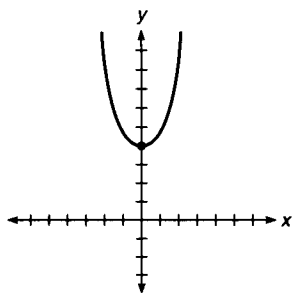
19. $y = (x - \frac{5}{2})^2 - 14\frac{1}{4}$
vertex: $(2\frac{1}{2}, -14\frac{1}{4})$; intercepts: (0, -8), $(\frac{5 \pm \sqrt{57}}{2}, 0) \approx (-1.3, 0)$, (6.3, 0)



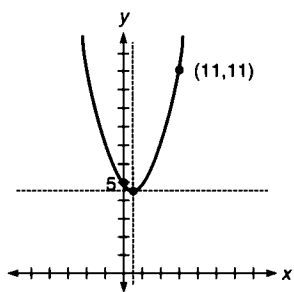
21. $y = 2(x - 1)^2 - 6$

vertex: $(1, -6)$; intercepts: $(0, -4)$,
 $(1 \pm \sqrt{3}, 0) \approx (-0.7, 0)$, $(2.7, 0)$


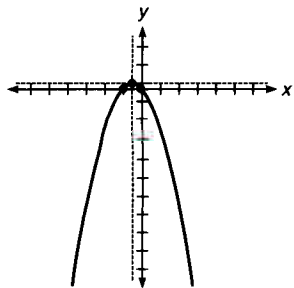
23. vertex: $(0, 4)$; intercepts: $(0, 4)$



25. $y = (x - \frac{1}{2})^2 + 4\frac{3}{4}$

vertex: $(\frac{1}{2}, 4\frac{3}{4})$; intercepts: $(0, 5)$


27. $y = -(x + \frac{1}{2})^2 + \frac{1}{4}$

vertex: $(-\frac{1}{2}, \frac{1}{4})$; intercepts: $(0, 0)$,
 $(-1, 0)$

29. 65 ft, 130 ft, 8,450 ft² 31. a square
with dimension 65 ft; area is 4,225 sq. ft

33. The maximum height of $s = 64$ feet is
reached after $t = 2$ seconds. The object is
thrown, and it returns to earth after 4
seconds.

35. The maximum velocity is 9
m/s, 3 meters from the inside wall.

37. A production of 50 units will produce
the maximum profit of \$1,500. 39. The
numbers are 4 and -4 and the product is
-16. 41. circle

43. $4a(ax^2 + bx + c) = 4a(0)$

$$4a^2x^2 + 4abx + 4ac = 0$$

$$4a^2x^2 + 4abx + 4ac + b^2 = b^2$$

$$4a^2x^2 + 4abx + b^2 = b^2 - 4ac$$

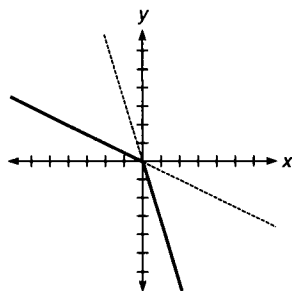
$$(2ax + b)^2 = b^2 - 4ac$$

$$2ax + b = \pm \sqrt{b^2 - 4ac}$$

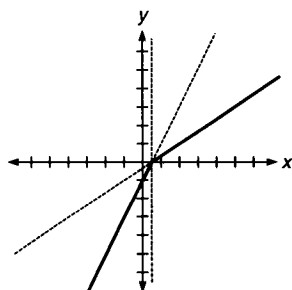
$$2ax = -b \pm \sqrt{b^2 - 4ac}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

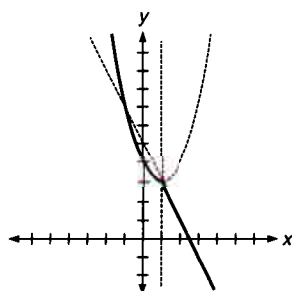
45.



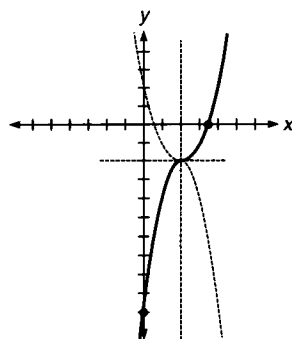
47.



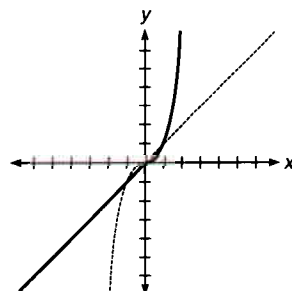
49.



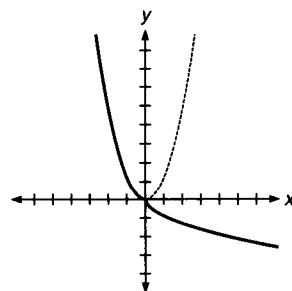
51.



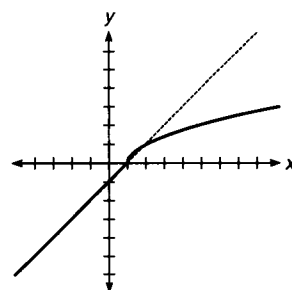
53.



55.



57.



Solutions to skill and review problems

1. Factor: $3x^2 + x - 10$.

$$(3x - 5)(x + 2)$$

2. Factor: $3x^2 + 13x - 10$.

$$(3x - 2)(x + 5)$$

3. Factor:
- $x^4 - 16$
- .

$$(x^2 - 4)(x^2 + 4)$$

$$(x - 2)(x + 2)(x^2 + 4)$$

4. List all the prime divisors of 96.

$$96 = 6 \cdot 16$$

$$= 2 \cdot 3 \cdot 2^4$$

$$= 2^5 \cdot 3$$

2 and 3 are the only prime divisors.

5. List all the positive integer divisors of 96.

$$2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96$$

6. If
- $f(x) = 2x^3 - x^2 - 6x + 20$
- , find
- $f(-2)$
- .

$$f(-2) = 2(-2)^3 - (-2)^2 - 6(-2) + 20$$

$$= 2(-8) - 4 + 12 + 20$$

$$= 12$$

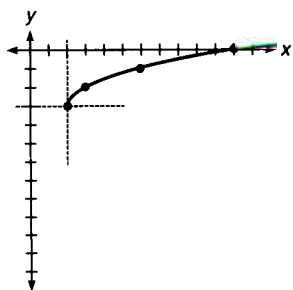
7. Use long division to divide

$$2x^3 - x^2 - 6x + 20 \text{ by } x^2 + 2.$$

$$\begin{array}{r} 2x - 1 \\ x^2 + 2 \overline{) 2x^3 - x^2 - 6x + 20} \\ \underline{2x^3 + 4x} \\ -x^2 - 10x + 20 \\ \underline{-x^2 - 2} \\ -10x + 22 \end{array}$$

Quotient is $2x - 1$, remainder is $-10x + 22$.

8. Graph
- $f(x) = \sqrt{x - 2} - 3$
- .

The graph of $y = \sqrt{x - 2} - 3$ is the graph of $y = \sqrt{x}$ but with the "origin" shifted to $(2, -3)$.

Intercepts:

$$x = 0: y = \sqrt{-2} - 3; \text{ no real solution}$$

so no y-intercept

$$y = 0: 0 = \sqrt{x - 2} - 3$$

$$3 = \sqrt{x - 2}$$

$$9 = x - 2$$

$$11 = x$$

Additional points:

x	3	6
y	-2	-1

9. Compute
- $f \circ g(x)$
- and
- $g \circ f(x)$
- if

$$f(x) = x^4 - 6x^2 + 8 \text{ and } g(x) = \sqrt{x + 1}.$$

$$f \circ g(x) = f(g(x)) = [g(x)]^4 - 6[g(x)]^2 + 8$$

$$= [\sqrt{x + 1}]^4 - 6[\sqrt{x + 1}]^2 + 8$$

$$= [(\sqrt{x + 1})^2]^2 - 6[\sqrt{x + 1}]^2 + 8$$

$$= [x + 1]^2 - 6(x + 1) + 8$$

$$= x^2 + 2x + 1 - 6x - 6 + 8$$

$$= x^2 - 4x + 3$$

$$g \circ f(x) = g(f(x)) = \sqrt{(x^4 - 6x^2 + 8) + 1}$$

$$= \sqrt{x^4 - 6x^2 + 9}$$

$$= \sqrt{(x^2 - 3)^2}$$

$$= |x^2 - 3|$$

Solutions to trial exercise problems

- 5.
- $y = -(x - 5)^2 - 1$

Vertex: $(5, -1)$

Intercepts:

$$x = 0: y = -(-5)^2 - 1 = -26,$$

$$(0, -26)$$

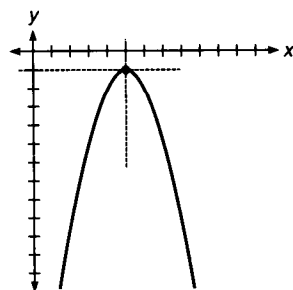
$$y = 0: 0 = -(x - 5)^2 - 1$$

$$1 = -(x - 5)^2; \text{ no real solution}$$

since the left side is positive and the right side is negative. Thus, no x -intercepts.

Additional points:

x	3	4	6	7
y	-5	-2	-2	-5



- 13.
- $y = -x^2 + 3x + 40$

$$y = -(x^2 - 3x) + 40$$

$$\frac{1}{2} \cdot (-3) = -\frac{3}{2}; \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y = -(x^2 - 3x + \frac{9}{4}) + 40 + 1(\frac{9}{4})$$

$$y = -(x - \frac{3}{2})^2 + 42\frac{1}{4}$$

$$\text{Vertex: } (1\frac{1}{2}, 42\frac{1}{4})$$

Intercepts:

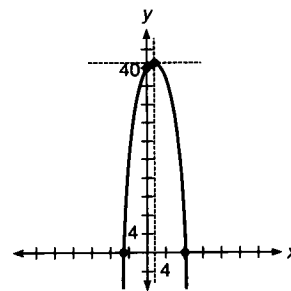
$$x = 0: y = -0^2 + 0 + 40 = 40; (0, 40)$$

$$y = 0: 0 = -x^2 + 3x + 40$$

$$0 = x^2 - 3x - 40$$

$$0 = (x - 8)(x + 5)$$

$$x = -5 \text{ or } 8; (-5, 0), (8, 0)$$



- 21.
- $y = 2x^2 - 4x - 4$

$$y = 2(x^2 - 2x) - 4$$

$$\frac{1}{2} \cdot (-2) = -1; (-1)^2 = 1$$

$$y = 2(x^2 - 2x + 1) - 4 - 2(1)$$

$$y = 2(x - 1)^2 - 6$$

$$\text{Vertex: } (1, -6)$$

Intercepts:

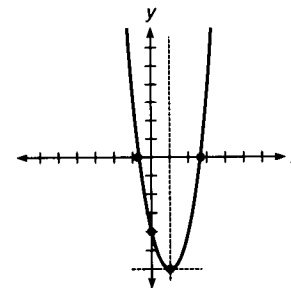
$$x = 0: y = 0 - 0 - 4 = -4; (0, -4)$$

$$y = 0: 0 = 2x^2 - 4x - 4$$

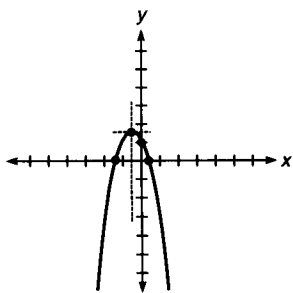
$$0 = x^2 - 2x - 2$$

$$x = 1 \pm \sqrt{3}; (1 \pm \sqrt{3}, 0) \approx$$

$$(-0.7, 0), (2.7, 0)$$

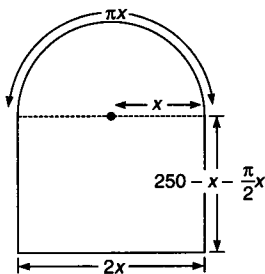


26. $y = -2x^2 - 2x + 1$
 $y = -2(x^2 + x) + 1$
 $\frac{1}{2} \cdot 1 = \frac{1}{2}; (\frac{1}{2})^2 = \frac{1}{4}$
 $y = -2(x^2 + x + \frac{1}{4}) + 1 + 2(\frac{1}{4})$
 $y = -2(x + \frac{1}{2})^2 + \frac{1}{2}$
Vertex: $(-\frac{1}{2}, \frac{1}{2})$
Intercepts:
 $x = 0: y = 0 - 0 + 1 = 1; (0, 1)$
 $y = 0: 0 = -2x^2 - 2x + 1$
 $x = \frac{-1 \pm \sqrt{3}}{2}$
 $(\frac{-1 - \sqrt{3}}{2}, 0), (\frac{-1 + \sqrt{3}}{2}, 0)$
 $\approx (-1.4, 0), (0.4, 0)$



32. The radius of the semicircle is x . The circumference of a circle is $C = 2\pi r$, so the circumference of the semicircle is half this:

$$\frac{C}{2} = \frac{2\pi r}{2} = \pi r = \pi x$$



The base of the figure has length $2x$. Since the total length of chain is 500 ft the other dimension of the rectangle is $\frac{1}{2}(500 - 2x - \pi x) = 250 - x - \frac{\pi}{2}x$. The area of a circle is $A = \pi r^2$, so the area of the semicircle is half this,

$\frac{1}{2}\pi r^2 = \frac{\pi}{2}x^2$. The area of the rectangular part is $2x(250 - x - \frac{\pi}{2}x)$.

Total area is:

$$\begin{aligned} A &= 2x(250 - x - \frac{\pi}{2}x) + \frac{\pi}{2}x^2 \\ &= 500x - 2x^2 - \pi x^2 + \frac{\pi}{2}x^2 \\ &= 500x - x^2(2 + \frac{\pi}{2} - \frac{\pi}{2}) \\ &= 500x - (2 + \frac{\pi}{2})x^2 \\ &\approx -3.571x^2 + 500x \\ &\approx -3.571(x^2 - \frac{500}{3.571}x) \\ &\approx -3.571(x^2 - 140.0x) \\ &\approx -3.571(x^2 - 140.0x + 4,900) + 3.571(4,900) \\ &\approx -3.571(x - 70.0)^2 + 17,496.9 \end{aligned}$$

Vertex: $(70.0, 17,496.9) = (x, A)$

The vertex is the maximum point:

$x = 70.0$, and $A = 17,496.9$.

Thus $x = 70$ ft and area = 17,497 ft².

36. $P = 14I - 0.20I^2$
 $= -0.2(I^2 - \frac{14}{0.2}I)$
 $= -0.2(I^2 - 70I)$
 $= -0.2(I^2 - 70I + 1,225) + 0.2(1,225)$
 $= -0.2(I - 35)^2 + 245$

Vertex: $(35, 245) = (I, P)$

Thus a current of 35 amperes produces a maximum power of 245 watts.

39. Let the numbers be x and $x - 8$. Then their product is $P = x(x - 8)$.

$$P = x^2 - 8x$$

$$P = x^2 - 8x + 16 - 16$$

$$P = (x - 4)^2 - 16$$

Vertex: $(4, -16) = (x, P)$. Thus, $x = 4$, $x - 8 = -4$, $P = -16$. Since the parabola opens upward this is a minimum. Thus, the numbers are 4 and -4 and the product is -16.

42. $ax^2 + bx + c = 0$

$$ax^2 + bx = -c$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$\frac{1}{2}\left(\frac{b}{a}\right) = \frac{b}{2a}, \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}, \text{ so}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = -\frac{c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2|a|}. \text{ Since } \pm |a|$$

$= \pm a$ (this can be believed with a few examples or proven using the

definition of $|a|$ (section 1-2)), we rewrite this as

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

50. $f(x) = \begin{cases} x^2 + 2, & x < 0 \\ -2x^2 + 2, & x \geq 0 \end{cases}$

Graph $y = x^2 + 2$:

Parabola with vertex at $(0, 2)$. Plot

additional points, say $(-2, 6)$ and $(2, 6)$.

Graph $y = -2x^2 + 2$:

Parabola with vertex at $(0, 2)$. Opens downward.

x -intercepts: ($y = 0$): $0 = -2x^2 + 2$

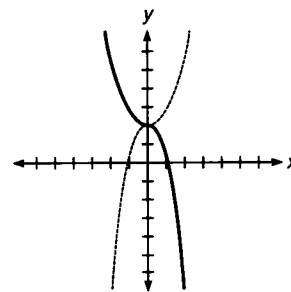
$$2x^2 = 2$$

$$x^2 = 1$$

$$x = \pm 1$$

Darken in the first line for $x < 0$;

darken in the second line for $x \geq 0$.



58. $f(x) = ax^2 + bx + c$
 $= a\left(x^2 + \frac{b}{a}x\right) + c$
 $= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) + c - a\left(\frac{b}{2a}\right)^2$
 $= a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
 $= a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \frac{4ac}{4a} - \frac{b^2}{4a}$
 $= a\left(x - \left(-\frac{b}{2a}\right)\right)^2 + \frac{4ac - b^2}{4a}$

Thus the vertex is at the point

$$\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right).$$

Exercise 4-2

Answers to odd-numbered problems

1. no zeros 3. $\frac{2}{3}$ 5. -11
 7. -2, 5 9. $\pm 6, \pm 3, \pm 2, \pm 1$
 11. $\pm \frac{4}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm 4, \pm 2, \pm 1$
 13. $\pm \frac{5}{2}, \pm \frac{1}{2}, \pm 5, \pm 1$
 15. $\pm \frac{1}{3}, \pm 9, \pm 3, \pm 1$
 17. $\pm 10, \pm 5, \pm 2, \pm 1, \pm \frac{1}{5}, \pm \frac{2}{5}$
 19. $\pm \frac{1}{2}, \pm \frac{1}{4}, \pm \frac{1}{8}, \pm 4, \pm 2, \pm 1$
 21. $\pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 1$
 23. $\pm \frac{4}{5}, \pm \frac{2}{5}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm \frac{1}{2}, \pm 4, \pm 2, \pm 1$
 25. $\pm 2, \pm 1$
 27. a. $3x^3 + 10x^2 + 41x + 164 + \frac{651}{x-4}$
 b. $f(4) = 651$
 29. a. $x^2 - x + 2 + \frac{3}{x-1}$
 b. $f(1) = 3$
 31. a. $x^4 - 3x^3 + 6x^2 - 19x + 57 + \frac{-166}{x+3}$
 b. $h(-2) = -166$
 33. a. $\frac{1}{2}x^2 + \frac{3}{4} + \frac{\frac{3}{2}}{x-6}$
 b. $f(6) = \frac{3}{2}$
 35. a. 0 or 2 positive zeros; 0 or 2 negative zeros
 b. $\pm 6, \pm 3, \pm 2, \pm 1$
 c. -2, -1, 1, 3
 d. $f(x) = (x-3)(x+2)(x-1)(x+1)$
 37. a. 1 or 3 positive zeros; no negative zeros
 b. $\frac{3}{4}, \frac{3}{2}, \frac{1}{4}, \frac{1}{2}, 3, 1$
 c. $\frac{3}{2}, \frac{1}{2}, 1$
 d. $f(x) = 4(x - \frac{3}{2})(x - \frac{1}{2})(x - 1)$
 39. a. 0, 2 or 4 positive zeros; no negative zeros
 b. 81, 27, 9, 3, 1
 c. 3, with multiplicity 2
 d. $f(x) = (x-3)^2(x^2 - 2x + 9)$
 41. a. one positive zero; one negative zero
 b. $\pm 1, \pm 5$ c. -1
 d. $f(x) = (x+1)(x^2 - x + 1)$
 $(x - \sqrt[3]{5})(x^2 + \sqrt[3]{5}x + (\sqrt[3]{5})^2)$
 e. irrational zero: $\sqrt[3]{5}$
 43. a. 0 or 2 positive zeros; 0 or 2 negative zeros
 b. $\pm \frac{2}{3}, \pm \frac{1}{3}, \pm 6, \pm 3, \pm 2, \pm 1$
 c. $\frac{1}{3}$ and -2
 d. $f(x) = 3(x - \frac{1}{3})(x + 2)$
 $(x - \sqrt{3})(x + \sqrt{3})$
 e. $\pm \sqrt{3}$

45. a. 0 or 2 positive zeros; one negative zero
 b. $\pm 1, \pm 2$
 e. The function f has one negative irrational zero between -1 and 0. It has 0 or 2 positive irrational zeros between 0 and 2.
 47. a. one positive zero; 1 or 3 negative zeros
 b. $\pm 1, \pm 3$ c. -3, 1
 d. $f(x) = 2(x+3)(x-1)(x^2 + x + 1)$
 49. a. 0 or 2 positive zeros; 0 or 2 negative zeros
 b. $\pm \frac{1}{3}, \pm 1$
 c. 1, -1 (mult 2), $\frac{1}{3}$
 d. $f(x) = 3(x-1)(x+1)^2(x - \frac{1}{3})$
 51. a. one positive zero; 0 or 2 negative zeros
 b. $\pm 1, \pm 2, \pm 5, \pm 10$ c. -2
 d. $f(x) = (x+2)\left(x - \frac{1-\sqrt{21}}{2}\right)\left(x - \frac{1+\sqrt{21}}{2}\right)$
 e. irrational zeros: $\frac{1 \pm \sqrt{21}}{2}$
 53. a. 0 or 2 positive zeros; 1 or 3 negative zeros
 b. $\pm 1, \pm 2, \pm 4$ c. -2
 d. $f(x) = (x+2)(x^4 - 2x^3 + x + 2)$
 e. There are 0 or 2 positive irrational zeros between the values 0 and 2. There are 0 or 2 negative irrational zeros between the values 0 and -1.
 55. a. 0 or 2 positive zeros; 0 or 2 negative zeros
 b. $\pm \frac{1}{3}, \pm \frac{1}{9}, \pm 1, \pm 3, \pm 9$
 c. $\pm \frac{1}{3}, \pm 3$
 d. $f(x) = 9(x - \frac{1}{3})(x + \frac{1}{3})(x-3)(x+3)$
 57. a. no positive zeros; 1, 3 or 5 negative zeros
 b. $-\frac{1}{4}, -\frac{1}{2}, -1, -2, -4$
 c. $-\frac{1}{2}$ (mult 2), -1
 d. $f(x) = 4(x+1)(x + \frac{1}{2})^2(x^2 + 2x + 4)$

59.

1	0	0	0	-3
	$\sqrt[4]{3}$	$\sqrt[4]{3^2}$	$\sqrt[4]{3^3}$	3
$\sqrt[4]{3}$	1	$\sqrt[4]{3}$	$\sqrt[4]{3^2}$	0

Note: $\sqrt[4]{3^4} = 3$

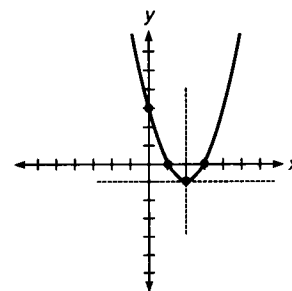
$$x^4 - 3 = (x - \sqrt[4]{3})(x^3 + \sqrt[4]{3}x^2 + \sqrt[4]{3^2}x + \sqrt[4]{3^3})$$

$$x^4 - 3 = (x - \sqrt[4]{3})(x^3 + \sqrt[4]{3}x^2 + \sqrt[4]{9}x + \sqrt[4]{27})$$

61. $\pm \frac{b}{a}, \pm \frac{d}{a}, \pm \frac{e}{a}, \pm \frac{bd}{a}, \pm \frac{be}{a}, \pm \frac{de}{a}, \pm \frac{bde}{a},$
 $\pm \frac{b}{c}, \pm \frac{d}{c}, \pm \frac{e}{c}, \pm \frac{bd}{c}, \pm \frac{be}{c}, \pm \frac{de}{c}, \pm \frac{bde}{c},$
 $\pm \frac{b}{ac}, \pm \frac{d}{ac}, \pm \frac{e}{ac}, \pm \frac{bd}{ac}, \pm \frac{be}{ac}, \pm \frac{de}{ac}, \pm \frac{bde}{ac},$
 $\pm b, \pm d, \pm e, \pm bd, \pm be, \pm de, \pm bde,$
 $\pm \frac{1}{a}, \pm \frac{1}{c}, \pm \frac{1}{ac}$
 63. $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0,$
 $a_4 \neq 0$

Solutions to skill and review problems

1. $f(x) = (x-2)^2 - 1$
 Graph of $y = x^2$ shifted right 2 units and down 1 unit.
 Vertex: (2, -1)
 Intercepts:
 $x = 0: y = (-2)^2 - 1 = 3; (0, 3)$
 $y = 0: 0 = (x-2)^2 - 1$
 $1 = (x-2)^2$
 $\pm 1 = x-2$
 $2 \pm 1 = x$
 $1, 3 = x; (1, 0), (3, 0)$



2. $f(x) = x^2 + x - 4$

$$f(x) = x^2 + x + \frac{1}{4} - 4 - \frac{1}{4}$$

$$f(x) = (x + \frac{1}{2})^2 - 4\frac{1}{4}$$

Graph of $y = x^2$ shifted left $\frac{1}{2}$ unit,
down $4\frac{1}{4}$ units.

Vertex: $(-\frac{1}{2}, -4\frac{1}{4})$

Intercepts:

$$x = 0: y = 0^2 + 0 - 4 = -4; (0, -4)$$

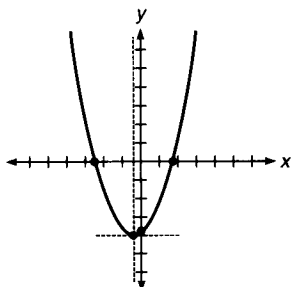
$$y = 0: 0 = (x + \frac{1}{2})^2 - 4\frac{1}{4}$$

$$\frac{17}{4} = (x + \frac{1}{2})^2$$

$$\pm\sqrt{\frac{17}{4}} = x + \frac{1}{2}$$

$$-\frac{1}{2} \pm \frac{\sqrt{17}}{2} = x$$

$$-2.6, 1.6 = x; (-2.6, 0), (1.6, 0)$$



3. $f(x) = |x - 2| - 3$

Graph of $y = |x|$ translated; origin
(2, -3).

Intercepts:

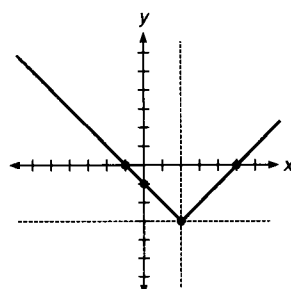
$$x = 0: y = |-2| - 3 = -1; (0, -1)$$

$$y = 0: 0 = |x - 2| - 3$$

$$3 = |x - 2|$$

$$x - 2 = 3 \text{ or } x - 2 = -3$$

$$x = 5 \text{ or } x = -1; (-1, 0), (5, 0)$$



4. $f(x) = x^3 - 1$

Graph of $y = x^3$ shifted down 1 unit;
origin (0, -1).

Intercepts:

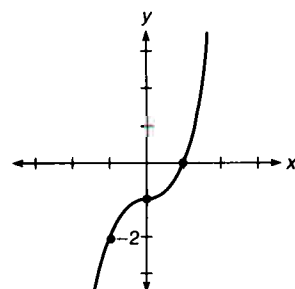
$$x = 0: y = 0^3 - 1 = -1; (0, -1)$$

$$y = 0: 0 = x^3 - 1$$

$$1 = x^3$$

$$1 = x; (1, 0)$$

Additional point: $(-1, -2)$



5. Find all zeros of $f(x) = 2x^5 + 7x^4 + 2x^3 - 11x^2 - 4x + 4$

If $\frac{p}{q}$ is a rational zero, p divides 4 and q divides 2:

$\pm\frac{4}{2}, \pm\frac{4}{1}, \pm\frac{2}{2}, \pm\frac{2}{1}, \pm\frac{1}{2}, \pm\frac{1}{1}$ or $\pm 1, \pm 2, \pm 4, \pm\frac{1}{2}$.

2	7	2	-11	-4	4
		2	9	11	-4
1	2	9	11	0	-4
					0

$(x - 1)$ is a factor of $f(x)$

$$f(x) = (x - 1)(2x^4 + 9x^3 + 11x^2 - 4)$$

	2	9	11	0	-4
		-2	-7	-4	4
-1	2	7	4	-4	0

$(x + 1)$ is a factor of $f(x)$

$$f(x) = (x - 1)(x + 1)(2x^3 + 7x^2 + 4x - 4)$$

	2	7	4	-4
		-4	-6	4
-2	2	3	-2	0

$(x + 2)$ is a factor of $f(x)$

$$f(x) = (x - 1)(x + 1)(x + 2)(2x^2 + 3x - 2)$$

$$f(x) = (x - 1)(x + 1)(x + 2)(2x - 1)(x + 2)$$

$$f(x) = 2(x - 1)(x + 1)(x - \frac{1}{2})(x + 2)^2$$

Rational zeros are $\pm 1, \frac{1}{2}, -2$

(multiplicity 2), -2 (multiplicity 2)

6. Solve $|2x - 3| < 9$

$$-9 < 2x - 3 < 9$$

$$-6 < 2x < 12$$

$$-3 < x < 6$$

Solutions to trial exercise problems

21. $2 - 3x^2 + 4x^3$; rewrite as $4x^3 - 3x^2 + 2$: In $\frac{p}{q}$ p divides 2 and q divides 4, so we have $\pm\frac{2}{4}, \pm\frac{2}{2}, \pm\frac{2}{1}, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{1}{1}$ or $\pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 1$.

25. $8x^3 - 8x + 16$; rewrite as $8(x^3 - x + 2)$ and focus on $x^3 - x + 2$. In $\frac{p}{q}$ p divides 2 and q divides 1, so we have $\pm\frac{2}{1}, \pm\frac{1}{1}$ or $\pm 2, \pm 1$.

33. $f(x) = \frac{1}{2}x^3 - 3x^2 + \frac{3}{4}x - 3$

a. $x - 6$; b. $f(6)$

a.

	$\frac{1}{2}$	-3	$\frac{3}{4}$	-3
		3	0	$\frac{9}{2}$
6	$\frac{1}{2}$	0	$\frac{3}{4}$	$\frac{3}{2}$

$$\frac{1}{2}x^3 - 3x^2 + \frac{3}{4}x - 3$$

$$\text{So } \frac{1}{2}x^3 - 3x^2 + \frac{3}{4}x - 3$$

$$x - 6$$

$$= \frac{1}{2}x^2 + \frac{3}{4} + \frac{3}{2}$$

b. $f(6) = \frac{3}{2}$

35. a. $f(x) = x^4 - x^3 - 7x^2 + x + 6$

There are 2 sign changes in $f(x)$; the number of positive roots is 0 or 2.

$$f(-x) = x^4 + x^3 - 7x^2 - x + 6$$

There are 2 sign changes in $f(-x)$; the number of negative roots is 0 or 2.

b. Possible rational zeros are $\pm 6, \pm 3, \pm 2, \pm 1$.

Test the rational zeros and factor.

	1	-1	-7	1	6
		3	6	-3	-6
3	1	2	-1	-2	0

$(x - 3)$ is a factor of $f(x)$

$$f(x) = (x - 3)(x^3 + 2x^2 - x - 2)$$

	1	2	-1	-2
		-2	0	2
-2	1	0	-1	0

$(x + 2)$ is a factor of $f(x)$

$$f(x) = (x - 3)(x + 2)(x^2 - 1)$$

$$f(x) = (x - 3)(x + 2)(x - 1)(x + 1)$$

Factor $x^2 - 1$

c. The rational zeros are $-2, -1, 1, 3$
(From the factors of part d).

d. $f(x) = (x - 3)(x + 2)(x - 1)(x + 1)$

41. a. $f(x) = x^6 - 4x^3 - 5$

There is one sign change so there is one positive root.

$$f(-x) = x^6 + 4x^3 - 5$$

There is one sign change so there is one negative root.

b. possible rational zeros: $\pm 1, \pm 5$. We can factor the expression for $f(x)$.

$$x^6 - 4x^3 - 5 = (x^3 - 5)(x^3 + 1)$$

$$= (x^3 - 5)(x + 1)$$

$$(x^2 - x + 1)$$

$x^3 - 5$ has the irrational zero $\sqrt[3]{5}$.

and $x^2 - x + 1$ has only complex

zeros, so this expression cannot be factored further using rational zeros.

c. rational zeros: -1 ; irrational zero: $\sqrt[3]{5}$

d. $f(x) = (x^3 - 5)(x + 1)(x^2 - x + 1)$

57. a. $f(x) = 4x^5 + 16x^4 + 37x^3 + 43x^2 + 22x + 4$

no sign changes; no positive roots.

$$f(-x) = -4x^5 + 16x^4 - 37x^3 + 43x^2 - 22x + 4$$

five sign changes; 1, 3, or 5 negative roots.

b. possible rational zeros: $-\frac{1}{4}, -\frac{1}{2}, -1, -2, -4$; test rational zeros and factor

	4	16	37	43	22	4
		-4	-12	-25	-18	-4
-1	4	12	25	18	4	0

$(x + 1)$ is a factor of $f(x)$

$$f(x) = (x + 1)$$

$$(4x^4 + 12x^3 + 25x^2 + 18x + 4)$$

	4	12	25	18	4
		-2	-5	-10	-4
$-\frac{1}{2}$	4	10	20	8	0

$(x + \frac{1}{2})$ is a factor of $f(x)$

$$f(x) = (x + 1)(x + \frac{1}{2})$$

$$(4x^3 + 10x^2 + 20x + 8)$$

$$f(x) = (2)(x + 1)(x + \frac{1}{2})$$

$$(2x^3 + 5x^2 + 10x + 4)$$

Common factor of 2

	2	5	10	4
		-1	-2	-4
$-\frac{1}{2}$	2	4	8	0

$(x + \frac{1}{2})$ is a factor for the second time

$(x + \frac{1}{2})^2$ is a factor of $f(x)$

$$f(x) = 2(x + 1)(x + \frac{1}{2})^2(2x^2 + 4x + 8)$$

$$f(x) = 2(2)(x + 1)(x + \frac{1}{2})^2(x^2 + 2x + 4)$$

Common factor of 2

$x^2 + 2x + 4$ is prime on R

c. rational zeros: $-\frac{1}{2}$ (mult 2), -1

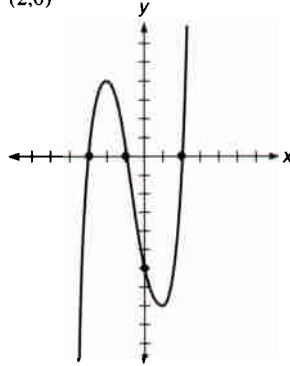
d. $f(x) = 4(x + 1)(x + \frac{1}{2})^2(x^2 + 2x + 4)$

Exercise 4-3

Answers to odd-numbered problems

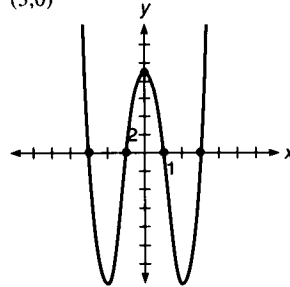
1. $y = (x - 2)(x + 1)(x + 3)$

intercepts: $(0, -6), (-3, 0), (-1, 0), (2, 0)$



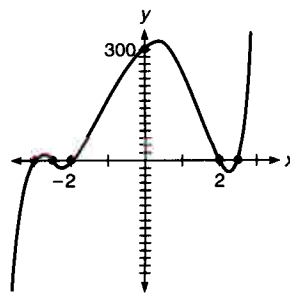
3. $y = (x - 1)(x + 1)(x - 3)(x + 3)$

intercepts: $(0, 9), (-3, 0), (-1, 0), (1, 0), (3, 0)$



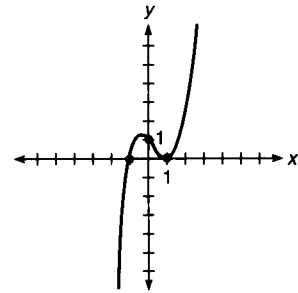
5. $y = (x - 2)(x + 2)(2x - 5)(2x + 5)$

$(x + 3)$
intercepts: $(300, 0), (-3, 0), (-2.5, 0), (-2, 0), (2, 0), (2.5, 0)$



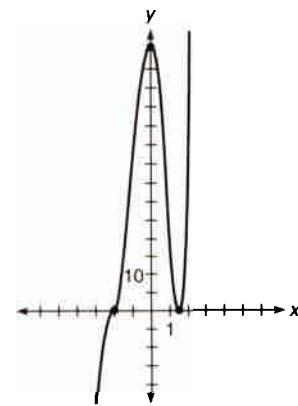
7. $y = (x - 1)^2(x + 1)$

intercepts: $(0, 1), (-1, 0), (1, 0)$



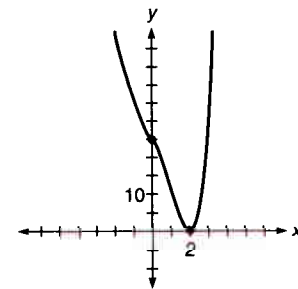
9. $y = (x + 2)^3(2x - 3)^2$

intercepts: $(0, 72), (-2, 0), (1\frac{1}{2}, 0)$

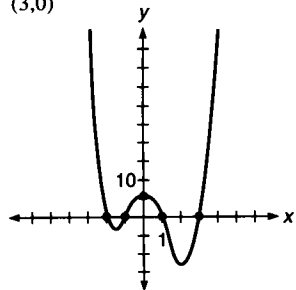


11. $y = (x - 2)^2(x^2 + 3x + 6)$

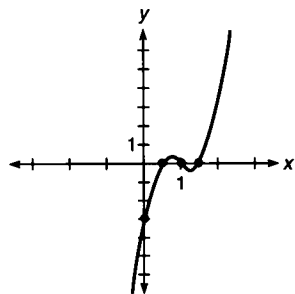
intercepts: $(0, 24), (2, 0)$



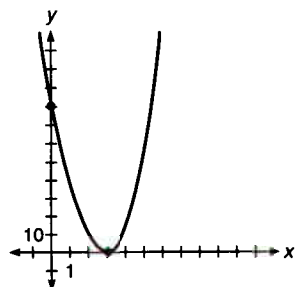
13. $y = (x - 3)(x - 1)(x + 1)(x + 2)$
intercepts: $(0, 6)$, $(-2, 0)$, $(-1, 0)$, $(1, 0)$, $(3, 0)$



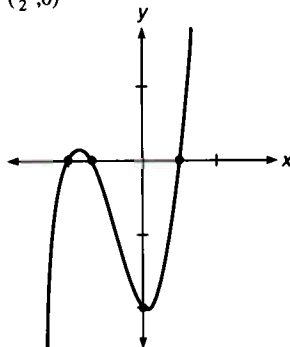
15. $y = (x - 1)(2x - 1)(2x - 3)$
intercepts: $(0, -3)$, $(1, 0)$, $(\frac{1}{2}, 0)$, $(\frac{3}{2}, 0)$



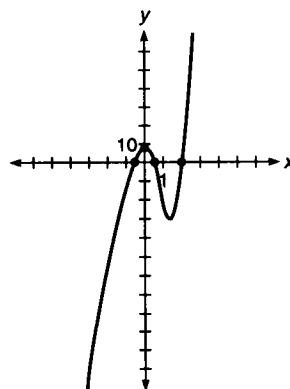
17. $y = (x - 3)^2(x^2 - 2x + 9)$
intercepts: $(0, 81)$, $(3, 0)$



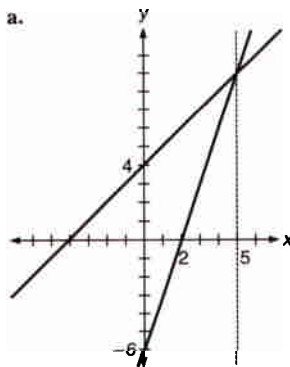
19. $y = (x + 1)(3x + 2)(2x - 1)$
intercepts: $(0, -2)$, $(-1, 0)$, $(-\frac{2}{3}, 0)$, $(\frac{1}{2}, 0)$



21. $y = (2x - 1)(2x + 1)(x - 2)$
 $(x^2 + 2x + 4)$
intercepts: $(0, 8)$, $(-\frac{1}{2}, 0)$, $(\frac{1}{2}, 0)$, $(2, 0)$

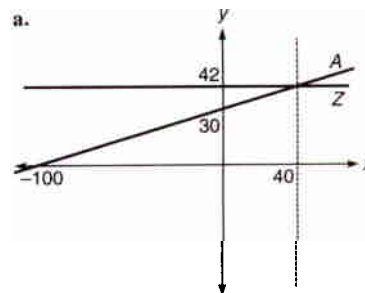


23. a.



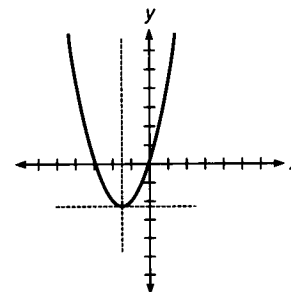
- b. At least five items must be produced to break even or make a profit.

25. a.

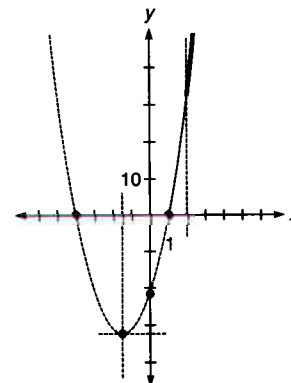


- b. When $x < 40$ A is cheaper; when $x > 40$ Z is cheaper.

27. $A(x) = x(x + 3)$, or $A(x) = x^2 + 3x$ describes area A as a function of width x.



29. $A(x) = 6x^2 + 16x - 24$ in²
Graph $y = 6x^2 + 16x - 24$.

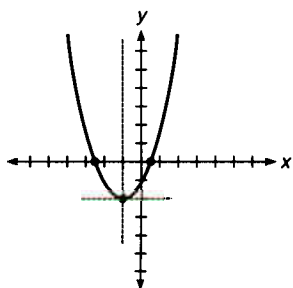


The value of x must be greater than 2 because width is $x - 2$, and must be a positive quantity.

31. a. $f(x) = 5x$
 $2f(x) = 2(5x) = 10x$
 $f(2x) = 5(2x) = 10x$
 Thus, $2f(x) = f(2x)$ for this function.
- b. $f(x) = -3x$
 $kf(x) = k(-3x) = -3kx$
 $f(kx) = -3(kx) = -3kx$
 Thus, $kf(x) = f(kx)$.
- c. $f(x) = x^2 - 2x - 8$; assume f is 3-scalable.
 Then, $3f(x) = 3(x^2 - 2x - 8) = 3x^2 - 6x - 24$
 Also, $f(3x) = (3x)^2 - 2(3x) - 8 = 9x^2 - 6x - 8$.
 If $3f(x) = f(3x)$, then
 $3x^2 - 6x - 24 = 9x^2 - 6x - 8$
 $6x^2 + 16 = 0$
 $6x^2 = -16$
 This has no real solutions for x , and in any case even if there were solutions the solution set would have to be all real numbers if $3f(x) = f(3x)$ is to be true for all real numbers. Thus, f is not 3-scalable.
33. 0.46682
35. 0.69996, -1.72043
37. 0.69091, -0.72720, -2.65081
39. ± 0.61803 , ± 1 , ± 1.30278 , ± 1.61803 , ± 2.30278

Solutions to skill and review problems

1. Graph $f(x) = x^2 + 2x - 1$.
 $y = x^2 + 2x - 1$
 $y = x^2 + 2x + 1 - 1 - 1$
 $y = (x + 1)^2 - 2$
 parabola; vertex: $(-1, -2)$; intercepts:
 $x = 0$: $y = 0^2 + 2(0) - 1 = -1$;
 $(0, -1)$
 $y = 0$: $0 = (x + 1)^2 - 2$
 $(x + 1)^2 = 2$
 $x + 1 = \pm\sqrt{2}$
 $x = -1 \pm\sqrt{2}$; $\approx (-2.4, 0)$, $(0.4, 0)$



2. Solve $\left| \frac{2x-3}{4} \right| \geq \frac{1}{2}$.
 $\frac{2x-3}{4} \geq \frac{1}{2}$ or $\frac{2x-3}{4} \leq -\frac{1}{2}$
 $4\left(\frac{2x-3}{4}\right) \geq 4\left(\frac{1}{2}\right)$ or $4\left(\frac{2x-3}{4}\right) \leq 4\left(-\frac{1}{2}\right)$
 $2x-3 \geq 2$ or $2x-3 \leq -2$
 $2x \geq 5$ or $2x \leq 1$
 $x \geq \frac{5}{2}$ or $x \leq \frac{1}{2}$

3. Solve $x^{-2} - x^{-1} - 12 = 0$.
 $x^2(x^{-2} - x^{-1} - 12) = x^2(0)$
 $x^0 - x^1 - 12x^2 = 0$
 $12x^2 + x - 1 = 0$
 $(4x-1)(3x+1) = 0$
 $4x-1 = 0$ or $3x+1 = 0$
 $4x = 1$ or $3x = -1$
 $x = \frac{1}{4}$ or $x = -\frac{1}{3}$

4. Solve $\sqrt{2x-2} = x-5$.
 $(\sqrt{2x-2})^2 = (x-5)^2$
 $2x-2 = x^2 - 10x + 25$
 $0 = x^2 - 12x + 27$
 $0 = (x-3)(x-9)$
 $x-3 = 0$ or $x-9 = 0$
 $x = 3$ or $x = 9$
 The value 3 does not check, so the answer is 9.

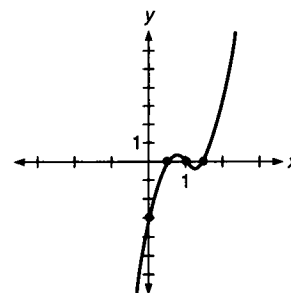
5. Combine $\frac{3}{x-1} - \frac{2}{x+1} + \frac{1}{x}$.
 $\frac{3(x+1) - 2(x-1) + 1}{(x-1)(x+1)} + \frac{1}{x}$
 $\frac{x+5}{x^2-1} + \frac{1}{x}$
 $\frac{x(x+5) + (x^2-1)}{x(x^2-1)}$
 $\frac{2x^2+5x-1}{x^3-x}$

6. Simplify $\sqrt[3]{\frac{4x^2y^7}{3z^8}}$
 $= \frac{y^2\sqrt[3]{4x^2y}}{z^2\sqrt[3]{3z^2}} \cdot \frac{\sqrt[3]{3^2z}}{\sqrt[3]{3^2z}} =$
 $\frac{y^2\sqrt[3]{4(3^2)x^2yz}}{z^2\sqrt[3]{3^3z^3}} =$
 $\frac{y^2\sqrt[3]{36x^2yz}}{z^2(3z)} = \frac{y^2\sqrt[3]{36x^2yz}}{3z^3}$

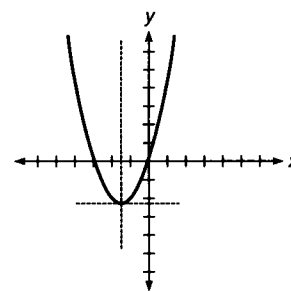
7. Rewrite $|5 - 2\pi|$ without absolute value symbols.
 $5 - 2\pi < 0$ so $|5 - 2\pi|$
 $= -(5 - 2\pi) = 2\pi - 5$

Solutions to trial exercise problems

15. $g(x) = 4x^3 - 12x^2 + 11x - 3$
 Using possible rational zeros and synthetic division we find that $y = (x-1)(2x-1)(2x-3)$.
 Intercepts:
 $x = 0$: $y = 0 - 0 + 0 - 3 = -3$;
 $(0, -3)$
 $y = 0$: $0 = (x-1)(2x-1)(2x-3)$
 $x = 1, \frac{1}{2}, \frac{3}{2}$; $(1, 0)$, $(\frac{1}{2}, 0)$, $(\frac{3}{2}, 0)$
 Additional points: $(0.75, 0.19)$, $(1.25, -0.19)$, $(2, 3)$



27. Let $x =$ width; then length is $x + 3$.
 The area A is the product of length and width. Thus, $A(x) = x(x + 3)$, or $A(x) = x^2 + 3x$ describes area A as a function of width x .
 Graph: $y = x^2 + 3x + \frac{9}{4} - \frac{9}{4}$
 $y = (x + 1\frac{1}{2})^2 - 2\frac{1}{4}$
 Parabola; vertex at $(-1\frac{1}{2}, -2\frac{1}{4})$;
 intercepts:
 $x = 0$: $y = x^2 + 3x$
 $y = 0^2 + 0 = 0$; $(0, 0)$
 $y = 0$: $0 = x(x + 3)$
 $x = 0$ or -3 ; $(0, 0)$, $(-3, 0)$



32. a.
- $f(a) = 5a$
- , and
- $f(b) = 5b$

$$f(a) + f(b) = 5a + 5b$$

$$f(a + b) = 5(a + b) = 5a + 5b$$

$$\text{Thus, } f(a) + f(b) = f(a + b)$$

- b.
- $f(a) = -3a + 1$
- ,
- $f(b) = -3b + 1$

$$f(a) + f(b) = -3a + 1 - 3b + 1 \\ = -3a - 3b + 2$$

$$f(a + b) = -3(a + b) + 1 = -3a \\ - 3b + 1$$

$$\text{Thus } f(a + b) \neq f(a) + f(b)$$

- c. Show that the function

$$f(x) = x^2 - 2x - 8 \text{ is not additive.}$$

$$f(a) = a^2 - 2a - 8,$$

$$f(b) = b^2 - 2b - 8$$

$$f(a) + f(b)$$

$$= (a^2 - 2a - 8) + (b^2 - 2b - 8)$$

$$= a^2 - 2a + b^2 - 2b - 16$$

$$f(a + b)$$

$$= (a + b)^2 - 2(a + b) - 8$$

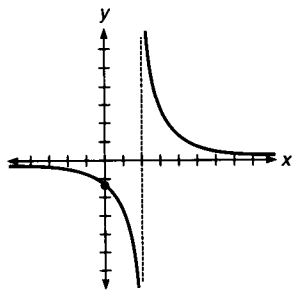
$$= a^2 + 2ab + b^2 - 2a - 2b - 8,$$

$$\text{which is not equal to } f(a) + f(b).$$

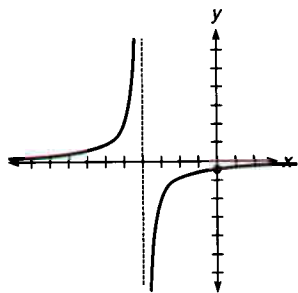
Exercise 4-4

Answers to odd-numbered problems

1. graph of
- $y = \frac{1}{x}$
- shifted right 2 units,
-
- vertically scaled 3 units; intercepts:
-
- $(0, -1\frac{1}{2})$
- ; asymptotes:
- $x = 2$
- ,
- $y = 0$

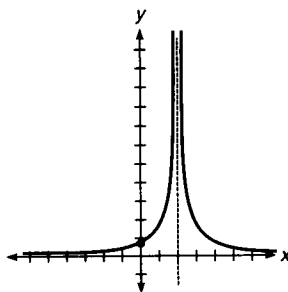


3. graph of
- $y = \frac{1}{x}$
- shifted left 4 units,
-
- vertically scaled
- -2
- units; intercepts:
-
- $(0, -\frac{1}{2})$
- ; asymptotes:
- $x = -4$
- ,
- $y = 0$

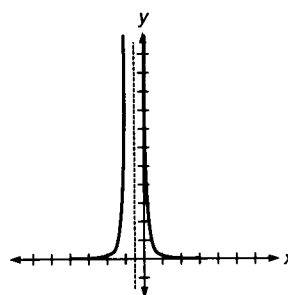


5. graph of
- $y = \frac{1}{x^2}$
- shifted right 2 units,

vertically scaled 3 units; intercepts:

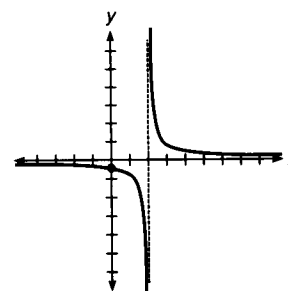
 $(0, \frac{3}{4})$; asymptotes: $x = 2$, $y = 0$ 

7. graph of
- $y = \frac{1}{x^4}$
- shifted left
- $\frac{1}{2}$
- unit;

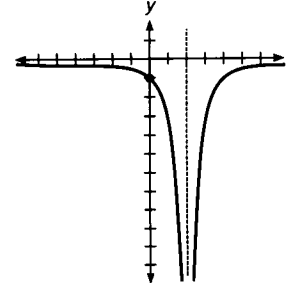
intercepts: $(0, 16)$;asymptotes: $x = -\frac{1}{2}$, $y = 0$ 

9. graph of
- $y = \frac{1}{x^3}$
- shifted right 2 units,

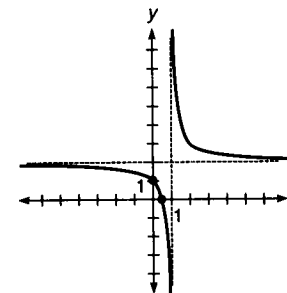
vertically scaled 3 units; intercepts:

 $(0, -\frac{3}{8})$; asymptotes: $x = 2$, $y = 0$ 

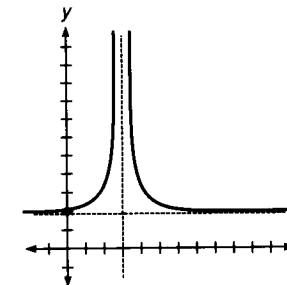
11. graph of
- $y = \frac{1}{x^2}$
- shifted right 2 units,

vertically scaled -4 units; intercepts: $(0, -1)$; asymptotes: $x = 2$, $y = 0$ 

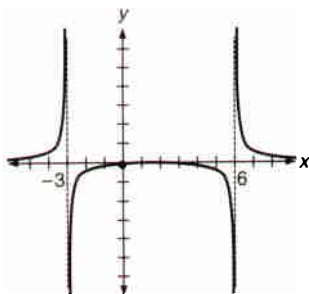
13. graph of
- $y = \frac{1}{x}$
- shifted right 1 unit, up

2 units; intercepts: $(0, 1)$, $(\frac{1}{2}, 0)$;asymptotes: $x = 1$, $y = 2$ 

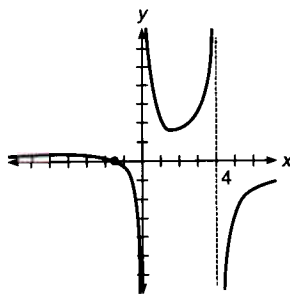
15. graph of
- $y = \frac{1}{x^2}$
- shifted right 3 units, up

2 units; intercepts: $(0, 2\frac{1}{9})$;asymptotes: $x = 3$, $y = 2$ 

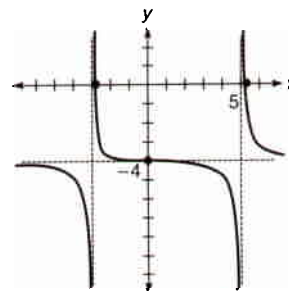
17. intercepts: $(0, -\frac{1}{6})$;
asymptotes: $x = -3, x = 6, y = 0$



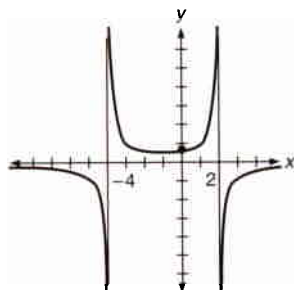
25. intercepts: $(-1\frac{1}{2}, 0)$;
asymptotes: $x = 0, x = 4, y = 0$



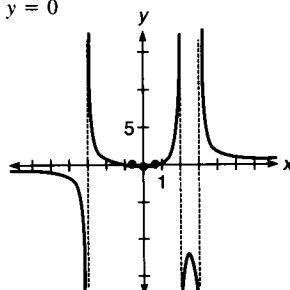
33. intercepts: $(0, -4), (\frac{9 \pm \sqrt{1,041}}{8}, 0)$;
asymptotes: $x = -3, x = 5, y = -4$



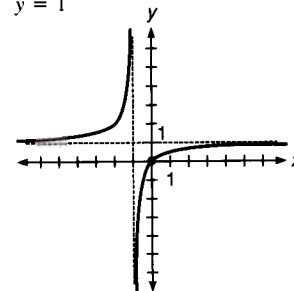
19. intercepts: $(0, \frac{1}{2})$; asymptotes: $x = -4, x = 2, y = 0$



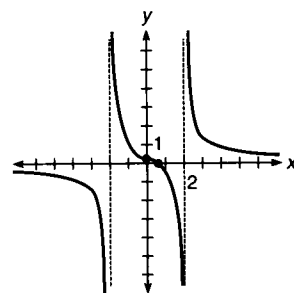
27. intercepts: $(0, -\frac{1}{18}), (\pm 0.6, 0)$;
asymptotes: $x = -3, x = 2, x = 3, y = 0$



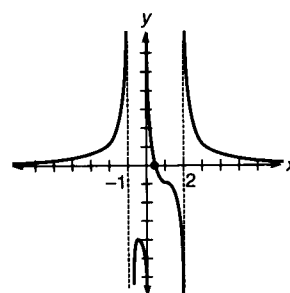
35. intercepts: $(0, 0)$; asymptotes: $x = -1, y = 1$



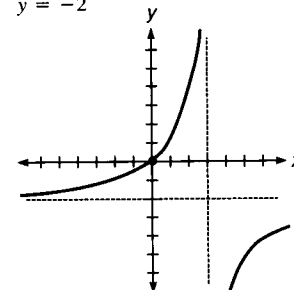
21. intercepts: $(0, \frac{1}{4}), (\frac{1}{2}, 0)$;
asymptotes: $x = \pm 2, y = 0$



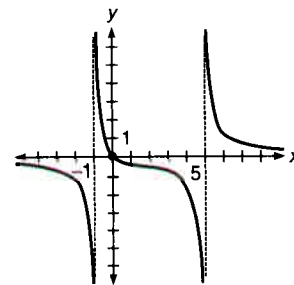
29. intercepts: $(\frac{1}{3}, 0)$; asymptotes: $x = -1, x = 0, x = 2, y = 0$



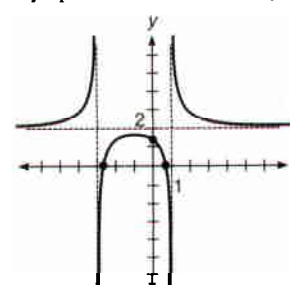
37. intercepts: $(0, 0)$; asymptotes: $x = 3, y = -2$



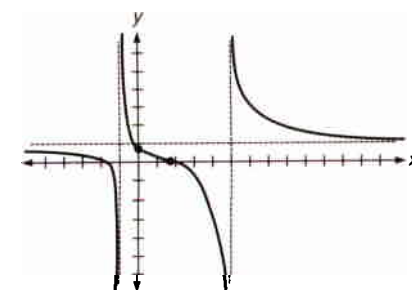
23. intercepts: $(0, 0)$; asymptotes: $x = -1, x = 5, y = 0$



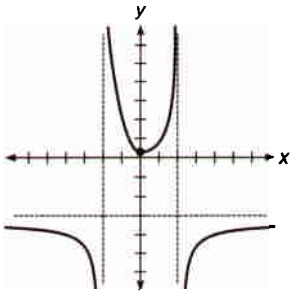
31. intercepts: $(0, 1\frac{1}{3}), (-1 \pm \sqrt{3}, 0)$;
asymptotes: $x = -3, x = 1, y = 2$



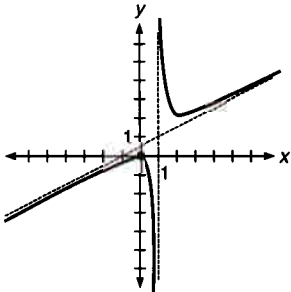
39. intercepts: $(0, \frac{2}{3}), (\pm \sqrt{3}, 0)$; asymptotes: $x = -1, x = 5, y = 1$



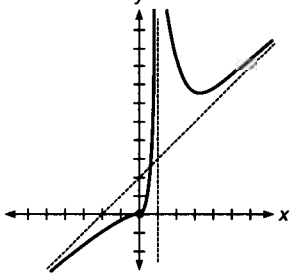
41. intercepts: $(0, 0.25)$;
asymptotes: $x = \pm 2$, $y = -3$



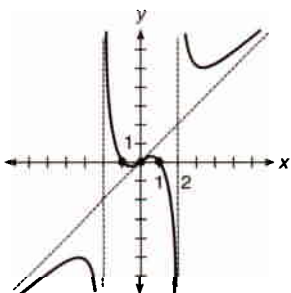
43. intercepts: $(0, 0)$; asymptotes: $x = 1$,
 $y = \frac{1}{2}x + \frac{1}{2}$



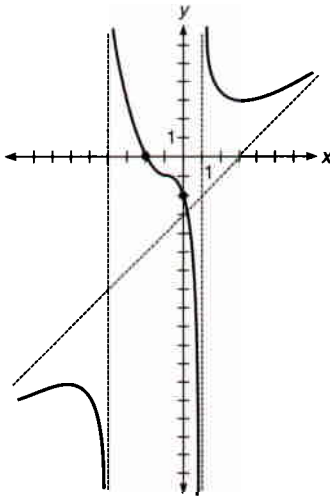
45. intercepts: $(0, 0)$; asymptotes: $x = 1$,
 $y = x + 2$



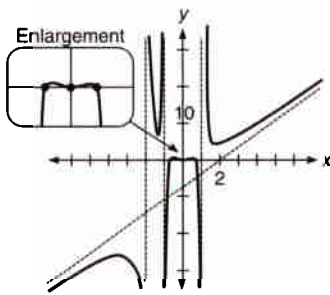
47. intercepts: $(0, 0)$, $(\pm 1, 0)$, $(0, 0)$;
asymptotes: $x = \pm 2$, $y = x$



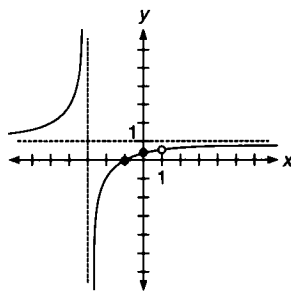
49. intercepts: $(0, -2)$, $(-2, 0)$; asymptotes:
 $x = -4$, $x = 1$, $y = x - 3$



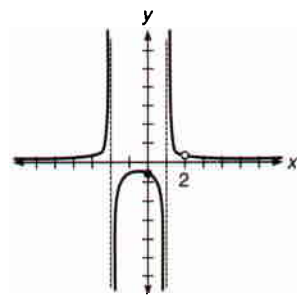
51. intercepts: $(0, 0)$, $(\pm \frac{\sqrt{2}}{2}, 0)$;
asymptotes: $x = \pm 1$, $x = -2$, $y = 2x - 4$



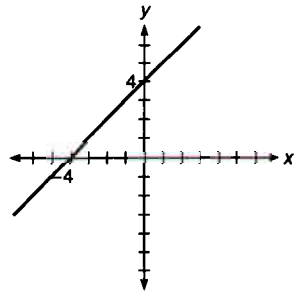
53. intercepts: $(0, \frac{1}{3})$, $(-1, 0)$;
asymptotes: $x = -3$, $y = 1$



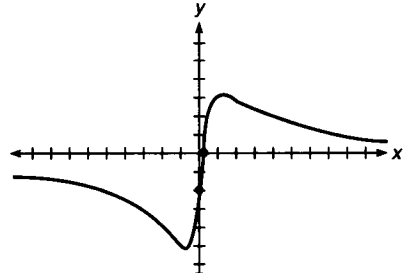
55. intercepts: $(0, -\frac{1}{2})$;
asymptotes: $x = -2$, $x = 1$, $y = 0$



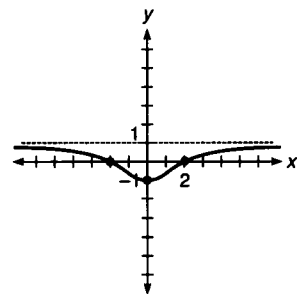
57. $y = x + 4$
intercepts: $(-4, 0)$, $(0, 4)$



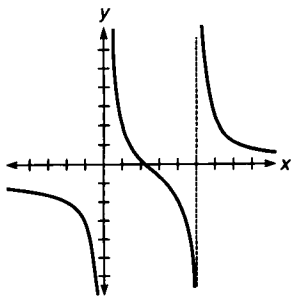
59. intercepts: $(0, -2)$, $(\frac{1}{4}, 0)$



61. intercepts: $(0, -1)$, $(\pm 2, 0)$

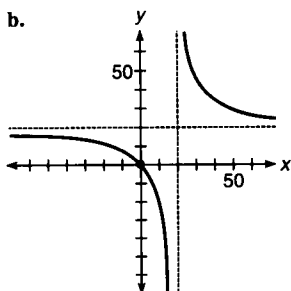


63.

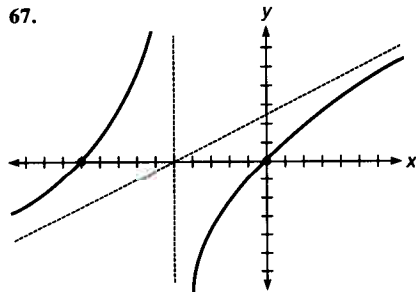


65. a. $y = \frac{20x}{x-20}$

b.



67.



69. a. \$750 b. \$250 c. \$125

Solutions to skill and review problems

- $f(5) = 2(5) - 3 = 7$
- $g(-4) = (-4)^2 + 2(-4) + 3 = 11$
- $g(2) = 2^2 + 2(2) + 3 = 11$
 $f(g(2)) = f(11) = 2(11) - 3 = 19$
- $f(-1) = 2(-1) - 3 = -5$
 $g(f(-1)) = g(-5) = (-5)^2 + 2(-5) + 3 = 18$
- Solve $x = 2y + 7$ for y .
 $x = 2y + 7$
 $2y = x - 7$
 $y = \frac{x-7}{2}$

6. Solve $x = \frac{1}{y-2}$ for y .

$$x = \frac{1}{y-2}$$

$$x(y-2) = 1$$

$$xy - 2x = 1$$

$$xy = 2x + 1$$

$$y = \frac{2x+1}{x}$$

7. Graph $f(x) = 2x^4 - x^3 - 14x^2 + 19x - 6$

$$y = 2(x-1)(x-2)(x+3)(x-\frac{1}{2})$$

(using possible rational zeros and synthetic division)

Intercepts:

$x = 0: y = -6; (0, -6)$

$y = 0:$

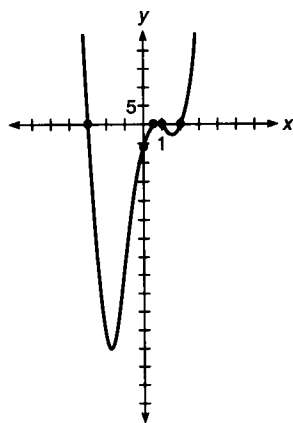
$0 = 2(x-1)(x-2)(x+3)(x-\frac{1}{2})$

$x = -3, \frac{1}{2}, 1; (-3, 0), (\frac{1}{2}, 0), (1, 0),$

$(2, 0)$

Additional points: $(-2, -60),$

$(-1, -36), (1.5, -2.25), (2.5, 16.5)$



8. Solve $|4 - 3x| = 16$

$4 - 3x = 16$ or $4 - 3x = -16$

$-12 = 3x$ or $20 = 3x$

$-4 = x$ or $6\frac{2}{3} = x$

$\{-4, 6\frac{2}{3}\}$

Solutions to trial exercise problems

11. $y = \frac{-4}{(x-2)^2}$

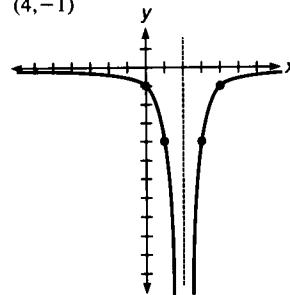
Same as $y = \frac{1}{x^2}$ translated right 2 units
and scaled vertically by -4 . Vertical

asymptote: $x = 2$. Horizontalasymptote: $y = 0$ (x -axis).

Intercepts:

$x = 0: y = \frac{-4}{(-2)^2} = -1; (0, -1)$

$y = 0: 0 = \frac{-4}{(x-2)^2}$ has no solutions

Additional points: $(1, -4), (3, -4), (4, -1)$ 

15. $y = \frac{1}{(x-3)^2} + 2$

Same as $y = \frac{1}{x^2}$, translated. Verticalasymptote at $x = 3$; horizontalasymptote at $y = 2$.Origin: $(3, 2)$

Intercepts:

$x = 0: y = \frac{1}{(-3)^2} + 2 = 2\frac{1}{9}; (0, 2\frac{1}{9})$

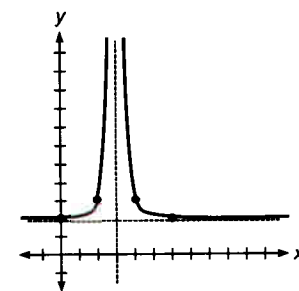
$y = 0: 0 = \frac{1}{(x-3)^2} + 2$

$-2 = \frac{1}{(x-3)^2}$

$-2(x^2 - 6x + 9) = 1$

$-2x^2 + 12x - 19 = 0$

$2x^2 - 12x + 19 = 0$; no real solutions

Additional points: $(2, 3), (4, 3), (6, 2\frac{1}{9})$ 

$$27. y = \frac{3x^2 - 1}{(x-2)(x^2-9)}$$

$$= \frac{3x^2 - 1}{(x-2)(x-3)(x+3)}$$

Vertical asymptotes: $x = 2, \pm 3$;
horizontal asymptote: $y = 0$ (x -axis).
Intercepts:

$$x = 0: y = \frac{-1}{(-2)(-9)} = -\frac{1}{18};$$

$$\left(0, -\frac{1}{18}\right)$$

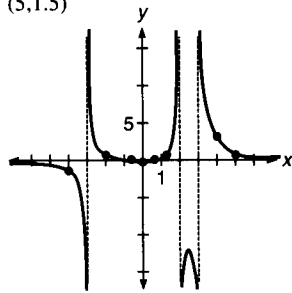
$$y = 0: 0 = \frac{3x^2 - 1}{(x-2)(x^2-9)}$$

$$0 = 3x^2 - 1$$

$$\frac{1}{3} = x^2$$

$$\pm \frac{\sqrt{3}}{3} = x; (\pm 0.6, 0)$$

Additional points: $(-4, -1.1)$,
 $(-2.0, 5.5)$, $(1.5, 1.7)$, $(2.25, -14.4)$,
 $(2.5, -12.9)$, $(2.75, -20.1)$, $(4, 3.4)$,
 $(5, 1.5)$



$$41. y = \frac{-3x^2 + 2x - 1}{x^2 - 4}$$

$$= -3 + \frac{2x - 13}{(x-2)(x+2)}$$

Horizontal asymptote: $y = -3$; vertical asymptotes: $x = \pm 2$

Intercepts:

$$x = 0: y = \frac{-1}{-4} = \frac{1}{4}; (0, 0.25)$$

$$y = 0: 0 = \frac{-3x^2 + 2x - 1}{x^2 - 4}$$

$$0 = -3x^2 + 2x - 1$$

$$0 = 3x^2 - 2x + 1$$

No real solutions.

Additional points: $(-5, -4.1)$,
 $(-3, -6.8)$, $(-1, 2)$, $(3, -4.4)$, $(5, -3.1)$,
 $(9, -2.94)$, $(11, -2.92)$

The value of y at $x = 5$ is less than -3 and at $x = 9$ is more than -3 . The

coordinate where $y = -3$ can be found by replacing y by -3 and solving.

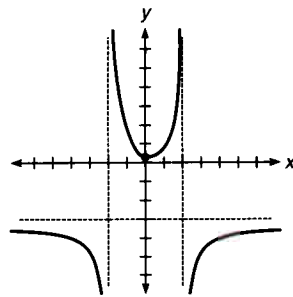
$$-3 = \frac{-3x^2 + 2x - 1}{x^2 - 4}$$

$$-3x^2 + 12 = -3x^2 + 2x - 1$$

$$13 = 2x$$

$$6.5 = x$$

The point $(6.5, -3)$ is plotted.



$$51. y = \frac{2x^4 - x^2}{(x+2)(x^2-1)}$$

$$= \frac{2x^4 - x^2}{x^3 + 2x^2 - x - 2}$$

$$= 2x - 4 + \frac{9x^2 - 8}{(x+2)(x-1)(x+1)}$$

Slant asymptote: $y = 2x - 4$; vertical asymptotes: $x = -2, \pm 1$.

Intercepts:

$$x = 0: y = \frac{0}{-2} = 0; (0, 0)$$

$$y = 0: 0 = \frac{2x^4 - x^2}{(x+2)(x^2-1)}$$

$$0 = 2x^4 - x^2$$

$$0 = x^2(2x^2 - 1)$$

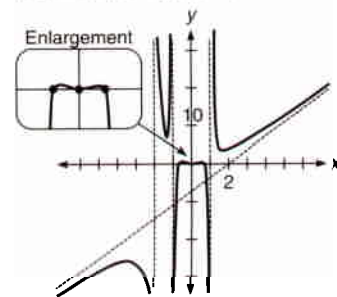
$$x^2 = 0 \text{ or } 2x^2 - 1 = 0$$

$$x = 0 \text{ or } x^2 = \frac{1}{2}$$

$$x = 0 \text{ or } x = \pm \frac{\sqrt{2}}{2}; (0, 0),$$

$$(\pm 0.7, 0)$$

Additional points: $(-5, -17.0)$,
 $(-4, -16.5)$, $(-3, -19.1)$,
 $(-1.75, 30.4)$, $(-1.5, 12.6)$,
 $(-1.25, 7.9)$, $(-0.5, 0.1)$, $(0.5, 0.07)$,
 $(1.5, 1.8)$, $(2.2, 3)$, $(3, 3.8)$



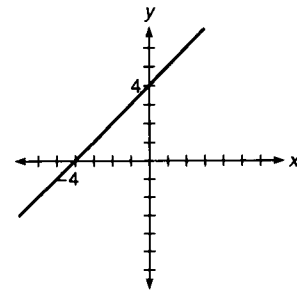
$$57. y = \frac{x^3 + 4x^2 + 3x + 12}{x^2 + 3}$$

$$= \frac{x^2(x+4) + 3(x+4)}{x^2 + 3}$$

$$= \frac{(x+4)(x^2+3)}{x^2+3}$$

$$= x + 4$$

This is a straight line with intercepts at $(0, 4)$ and $(-4, 0)$. Since $x^2 + 3 \neq 0$ for all real values of x there are no restrictions on the domain.



Exercise 4-5

Answers to odd-numbered problems

1. $x + 3$; $5x - 13$; $-6x^2 + 34x - 40$;

$$\frac{3x-5}{-2x+8}; -6x+19; -6x+18$$

3. $x + 4 + \sqrt{x-4}$; $x + 4 - \sqrt{x-4}$;

$$(x+4)\sqrt{x-4}; \frac{(x+4)\sqrt{x-4}}{x-4};$$

$$\sqrt{x-4} + 4; \sqrt{x}$$

5. $\frac{3x^2-4x+3}{2x^2-2x}; \frac{-x^2-4x+3}{2x^2-2x}; \frac{x-3}{2x-2};$

$$\frac{x^2-4x+3}{2x^2}; \frac{-2x+3}{2x}; -\frac{x-3}{x+3}$$

7. $x^4 - x^2 + 3 + \sqrt{\frac{x}{x+1}};$

$$x^4 - x^2 + 3 - \sqrt{\frac{x}{x+1}};$$

$$(x^4 - x^2 + 3) \left(\sqrt{\frac{x}{x+1}} \right);$$

$$(x+1)(x^4 - x^2 + 3) \sqrt{\frac{x}{x+1}};$$

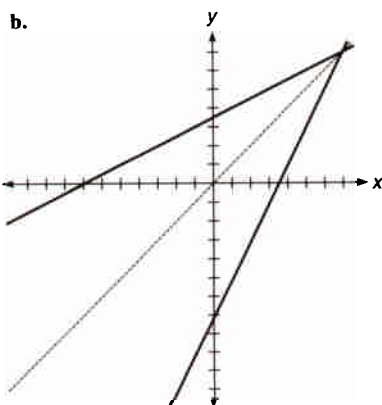
$$\frac{x}{(x+1)(x^4 - x^2 + 3)};$$

$$\frac{3x^2+5x+3}{x^2+2x+1}; \sqrt{\frac{x^4-x^2+3}{x^4-x^2+4}}$$

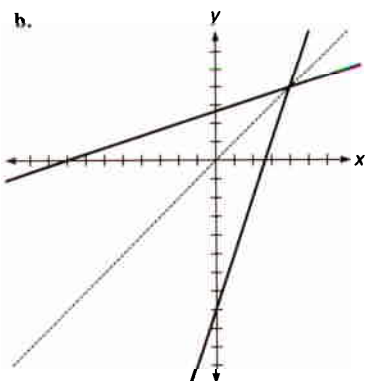
9. $x + 3$; $x - 3$; $3x$; $\frac{x}{3}$; 3 ; 3

11. $\sqrt[3]{x-5} + (x^3+5)$; $\sqrt[3]{x-5} - x^3 - 5$;
 $(x^3+5)\sqrt[3]{x-5}$; $\frac{\sqrt[3]{x-5}}{x^3+5}$; x ; x

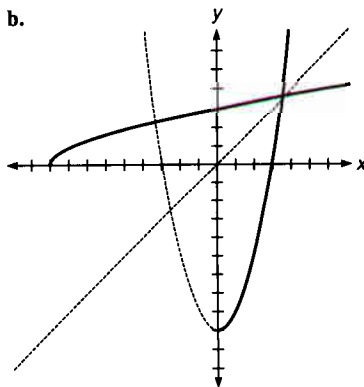
13. a. $f(x) = 2x - 7$; $g(x) = \frac{1}{2}x + 3\frac{1}{2}$
 $f(g(x)) = 2(g(x)) - 7$
 $= 2(\frac{1}{2}x + 3\frac{1}{2}) - 7$
 $= x + 7 - 7 = x$
 $g(f(x)) = \frac{1}{2}(2x - 7) + 3\frac{1}{2} = x$



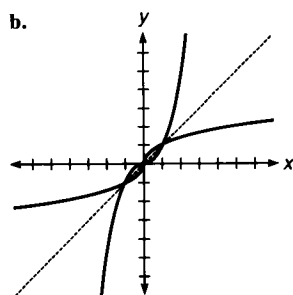
15. a. $f(x) = \frac{1}{3}x + \frac{8}{3}$; $g(x) = 3x - 8$
 $f(g(x)) = \frac{1}{3}(3x - 8) + \frac{8}{3}$
 $= x - \frac{8}{3} + \frac{8}{3} = x$
 $g(f(x)) = 3(\frac{1}{3}x + \frac{8}{3}) - 8$
 $= x + 8 - 8 = x$



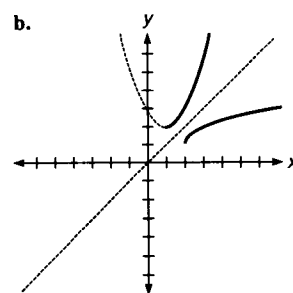
17. a. $f(x) = x^2 - 9$, $x \geq 0$;
 $g(x) = \sqrt{x+9}$
 $f(g(x)) = (\sqrt{x+9})^2 - 9$
 $= x + 9 - 9 = x$
 $g(f(x)) = \sqrt{(x^2 - 9) + 9} = \sqrt{x^2} = x$



19. a. $f(x) = x^3$; $g(x) = \sqrt[3]{x}$
 $f(g(x)) = (\sqrt[3]{x})^3 = x$
 $g(f(x)) = \sqrt[3]{x^3} = x$



21. a. $f(x) = x^2 - 2x + 3$, $x \geq 1$
 $g(x) = \sqrt{x-2} + 1$
 $f(g(x)) = (\sqrt{x-2} + 1)^2 - 2$
 $= 2(\sqrt{x-2} + 1) + 3$
 $= ((x-2) + 2\sqrt{x-2} + 1) + 3$
 $= x - 2 + 2\sqrt{x-2} - 2 + 3$
 $= x$
 $g(f(x)) = \sqrt{(x^2 - 2x + 3) - 2} + 1$
 $= \sqrt{x^2 - 2x + 1} + 1$
 $= \sqrt{(x-1)^2} + 1$
 $= (x-1) + 1 = x$



23. a. $f(x) = \frac{2x}{x-3}$; $g(x) = \frac{3x}{x-2}$

$$f(g(x)) = \frac{2\left(\frac{3x}{x-2}\right)}{\frac{3x}{x-2} - 3}$$

$$= \frac{\frac{6x}{x-2}}{\frac{3x - 3(x-2)}{x-2}} \cdot \frac{x-2}{x-2}$$

$$= \frac{6x}{3x - 3(x-2)} \cdot \frac{x-2}{x-2}$$

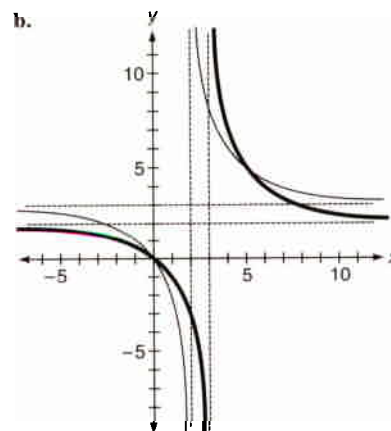
$$= \frac{6x}{x-2} \cdot \frac{x-2}{x-2} = x$$

$$g(f(x)) = \frac{3\left(\frac{2x}{x-3}\right)}{\frac{2x}{x-3} - 2}$$

$$= \frac{\frac{6x}{x-3}}{\frac{2x - 2(x-3)}{x-3}} \cdot \frac{x-3}{x-3}$$

$$= \frac{6x}{2x - 2(x-3)} \cdot \frac{x-3}{x-3}$$

$$= \frac{6x}{x-3} \cdot \frac{x-3}{x-3} = x$$



25. $f^{-1}(x) = \frac{1}{4}x + \frac{5}{4}$

27. $h^{-1}(x) = -\frac{2}{5}x + \frac{24}{5}$

29. $g^{-1}(x) = \sqrt{x+9}$

31. $f^{-1}(x) = \sqrt{9-x^2}$

33. $h^{-1}(x) = x^2 + 4$, $x \geq 0$

35. $g^{-1}(x) = \frac{\sqrt[3]{4(x+9)}}{2}$

37. $f^{-1}(x) = \frac{x^3+5}{4}$

39. $f^{-1}(x) = \frac{3}{4x+5}$

41. $g^{-1}(x) = \frac{-x}{x-1}$

43. $h^{-1}(x) = \frac{-x-1}{x-1}$

45. $h^{-1}(x) = 1 + \sqrt{x+10}$

47. $g^{-1}(x) = \frac{-3 + \sqrt{8x+25}}{4}$

49. $C(x) = \frac{x^3}{2}$ 51. $V_e(t) = \frac{1}{4}t - 2$
 53. $A(t) = 80t^3$ 55. $A^{-1}(x) = \frac{1}{4}x - 4$
 57. $R^{-1}(x) = \frac{20x}{20-x}$
 59. $f(g(x)) = (-\sqrt{x+9})^2 - 9$
 $= (x+9) - 9 = x$
 $g(f(x)) = -\sqrt{(x^2-9)+9} = -\sqrt{x^2}$
 $= -|x|$; since $x \leq 0$, $|x|$
 $= -x$, so $-|x| = -(-x) = x$
 61. $f^{-1}(x) = \frac{1}{a}x - \frac{b}{a}$; the inverse does
 not exist if $a = 0$.

Solutions to skill and review problems

1. Combine $\frac{2}{x+3} - \frac{3}{x-2}$.

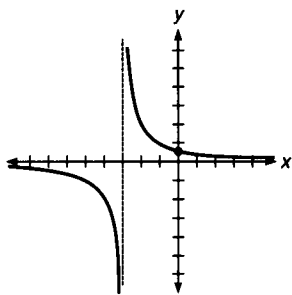
$$\frac{2}{x+3} - \frac{3}{x-2}$$

$$\frac{2(x-2) - 3(x+3)}{(x+3)(x-2)}$$

$$\frac{-x-13}{x^2+x-6}$$

2. Graph $f(x) = \frac{2}{x+3}$.

Vertical asymptote: $x = -3$; horizontal asymptote: $y = 0$ (x -axis); intercepts: $x = 0$: $y = \frac{2}{3}$; $(0, \frac{2}{3})$
 $y = 0$: $0 = \frac{2}{x+3}$; no solution
 Additional points: $(-6, -0.67)$, $(-4, -2)$, $(-2, 2)$, $(1, 0.5)$



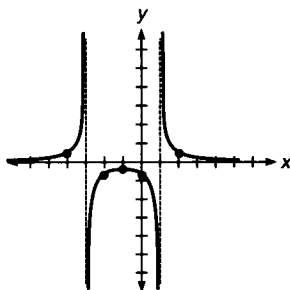
3. Graph $f(x) = \frac{2}{(x+3)(x-1)}$.

Vertical asymptotes: $x = -3, 1$;
 horizontal asymptote: $y = 0$ (x -axis);
 intercepts:

$$x = 0: y = \frac{2}{3(-1)} = -\frac{2}{3}; (0, -\frac{2}{3})$$

$$y = 0: 0 = \frac{2}{(x+3)(x-1)}; \text{no solution}$$

Additional points: $(-4, 0.4)$,
 $(-2, -0.67)$, $(-1, -0.5)$, $(2, 0.4)$



4. Graph $f(x) = \frac{2x^2}{(x+3)(x-1)}$.

$$= \frac{2x^2}{x^2+2x-3}$$

$$= 2 + \frac{-4x+6}{x^2+2x-3}$$

$$= 2 + \frac{-4x+6}{(x+3)(x-1)}$$

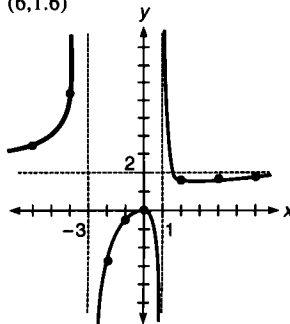
Vertical asymptotes: $x = -3, 1$;
 horizontal asymptote: $y = 2$;
 intercepts:

$$x = 0: y = \frac{0}{-3} = 0; (0, 0)$$

$$y = 0: 0 = \frac{2x^2}{(x+3)(x-1)}$$

$$0 = x; (0, 0)$$

Additional points: $(-6, 3.4)$, $(-4, 6.4)$,
 $(-2, -2.7)$, $(-1, -0.5)$, $(2, 1.6)$, $(4, 1.5)$,
 $(6, 1.6)$



5. Solve $\left| \frac{5x-2}{x+1} \right| < 2$.

This inequality is nonlinear so we must use the critical point/test point method. Find critical points:

a. Solve the corresponding equality.

$$\left| \frac{5x-2}{x+1} \right| = 2$$

$$\frac{5x-2}{x+1} = 2 \quad \text{or} \quad \frac{5x-2}{x+1} = -2$$

$$5x-2 = 2x+2 \quad \text{or} \quad 5x-2 = -2x-2$$

$$3x = 4 \quad \text{or} \quad 7x = 0$$

$$x = \frac{4}{3} = 1\frac{1}{3} \quad \text{or} \quad x = 0$$

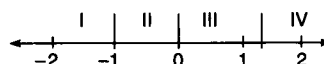
These are critical points.

b. Find zeros of denominators.

$$x+1 = 0$$

$$x = -1$$

Critical points are $-1, 0, 1\frac{1}{3}$. They form the 4 intervals shown.



Choose a test point from each interval, such as $-2, -\frac{1}{2}, 1, 2$. Try these in the original inequality.

$$x = -2: \left| \frac{5(-2)-2}{-2+1} \right| < 2; 12 < 2;$$

false

$$x = -\frac{1}{2}: \left| \frac{5(-\frac{1}{2})-2}{-\frac{1}{2}+1} \right| < 2; 9 < 2;$$

false

$$x = 1: \left| \frac{5(1)-2}{1+1} \right| < 2; 1\frac{1}{2} < 2; \text{true}$$

$$x = 2: \left| \frac{5(2)-2}{2+1} \right| < 2; 2\frac{2}{3} < 2; \text{false}$$

Only interval III forms the solution:

$$\{x \mid 0 < x < 1\frac{1}{3}\}.$$

6. Graph $f(x) = x^3 - x^2 - x + 1$.

$$\begin{aligned} y &= x^3 - x^2 - x + 1 \\ &= x^2(x-1) - 1(x-1) \\ &= (x-1)(x^2-1) \\ &= (x-1)(x-1)(x+1) \\ y &= (x-1)^2(x+1) \end{aligned}$$

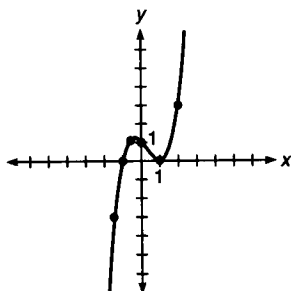
Intercepts:

$$x = 0: y = 1; (0, 1)$$

$$y = 0: 0 = (x-1)^2(x+1)$$

$$x = -1 \text{ or } 1; (-1, 0), (1, 0)$$

The zero 1 has multiplicity 2 (even multiplicity) so the graph does not cross the x -axis at 1. Additional points: $(-1.5, -3.1), (-0.5, 1.1), (2, 3)$



Solutions to trial exercise problems

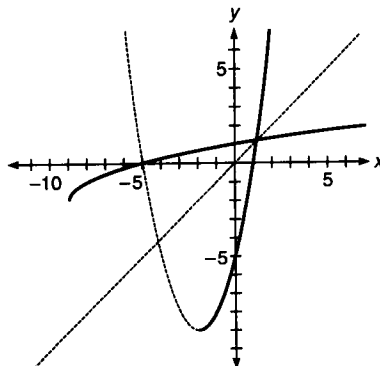
5. $f(x) = \frac{x-3}{2x}; g(x) = \frac{x}{x-1}$

$$\begin{aligned} \frac{x-3}{2x} + \frac{x}{x-1} &= \frac{(x-3)(x-1) + x(2x)}{2x(x-1)} \\ &= \frac{x^2 - 4x + 3 + 2x^2}{2x^2 - 2x} = \frac{3x^2 - 4x + 3}{2x^2 - 2x} \\ \frac{x-3}{2x} - \frac{x}{x-1} &= \frac{(x-3)(x-1) - x(2x)}{2x(x-1)} \\ &= \frac{x^2 - 4x + 3 - 2x^2}{2x^2 - 2x} = \frac{-x^2 - 4x + 3}{2x^2 - 2x} \\ \frac{x-3}{2x} \cdot \frac{x}{x-1} &= \frac{x-3}{2} \cdot \frac{1}{x-1} = \frac{x-3}{2x-2} \\ \frac{x-3}{2x} \div \frac{x}{x-1} &= \frac{x-3}{2x} \cdot \frac{x-1}{x} = \frac{x^2 - 4x + 3}{2x^2} \\ f(g(x)) &= f\left(\frac{x}{x-1}\right) = \frac{\frac{x}{x-1} - 3}{2 \cdot \frac{x}{x-1}} \cdot \frac{x-1}{x-1} \\ &= \frac{x - 3(x-1)}{2x} = \frac{-2x + 3}{2x} \\ g(f(x)) &= g\left(\frac{x-3}{2x}\right) = \frac{\frac{x-3}{2x}}{\frac{x-3}{2x} - 1} \cdot \frac{2x}{2x} \\ &= \frac{x-3}{(x-3) - 2x} = \frac{x-3}{-x-3} = -\frac{x-3}{x+3} \end{aligned}$$

22. a. $f(x) = \sqrt{x+9} - 2; g(x) = x^2 + 4x - 5, x \geq -2$

$$\begin{aligned} f(g(x)) &= \sqrt{(x^2 + 4x - 5) + 9} - 2 \\ &= \sqrt{x^2 + 4x + 4} - 2 \\ &= \sqrt{(x+2)^2} - 2 = (x+2) - 2 = x \\ g(f(x)) &= (\sqrt{x+9} - 2)^2 + 4(\sqrt{x+9} - 2) - 5 \\ &= (x+9) - 4\sqrt{x+9} + 4 + 4\sqrt{x+9} - 8 - 5 = x \end{aligned}$$

b.

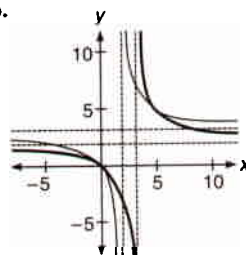


24. a. $f(x) = \frac{x-3}{x-2}; g(x) = 2 - \frac{1}{x-1}$

$$\begin{aligned} f(g(x)) &= \frac{\left(2 - \frac{1}{x-1}\right) - 3}{\left(2 - \frac{1}{x-1}\right) - 2} \\ &= \frac{-1 - \frac{1}{x-1}}{-\frac{1}{x-1}} \cdot \frac{x-1}{x-1} \\ &= \frac{(-x+1) - 1}{-1} = x \end{aligned}$$

$$\begin{aligned} g(f(x)) &= 2 - \frac{1}{\frac{x-3}{x-2} - 1} \\ &= 2 - \frac{1}{\frac{x-3}{x-2} - \frac{x-2}{x-2}} \cdot \frac{x-2}{x-2} \\ &= 2 - \frac{x-2}{(x-3) - (x-2)} = 2 - \frac{x-2}{-1} \\ &= 2 + (x-2) = x \end{aligned}$$

b.



39. $f(x) = \frac{3-5x}{4x}$

$$y = \frac{3-5x}{4x}$$

$$x = \frac{3-5y}{4y}$$

$$4xy = 3 - 5y$$

$$y(4x + 5) = 3$$

$$y = \frac{3}{4x+5}$$

$$f^{-1}(x) = \frac{3}{4x+5}$$

47. $g(x) = 2x^2 + 3x - 2, x \geq -\frac{3}{4}$

$$y = 2x^2 + 3x - 2, x \geq -\frac{3}{4}$$

$$x = 2y^2 + 3y - 2, y \geq -\frac{3}{4}$$

$$0 = 2y^2 + 3y + (-x-2)$$

$$y = \frac{-3 \pm \sqrt{8x+25}}{4}, y \geq -\frac{3}{4}$$

$$y = -\frac{3}{4} \pm \frac{\sqrt{8x+25}}{4}$$

$$\text{Since } y \geq -\frac{3}{4},$$

$$g^{-1}(x) = \frac{-3 + \sqrt{8x+25}}{4}$$

62. $f(x) = ax^2 + bx + c$

$$y = ax^2 + bx + c$$

$$x = ay^2 + by + c$$

$$0 = ay^2 + by + c - x$$

$$y = \frac{-b \pm \sqrt{b^2 - 4a(c-x)}}{2a}$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac + 4acx}}{2a}$$

$$\text{If } x \geq \frac{-b}{2a} \text{ in the domain we want}$$

$$y \geq \frac{-b}{2a} \text{ so we choose}$$

$$f^{-1}(x) = \frac{-b + \sqrt{b^2 - 4ac + 4acx}}{2a}$$

Exercise 4-6**Answers to odd-numbered problems**

1. $\frac{-2}{x-4} + \frac{3}{x-1}$

3. $\frac{-4}{x-1} + \frac{-2}{x+1}$

5. $2x + \frac{4}{2x-1} + \frac{-3}{x-1}$

7. $3x - 2 + \frac{-3}{3x+1} + \frac{3}{2x-5}$

9. $\frac{2}{(x-1)^2} + \frac{3}{x-2}$

11. $\frac{3}{2x} + \frac{-3}{x^2} + \frac{5}{x-2}$

13. $\frac{2}{x-3} + \frac{-2}{(x-3)^2} + \frac{1}{x+1} + \frac{-2}{(x+1)^2}$

15. $\frac{5}{(x-3)^2} + \frac{-5}{x+1} + \frac{2}{(x+1)^2}$

17. $\frac{3}{x-1} + \frac{-2x-1}{x^2+x+1}$

19. $\frac{-1}{x} + \frac{5}{x^2+2x+4}$

21. $\frac{3x}{x^2+x+1} - \frac{2}{x-3}$

23. $\frac{3x+1}{x^2+2x+4} - \frac{2}{x+3}$

25. $\frac{1}{x+5} + \frac{1}{x+10}$

27. $\frac{99}{100}$

29. $\frac{14,949}{10,100} \approx 1.4801$

Solutions to skill and review problems

1. Compute a. 8^3 b. $8^{1/3}$ c. 8^{-3} d. $8^{-1/3}$

a. $8^3 = 8 \cdot 8 \cdot 8 = 512$

b. $8^{1/3} = \sqrt[3]{8} = 2$

c. $8^{-3} = \frac{1}{8^3} = \frac{1}{512}$

d. $8^{-1/3} = \frac{1}{8^{1/3}} = \frac{1}{2}$

2. If $2^5 = a^5$, what is a ?

$a = 2$

3. If $2^a = 2^5$, what is a ?

$a = 5$

4. Graph $f(x) = 2x^2 - x - 6$.

This is a parabola; we complete the square.

$$y = 2(x^2 - \frac{1}{2}x) - 6$$

$$y = 2(x^2 - \frac{1}{2}x + \frac{1}{16}) - 6 - 2(\frac{1}{16})$$

$$\frac{1}{2}(-\frac{1}{2}) = -\frac{1}{4}; (-\frac{1}{4})^2 = \frac{1}{16}$$

$$y = 2(x - \frac{1}{4})^2 - 6\frac{1}{8}$$

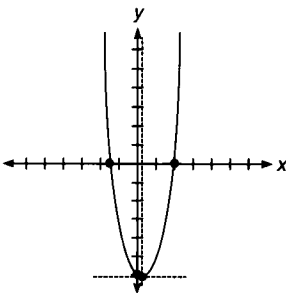
Vertex: $(\frac{1}{4}, -6\frac{1}{8})$; intercepts:

$x = 0: y = -6; (0, -6)$

$y = 0: 0 = 2x^2 - x - 6$

$0 = (2x+3)(x-2)$

$x = -\frac{3}{2}, 2; (-1\frac{1}{2}, 0), (2, 0)$



5. Graph $f(x) = (x-1)(x+2)(x-2)$.

Intercepts:

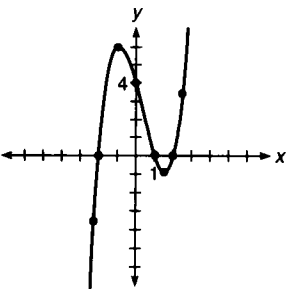
$x = 0: y = (-1)(2)(-2) = 4; (0, 4)$

$y = 0: 0 = (x-1)(x+2)(x-2)$

$x = -2, 1, 2; (-2, 0), (1, 0), (2, 0)$

Additional points: $(-2.25, -3.5)$,

$(-1.6), (1.5, -0.9), (2.5, 3.4)$



6. Solve $x^3 - x^2 + 1 > x$.

This is a nonlinear inequality; it must be solved using the critical point, test point method. Find critical points from (a) the corresponding equality and (b) zeros of denominators. Solve the corresponding equality:

$$x^3 - x^2 + 1 = x$$

$$x^3 - x^2 - x + 1 = 0$$

$$x^2(x-1) - 1(x-1) = 0$$

$$(x-1)(x^2-1) = 0$$

$$(x-1)(x+1)(x-1) = 0$$

$$x = \pm 1$$

Critical points: Find test points in each interval and test in the original inequality. We will use $\pm 2, 0$.

$$x^3 - x^2 + 1 > x$$

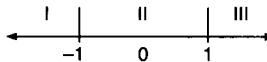
$$x = -2: -11 > -2; \text{false}$$

$$x = 0: 1 > 0; \text{true}$$

$$x = 2: 5 > 2; \text{true}$$

Thus the solution set is intervals II and III.

$$\{x \mid -1 < x < 1 \text{ or } x > 1\}$$



7. Graph $f(x) = \frac{x^2+1}{x^2-1}$.

$$y = \frac{x^2+1}{x^2-1} = 1 + \frac{2}{(x-1)(x+1)}$$

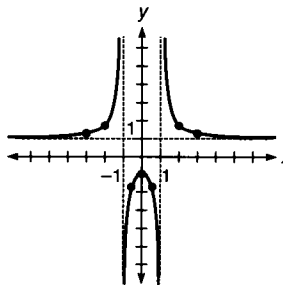
Horizontal asymptote: $y = 2$; vertical asymptotes: $x = \pm 1$; intercepts:

$$x = 0: y = \frac{1}{-1} = -1; (0, -1)$$

$$y = 0: 0 = \frac{x^2+1}{x^2-1}$$

$$0 = x^2 + 1; \text{no real solutions so no } x\text{-intercepts}$$

Additional points: $(\pm 3, 1.25), (\pm 2, 1.7), (\pm 0.5, -1.7)$



Solutions to trial exercise problems

$$13. \frac{3x^3 - 11x^2 + x - 17}{(x-3)^2(x+1)^2} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{x+1} + \frac{D}{(x+1)^2}$$

$$\frac{3x^3 - 11x^2 + x - 17}{(x-3)^2(x+1)^2} \cdot (x-3)^2(x+1)^2$$

$$= \frac{A}{x-3} \cdot (x-3)^2(x+1)^2 + \frac{B}{(x-3)^2} \cdot (x-3)^2(x+1)^2$$

$$+ \frac{C}{x+1} \cdot (x-3)^2(x+1)^2 + \frac{D}{(x+1)^2} \cdot (x-3)^2(x+1)^2$$

$$3x^3 - 11x^2 + x - 17 = A(x-3)(x+1)^2 + B(x+1)^2$$

$$+ C(x-3)^2(x+1) + D(x-3)^2$$

$$\text{Let } x = 3: -32 = A(0) + B(16) + C(0) + D(0)$$

$$-2 = B$$

$$\text{Let } x = -1: -32 = A(0) + B(0) + C(0) + D(16)$$

$$-2 = D$$

We now make any other two choices for x .

$$\text{Let } x = 0: -17 = -3A + (-2) + 9C + 9(-2)$$

$$B = -2, D = -2$$

$$-17 = -3A - 2 + 9C - 18$$

$$3 = -3A + 9C$$

$$[1] \quad 1 = -A + 3C$$

$$\text{Let } x = 1: -24 = -8A + 4(-2) + 8C + 4(-2)$$

$$-24 = -8A - 8 + 8C - 8$$

$$B = -2, D = -2$$

$$-8 = -8A + 8C$$

$$[2] \quad 1 = A - C$$

By equation [1], $A = 3C - 1$; plugging this into equation [2] we obtain

$$1 = (3C - 1) - C$$

$$1 = 2C - 1$$

$$2 = 2C$$

$$C = 1$$

$$\text{Since } A = 3C - 1, A = 3 - 1 = 2.$$

$$\text{Thus, } \frac{3x^3 - 11x^2 + x - 17}{(x-3)^2(x+1)^2} =$$

$$\frac{2}{x-3} + \frac{-2}{(x-3)^2} + \frac{1}{x+1} + \frac{-2}{(x+1)^2}.$$

$$21. \frac{x^2 - 11x - 2}{(x-3)(x^2 + x + 1)} = \frac{A}{x-3} + \frac{Bx + C}{x^2 + x + 1}$$

$$\frac{x^2 - 11x - 2}{(x-3)(x^2 + x + 1)} \cdot (x-3)(x^2 + x + 1)$$

$$= \frac{A}{x-3} \cdot (x-3)(x^2 + x + 1)$$

$$+ \frac{Bx + C}{x^2 + x + 1} \cdot (x-3)(x^2 + x + 1)$$

$$x^2 - 11x - 2 = A(x^2 + x + 1) + (Bx + C)(x-3)$$

$$\text{Let } x = 3: -26 = A(13)$$

$$-2 = A$$

$$\text{Let } x = 0: -2 = -2(1) + C(-3)$$

$$A = -2$$

$$C = 0$$

$$\text{Let } x = 1: -12 = -2(3) + B(-2)$$

$$A = -2, C = 0$$

$$B = 3$$

$$\frac{x^2 - 11x - 2}{(x-3)(x^2 + x + 1)} = \frac{3x}{x^2 + x + 1} - \frac{2}{x-3}$$

$$27. \frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$\frac{1}{n(n+1)} \cdot n(n+1) = \frac{A}{n} \cdot n(n+1) + \frac{B}{n+1} \cdot n(n+1)$$

$$1 = A(n+1) + Bn$$

$$\text{Let } n = 0: 1 = A$$

$$\text{Let } n = -1: 1 = -B; B = -1$$

$$\frac{1}{n(n+1)} = \frac{1}{n} + \frac{-1}{n+1}$$

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\text{Thus, for example, } \frac{1}{1 \cdot 2} = \frac{1}{1} - \frac{1}{2} \text{ and } \frac{1}{99 \cdot 100} = \frac{1}{99} - \frac{1}{100}.$$

$$\text{Thus, } \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{98 \cdot 99} + \frac{1}{99 \cdot 100} =$$

$$\left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{98} - \frac{1}{99}\right)$$

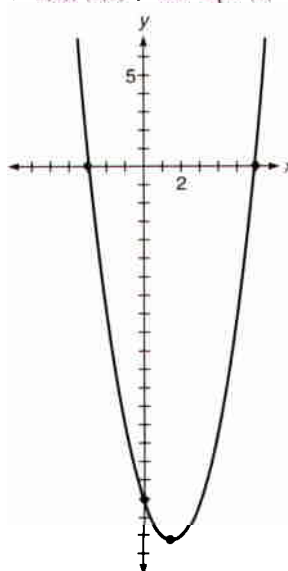
$$+ \left(\frac{1}{99} - \frac{1}{100}\right) =$$

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{98} - \frac{1}{99} + \frac{1}{99}$$

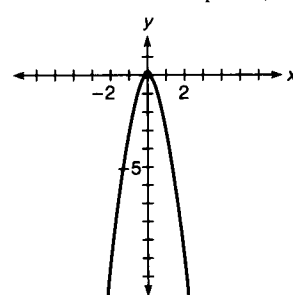
$$- \frac{1}{100} = \frac{1}{1} - \frac{1}{100} = \frac{99}{100}$$

Chapter 4 review

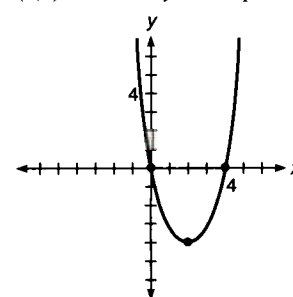
1. vertex: $(1\frac{1}{2}, -20\frac{1}{4})$; x-intercept: $(-3, 0), (6, 0)$; y-intercept: $(0, -18)$



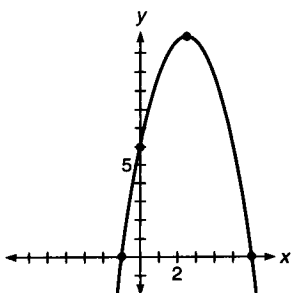
2. vertex and all intercepts at $(0, 0)$



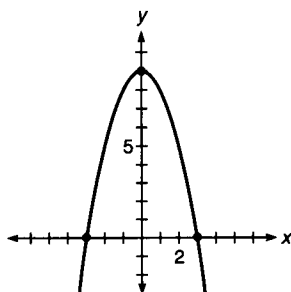
3. vertex: $(2, -4)$; x-intercept: $(0, 0), (4, 0)$; $(0, 0)$ is also the y-intercept



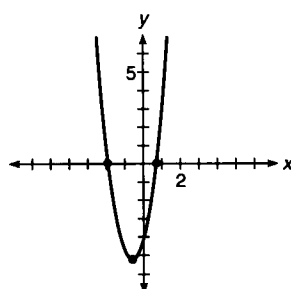
4. vertex: $(2\frac{1}{2}, 12\frac{1}{4})$; x-intercept: $(-1, 0)$, $(6, 0)$; y-intercept: $(0, 6)$



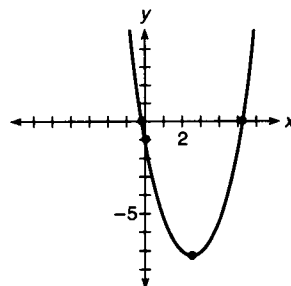
5. vertex: $(0, 9)$; x-intercept: $(-3, 0)$, $(3, 0)$; y-intercept: $(0, 9)$



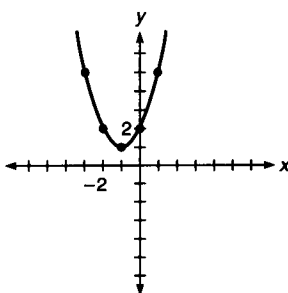
6. vertex: $(-\frac{2}{3}, -5\frac{1}{3})$; x-intercept: $(-2, 0)$, $(\frac{2}{3}, 0)$; y-intercept: $(0, -4)$



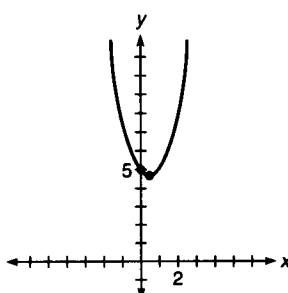
7. vertex: $(2\frac{1}{2}, -7\frac{1}{4})$; x-intercept: $(-0.2, 0)$, $(5.2, 0)$; y-intercept: $(0, -1)$



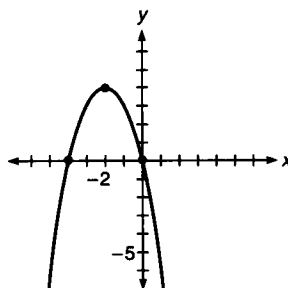
8. vertex: $(-1, 1)$; y-intercept: $(0, 2)$; additional points: $(-3, 5)$, $(-2, 2)$, $(1, 5)$



9. vertex: $(\frac{1}{2}, 4\frac{3}{4})$; y-intercept: $(0, 5)$



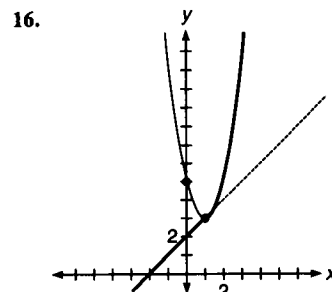
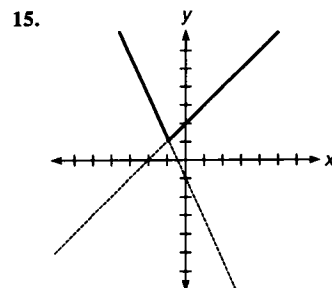
10. vertex: $(-2, 4)$; x-intercept: $(0, 0)$, $(-4, 0)$; y-intercept: $(0, 0)$



11. The dimensions are 50 and 100; in this case the area is 5,000 sq. ft.
12. The dimensions are 100 feet on a side, and the area is 10,000 sq. ft.

13. The rectangle (a square) will give a larger area for a given perimeter.

14. The object will rise to a maximum height of 4,096 ft after 16 seconds; the object returns to the ground after 32 seconds.



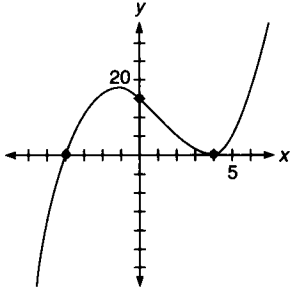
17. $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$
18. $\pm 1, \pm 2, \pm 5, \pm 10, \pm \frac{1}{2}, \pm \frac{5}{2}$
19. $\pm 1, \pm 2, \pm 4$
20. $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$
21. $f(x) \div (x - 3) = 2x^3 + x^2 + 5x + 15 + \frac{44}{x - 3}; f(3) = 44$
22. $g(x) \div (x + 4) = -2x^2 + 5x - 23 + \frac{94}{x + 4}; g(-4) = 94$
23. $f(x) \div (x - 4) = \frac{1}{2}x^2 - x - \frac{13}{4} - \frac{16}{x - 4}; f(4) = -16$

24. a. 0 or 2 positive real zeros; 0 or 2 negative real zeros
b. $\pm(1, 2, 3, 6, 9, 18, 27, 54, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2})$
c,d. $f(x) = (x + 2)(x - 3)(2x^2 + 3x - 9)$
 $= (x + 2)(x - 3)(x + 3)(2x - 3)$
All zeros are $-3, -2, \frac{3}{2}, 3$.
25. a. 0 or 2 positive real zeros; 0 or 2 negative real zeros
b. $\pm(1, 2, 4, \frac{1}{2})$
c,d. $f(x) = (x - 1)(x - 2)(2x^2 + 5x + 2)$
 $= (x - 1)(x - 2)(x + 2)(2x + 1)$
All zeros are $1, 2, -2, -\frac{1}{2}$

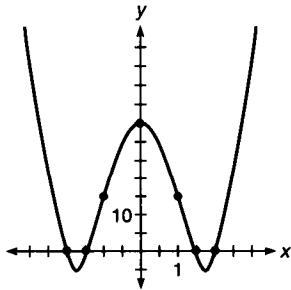
26. a. 0 or 2 positive real zeros; 0 or 2 negative real zeros
 b. $\pm(1, 2, 4, \frac{1}{2})$
 c, d. $h(x) = 2(x + \frac{1}{2})(x^3 - 5x^2 - 4x + 4)$; $-\frac{1}{2}$ is the only rational zero
 e. -2 is the greatest negative integer lower bound; 6 is the least positive integer upper bound.

27. a. 1 or 3 positive real zeros; 0 or 2 negative real zeros
 b. $\pm(1, 3, 9, 27, \frac{1}{2}, \frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \frac{1}{4}, \frac{3}{4}, \frac{9}{4}, \frac{27}{4}, \frac{1}{8}, \frac{3}{8}, \frac{9}{8}, \frac{27}{8}, \frac{1}{16}, \frac{3}{16}, \frac{9}{16}, \frac{27}{16})$
 c, d. $f(x) = (x - 3)(2x - 3)(2x + 3)(2x - 1)(2x + 1)$
 All the zeros for f are $3, \pm\frac{3}{2}, \pm\frac{1}{2}$.

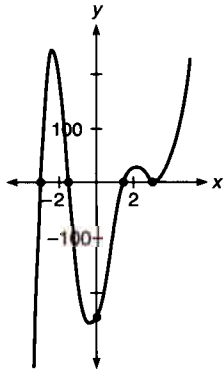
28. $g(x) = \frac{1}{4}(x - 4)^2(x + 4)$
 y-intercept: 16 ; x-intercept: ± 4



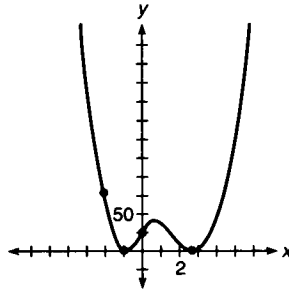
29. $h(x) = (x - 2)(x + 2)(2x - 3)(2x + 3)$
 x-intercepts: $\pm 2, \pm 1\frac{1}{2}$; y-intercept: 36



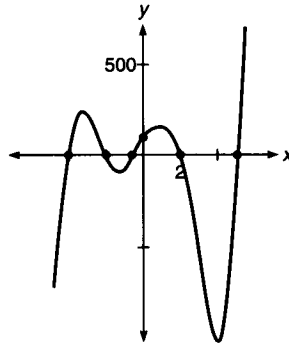
30. $g(x) = (x - 3)(x + 3)(2x - 3)$
 $(2x + 3)(x - 3)$
 x-intercepts: $\pm 3, \pm 1\frac{1}{2}$; y-intercept: -243



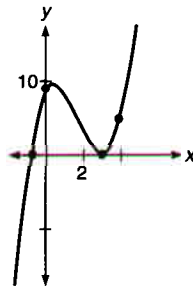
31. $f(x) = (2x - 5)^2(x + 1)^2$
 x-intercepts: $2\frac{1}{2}, -1$; y-intercept: 25



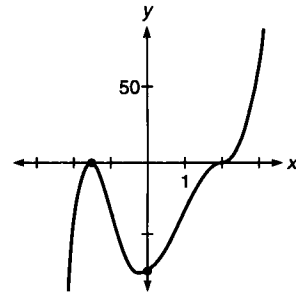
32. $h(x) = (x - 5)(x + 4)(x - 2)(x + 2)(2x + 1)$
 x-intercepts: $-4, -2, -\frac{1}{2}, 2, 5$
 y-intercept: 80



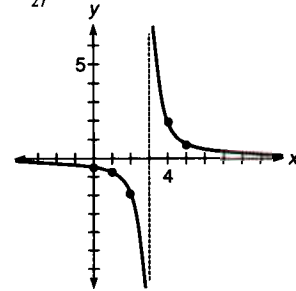
33. x-intercepts: $-1, 3$; y-intercept: 9



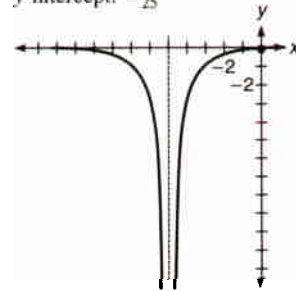
34. x-intercepts: $2, -1\frac{1}{2}$; y-intercept: -72



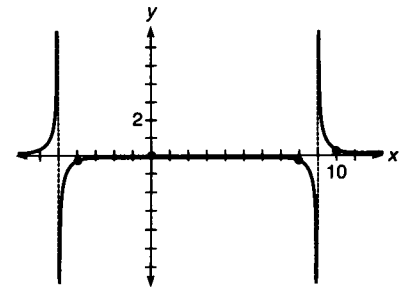
35. asymptotes: $x = 3, y = 0$; y-intercept: $-\frac{2}{27}$



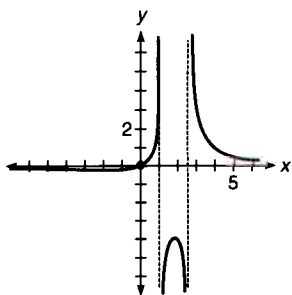
36. asymptotes: $x = -5, y = 0$
 y-intercept: $-\frac{3}{25}$



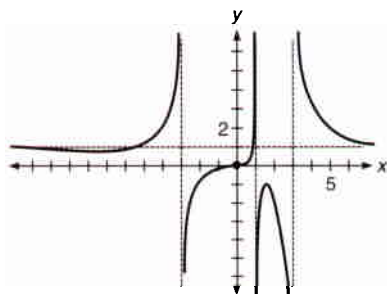
37. asymptotes: $x = -5, x = 9, y = 0$
 y-intercept: $-\frac{1}{15}$



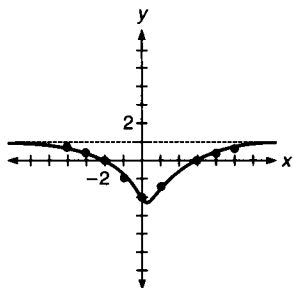
38. asymptotes: $x = 1$, $x = 2\frac{1}{2}$, $y = 0$;
intercepts at origin



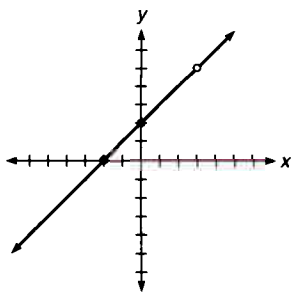
39. asymptotes: $x = -3$, $x = 1$, $x = 3$,
 $y = 1$; y-intercept: 0, x-intercept: 0



40. asymptote: $y = 1$; x-intercepts: -2 , 3 ,
y-intercept: -2



41. $f(x) = \frac{x^2 - x - 6}{x - 3} = x + 2$ when $x \neq 3$;
x-intercept: -2 y-intercept: $f(0) = 2$



$$42. 0; -x + 6; -\frac{1}{4}x^2 + 3x - 9; \frac{-(x-6)}{x-6}$$

$$= -1 \text{ if } x \neq 6; -\frac{1}{4}x + 4\frac{1}{2};$$

$$-\frac{1}{4}x - 1\frac{1}{2}$$

$$43. x^4 - 1 + \sqrt{8-x}; x^4 - 1 - \sqrt{8-x};$$

$$(x^4 - 1)(\sqrt{8-x}); \frac{x^4 - 1}{\sqrt{8-x}};$$

$$x^2 - 16x + 63; \sqrt{9-x^4}$$

$$44. \frac{4x^2 - 7x + 3}{2x(2x-1)}; \frac{-7x+3}{2x(2x-1)}; \frac{x-3}{2(2x-1)};$$

$$\frac{2x^2 - 7x + 3}{2x^2}; \frac{-5x+3}{2x}; \frac{-x+3}{6}$$

$$45. x; -5x; -6x^2; -\frac{2}{3}; -6x; -6x$$

$$46. x-3; x+3; -3x; -\frac{x}{3}; -3; -3$$

$$47. g^{-1}(x) = \frac{4x+5}{2}$$

$$48. h^{-1}(x) = \frac{-x+4}{2x-1}$$

$$49. g^{-1}(x) = \sqrt{x-8}$$

$$50. g^{-1}(x) = \frac{\sqrt[3]{x+27}}{2}$$

$$51. g^{-1}(x) = -x^3 - 9x^2 - 27x - 26$$

$$52. f^{-1}(x) = \frac{7 + \sqrt{4x+25}}{2}$$

$$53. f(x) = \frac{160}{7}x + \frac{120}{7}; 154 \text{ gallons}$$

$$54. \frac{5}{x-3} + \frac{-2}{2x+1}$$

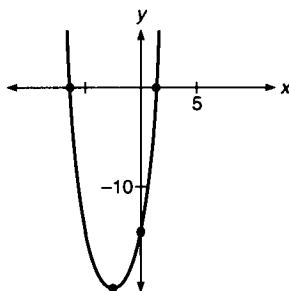
$$55. \frac{4}{x-3} + \frac{-1}{(x-3)^2} + \frac{5}{2x+1}$$

$$56. \frac{1}{x-2} + \frac{x-2}{x^2-x+4}$$

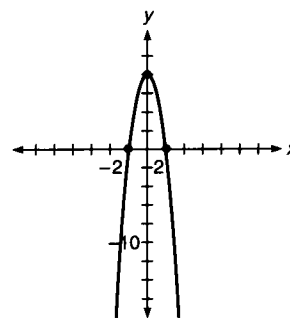
$$57. \frac{-4}{x+2} + \frac{2}{x-2} + \frac{-2}{(x-2)^2} + \frac{3}{(x-2)^3}$$

Chapter 4 test

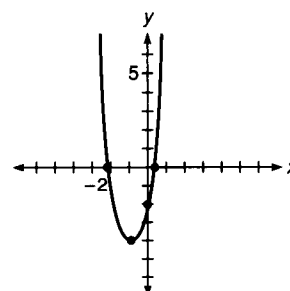
1. vertex: $(-2\frac{1}{2}, -20\frac{1}{4})$; x-intercept:
 $(-7, 0)$, $(2, 0)$; y-intercept: $(0, -14)$



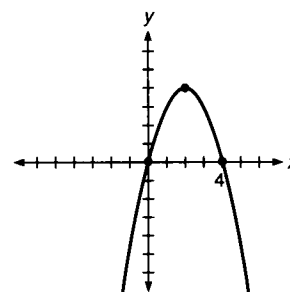
2. vertex: $(0, 8)$; x-intercept: $(-2, 0)$, $(2, 0)$;
y-intercept: $(0, 8)$



3. vertex: $(-\frac{5}{6}, -4\frac{1}{12})$; x-intercept:
 $(-2, 0)$, $(\frac{1}{3}, 0)$; y-intercept: $(0, -2)$

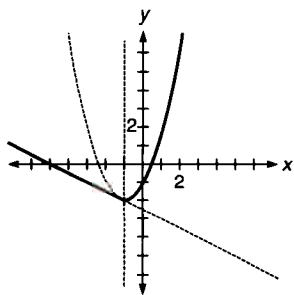


4. vertex: $(2, 4)$; x-intercept: $(0, 0)$, $(4, 0)$;
y-intercept is the origin also



5. The dimensions should be 12.5 ft by 25 ft, and the area will be 312.5 ft².
6. The object will be at its highest point after 1.5 seconds, and it will be 36 feet high at that time; it returns to its starting point after $t = 3$ seconds.

7.

8. $\pm(1, 2, 4, 8)$ 9. $\pm(1, 2, 3, 4, 6, 12, \frac{1}{2}, \frac{3}{2}, \frac{1}{4}, \frac{3}{4})$ 10. $f(x) \div (x+3) = 3x^3 - 11x^2 + 3x - 9$
 $+ \frac{7}{x+3}$, and $f(-3) = 7$

11. a. 0 or 2 positive real zeros; one negative real zero

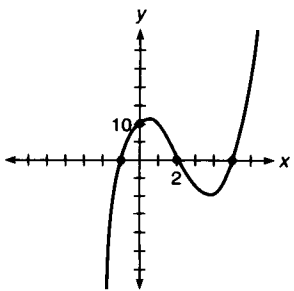
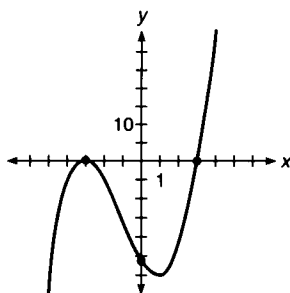
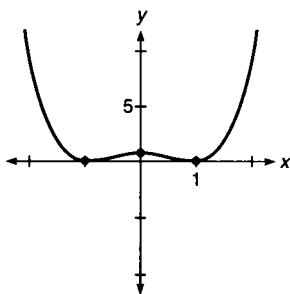
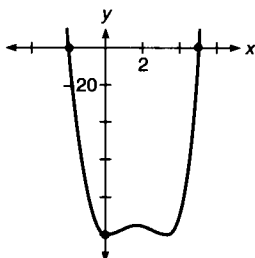
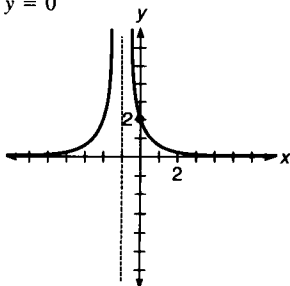
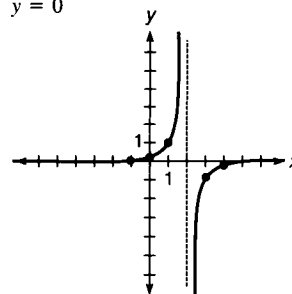
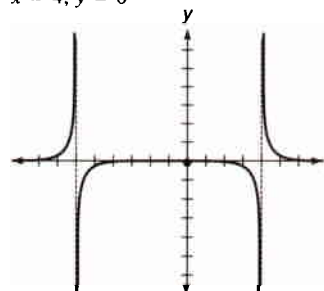
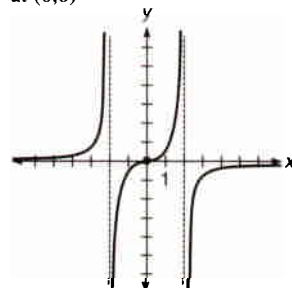
b. $\pm(1, \frac{1}{2}, \frac{1}{4})$ c,d. $f(x) = (x-1)(2x-1)(2x+1)$
Real zeros are $1, \pm\frac{1}{2}$.

12. a. no positive real zeros; 0, 2, or 4 real negative roots

b. possible rational zeros: ± 1 c,d. $f(x) = (x+1)^2(x^2+x+1)$
rational zeros are -1 , multiplicity 2

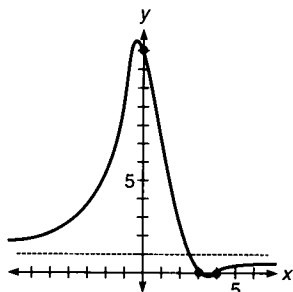
e. There are no possible irrational zeros.

13. a. 1 or 3 positive real zeros; 0 or 2 negative real zeros

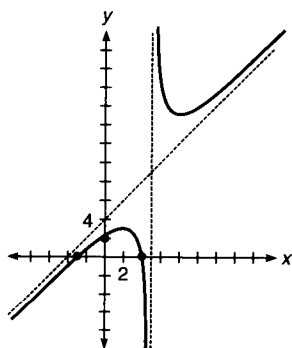
b. $\pm(1, 2, 3, 4, 6, 12, \frac{1}{3}, \frac{2}{3}, \frac{4}{3})$ c,d. $f(x) = (x-1)(x-2)^2(x+3)$
 $(3x+1)$
Real zeros are $1, 2, -3, -\frac{1}{3}$.
The zero 2 has multiplicity 2.14. $f(x) = (x-2)(x+1)(x-5)$
x-intercepts: $-1, 2, 5$; y-intercept: 1015. $g(x) = (x+3)^2(x-3)$
x-intercepts: $3, -3$; y-intercept: -27 16. $f(x) = (x-1)^2(x+1)^2$
x-intercepts at ± 1 ; y-intercept at 117. $h(x) = (x-5)(x+2)(x^2-3x+10)$
x-intercepts at $-2, 5$; y-intercept at -100 18. y-intercept: 2; asymptotes: $x = -1$,
 $y = 0$ 19. y-intercept: $\frac{1}{8}$; asymptotes: $x = 2$,
 $y = 0$ 20. $f(x) = \frac{1}{(x-4)(x+6)}$
y-intercept: $-\frac{1}{24}$; asymptotes: $x = -6$,
 $x = 4$, $y = 0$ 21. $f(x) = \frac{-x}{(x-2)(x+2)}$
asymptotes: $x = \pm 2$, $y = 0$; intercepts
at $(0,0)$ 

22. $g(x) = 1 + \frac{-7x + 11}{x^2 + 1}$

asymptote: $y = 1$; x -intercept: 3 or 4,
 y -intercept: 12



23. $f(x) = x + 4 + \frac{8}{x - 5}$ the line $x + 4$ is
a slant asymptote; vertical asymptote at
 $x = 5$; x -intercepts are at -3 and 4 ;
 y -intercept at $f(0) = 2\frac{2}{5}$



24. $x^2 + 1$; $-x^2 + 4x + 9$;

$2x^3 + x^2 - 18x - 20$; $\frac{2x + 5}{x^2 - 2x - 4}$;

$2x^2 - 4x - 3$; $4x^2 + 16x + 11$

25. $x^4 - 2 + 2\sqrt{x + 1}$;
 $x^4 - 2 - 2\sqrt{x + 1}$; $2(x^4 - 2)\sqrt{x + 1}$;
 $\frac{x^4 - 2}{2\sqrt{x + 1}}$; $16x^2 + 32x + 14$; $2\sqrt{x^4 - 1}$

26. $\frac{3x^2 + 3x - 2}{x(2x - 1)}$; $\frac{x^2 + 3x - 2}{x(2x - 1)}$; $\frac{x + 2}{2x - 1}$;
 $\frac{2x^2 + 3x - 2}{x^2}$; $\frac{5x - 2}{x}$; $\frac{x + 2}{x + 4}$

27. $g^{-1}(x) = \frac{x - 4}{5}$ 28. $f^{-1}(x) = \frac{1}{4x - 5}$

29. $g^{-1}(x) = \sqrt{x + 4}$

30. $f^{-1}(x) = \frac{1 - \sqrt{4x + 25}}{2}$

31. $f(x) = 2.5x - 100$; 62.5° F

32. $\frac{2}{x + 1} + \frac{-1}{(x + 1)^2} + \frac{3}{x - 2}$

33. $\frac{3}{x - 1} + \frac{x + 2}{x^2 + x + 4}$

Chapter 5

Exercise 5-1

Answers to odd-numbered problems

1. 13.417° 3. 0.2° 5. 25.555°

7. 165.783° 9. 33.099° 11. 159.983°

13. 48.2° 15. $71^\circ 48'$ 17. 106.40°

19. 15 21. 6 23. $6\sqrt{5}$ 25. $\sqrt{14}$

27. $50\sqrt{13}$ 29. 4 31. $\sqrt{185.31} \approx 13.6$

33. $7\sqrt{2}$ 35. $3\sqrt{47}$ 37. $\sqrt{2}$

39. 60.8 feet 41. 90.1 feet 43. 20.07

ohms 45. 3,770 ohms 47. 22.9

minutes 49. The reach of the ladder
decreases by about 1 foot, not 5 feet.

51. $\sec \alpha = 3$ 53. $\cot \beta = \frac{\sqrt{2}}{2}$

55. $\tan \theta = \frac{\sqrt{5}}{5}$ 57. $\sec \theta = \frac{4}{3}\sqrt{6}$

In problems 59 through 79 values are given
in the order sin, csc, cos, sec, tan, cot.

59. $\frac{b}{c} = \frac{12}{13}, \frac{13}{12}, \frac{a}{c} = \frac{5}{13}, \frac{13}{5}, \frac{b}{a} = \frac{12}{5}, \frac{5}{12}$

61. $\frac{\sqrt{65}}{13}, \frac{\sqrt{65}}{5}, \frac{2}{13}\sqrt{26}, \frac{\sqrt{26}}{4}, \frac{\sqrt{10}}{4}, \frac{2}{5}\sqrt{10}$

63. $\frac{2}{13}\sqrt{13}, \frac{\sqrt{13}}{2}, \frac{3}{13}\sqrt{13}, \frac{\sqrt{13}}{3}, \frac{2}{3}, \frac{3}{2}$

65. $\frac{4}{7}, \frac{7}{4}, \frac{\sqrt{33}}{7}, \frac{7}{33}\sqrt{33}, \frac{4}{33}\sqrt{33}, \frac{\sqrt{33}}{4}$

67. $\frac{b}{c} = \frac{12}{13}, \frac{13}{12}, \frac{a}{c} = \frac{5}{13}, \frac{13}{5}, \frac{b}{a} = \frac{12}{5}, \frac{5}{12}$

69. $\frac{2}{3}, \frac{3}{2}, \frac{\sqrt{5}}{3}, \frac{3}{5}\sqrt{5}, \frac{2}{5}\sqrt{5}, \frac{\sqrt{5}}{2}$

71. $\frac{\sqrt{2}}{2}, \sqrt{2}, \frac{\sqrt{2}}{2}, \sqrt{2}, 1, 1$

73. $\frac{\sqrt{39}}{8}, \frac{8}{39}\sqrt{39}, \frac{5}{8}, \frac{8}{5}, \frac{\sqrt{39}}{5}, \frac{5}{39}\sqrt{39}$

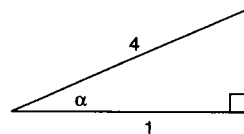
75. $\frac{1}{4}\sqrt{15}, \frac{4}{15}\sqrt{15}, \frac{1}{4}, 4, \sqrt{15}, \frac{\sqrt{15}}{15}$

77. $\frac{\sqrt{z^2 - x^2}}{z}, \frac{z}{\sqrt{z^2 - x^2}}, \frac{x}{z}, \frac{z}{x},$

$\frac{\sqrt{z^2 - x^2}}{x}, \frac{x}{\sqrt{z^2 - x^2}}$

79. $\frac{1}{3}, 3, \frac{2\sqrt{2}}{3}, \frac{3}{4}\sqrt{2}, \frac{\sqrt{2}}{4}, 2\sqrt{2}$

81.

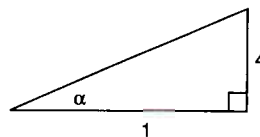


$\sec \alpha = 4, \sin \alpha = \frac{\sqrt{15}}{4},$

$\csc \alpha = \frac{4}{15}\sqrt{15}, \tan \alpha = \sqrt{15},$

$\cot \alpha = \frac{\sqrt{15}}{15}$

83.

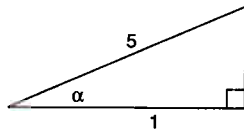


$\cot \alpha = \frac{1}{4}, \sin \alpha = \frac{4}{17}\sqrt{17},$

$\csc \alpha = \frac{\sqrt{17}}{4}, \cos \alpha = \frac{\sqrt{17}}{17},$

$\sec \alpha = \sqrt{17}$

85.

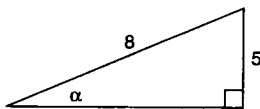


$\sec \alpha = 5, \sin \alpha = \frac{2\sqrt{6}}{5},$

$\csc \alpha = \frac{5}{12}\sqrt{6}, \tan \alpha = 2\sqrt{6},$

$\cot \alpha = \frac{\sqrt{6}}{12}$

87.

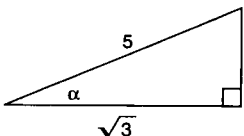


$\csc \alpha = 1.6 = \frac{8}{5}, \sin \alpha = \frac{5}{8},$

$\cos \alpha = \frac{\sqrt{39}}{8}, \sec \alpha = \frac{8}{39}\sqrt{39},$

$\tan \alpha = \frac{5}{39}\sqrt{39}, \cot \alpha = \frac{\sqrt{39}}{5}$

89.

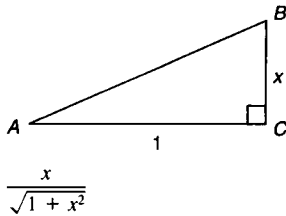


$\sec \alpha = \frac{5}{3}\sqrt{3}, \sin \alpha = \frac{\sqrt{22}}{5},$

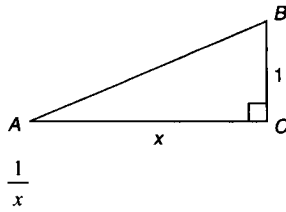
$\csc \alpha = \frac{5}{22}\sqrt{22}, \tan \alpha = \frac{\sqrt{66}}{3},$

$\cot \alpha = \frac{\sqrt{66}}{22}$

91.



93.



95. 107 knots 97. 16.6 knots

99. 27

Solutions to skill and review problems

1. Find the equation of the line that passes through the points $(-2, 5)$ and $(3, -10)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-10 - 5}{3 - (-2)} = \frac{-15}{5} = -3$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -3(x - (-2))$$

$$y = -3x - 1$$

2. Solve the inequality $|2x - 3| < 13$.

$$-13 < 2x - 3 < 13$$

$$-10 < 2x < 16$$

$$-5 < x < 8$$

3. Graph the parabola $y = x^2 - 3x - 4$.

$$y = x^2 - 3x + \frac{9}{4} - 4 - \frac{9}{4}$$

$$y = (x - \frac{3}{2})^2 - \frac{25}{4}; \text{ vertex at } (1\frac{1}{2}, -6\frac{1}{4})$$

$$(1\frac{1}{2}, -6\frac{1}{4})$$

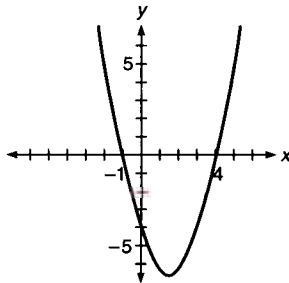
$$x\text{-intercept:}$$

$$0 = x^2 - 3x - 4$$

$$0 = (x - 4)(x + 1)$$

$$x = -1 \text{ or } 4$$

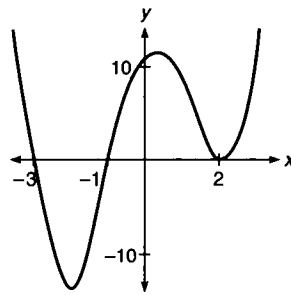
$$y\text{-intercept: } y = 0^2 - 3(0) - 4 = -4$$



4. Use the rational zero theorem and synthetic division to find all zeros of the polynomial $2x^4 + 5x^3 - 5x^2 - 5x + 3$. Synthetic division produces the zeros $-3, -1, \frac{1}{2}, 1$.

5. Graph the polynomial

$$f(x) = (x + 1)(x - 2)^2(x + 3).$$



Solutions to trial exercise problems

$$6. 87^\circ 2' 13'' = \left(87 + \frac{2}{60} + \frac{13}{3,600}\right)^\circ \approx 87.037^\circ$$

$$17. (180 - 43.45 - 30.15)^\circ = 106.40^\circ$$

$$25. c^2 = a^2 + b^2$$

$$c^2 = (\sqrt{5})^2 + 3^2$$

$$c^2 = 5 + 9$$

$$c^2 = 14$$

$$c = \sqrt{14}$$

$$34. c^2 = a^2 + b^2$$

$$(4\sqrt{5})^2 = (3\sqrt{2})^2 + b^2$$

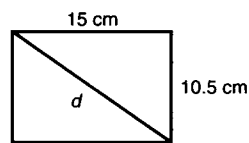
$$16 \cdot 5 = 9 \cdot 2 + b^2$$

$$62 = b^2$$

$$\sqrt{62} = b$$

$$47. d^2 = 15^2 + 10.5^2, \text{ so } d \approx 18.3 \text{ cm, and}$$

$$\frac{18.3 \text{ cm}}{0.8 \text{ cm/min}} = 22.9 \text{ minutes to make the cut.}$$



$$56. \tan \alpha = 2.25 = 2\frac{1}{4};$$

$$\cot \alpha = \frac{1}{\tan \alpha} = \frac{1}{2\frac{1}{4}} = \frac{1}{\frac{9}{4}} = \frac{4}{9}$$

$$63. a^2 + 2^2 = (\sqrt{13})^2$$

$$a^2 = 9$$

$$a = 3$$

$$\sin B = \frac{b}{c} = \frac{2}{\sqrt{13}} = \frac{2}{13}\sqrt{13},$$

$$\csc B = \frac{\sqrt{13}}{2}$$

$$\cos B = \frac{a}{c} = \frac{3}{\sqrt{13}} = \frac{3}{13}\sqrt{13},$$

$$\sec B = \frac{\sqrt{13}}{3}$$

$$\tan B = \frac{b}{a} = \frac{2}{3}, \cot B = \frac{3}{2}$$

$$79. \left(\frac{z}{3}\right)^2 + b^2 = z^2$$

$$b^2 = z^2 - \frac{z^2}{9} = \frac{8}{9}z^2$$

$$b = \frac{2\sqrt{2}}{3}z$$

$$\sin A = \frac{a}{c} = \frac{\frac{z}{3}}{\frac{2\sqrt{2}}{3}z} = \frac{z}{3} \cdot \frac{1}{z} = \frac{1}{3},$$

$$\csc A = 3$$

$$\cos A = \frac{b}{c} = \frac{\frac{2\sqrt{2}}{3}z}{\frac{2\sqrt{2}}{3}z} = \frac{2\sqrt{2}}{3}$$

$$\sec A = \frac{3}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3}{4}\sqrt{2}$$

$$\tan A = \frac{a}{b} = \frac{\frac{z}{3}}{\frac{2\sqrt{2}}{3}z} = \frac{z}{3} \cdot \frac{3}{2\sqrt{2}z}$$

$$= \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$$

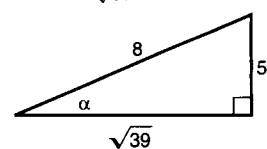
$$\cot A = \frac{1}{\frac{\sqrt{2}}{4}} = 2\sqrt{2}$$

$$87. \csc \alpha = 1.6 = \frac{1.6}{1} = \frac{16}{10} = \frac{8}{5}, \text{ so}$$

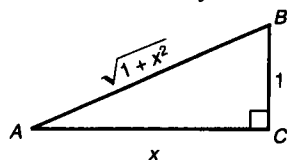
$$\sin \alpha = \frac{5}{8}, \cos \alpha = \frac{\sqrt{39}}{8},$$

$$\sec \alpha = \frac{8}{\sqrt{39}} = \frac{8}{39}\sqrt{39},$$

$$\tan \alpha = \frac{5}{\sqrt{39}} = \frac{5}{39}\sqrt{39}, \cot \alpha = \frac{\sqrt{39}}{5}$$

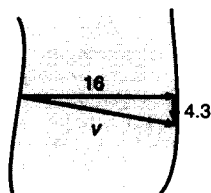


93. $\tan B = x = \frac{x}{1} = \frac{\text{opp}}{\text{adj}};$



$\tan A = \frac{\text{opp}}{\text{adj}} = \frac{1}{x}$

97. $v^2 = 16^2 + 4.3^2$
 $v \approx 16.6$ knots



Exercise 5-2

Answers to odd-numbered problems

1. 0.5192 3. 0.2116 5. 1.8137
 7. 0.6465 9. 2.5048 11. 0.9793
 13. 0.5868 15. 0.9524 17. 0.8596
 19. 0.9178 21. 4.3143 23. 0.6652
 25. 930.6 sq. ft 27. -7.56
 29. 195.2 watts 31. 48.5 mm
 33. 39.9° 35. 53.2° 37. 62.0°
 39. 31.6° 41. 31.7° 43. 63.4°
 45. 78.0° 47. 68.6°
 49. $A = 51.7^\circ$, $c \approx 19.4$, $b \approx 12.0$
 51. $B = 76.3^\circ$, $c \approx 46.9$, $b \approx 45.5$
 53. $B = 60.6^\circ$, $a \approx 0.379$, $c \approx 0.771$
 55. $A = 12.0^\circ$, $c \approx 22.3$, $a \approx 4.6$
 57. $B = 75.0^\circ$, $b \approx 9.7$, $a \approx 2.6$
 59. $A = 24.5^\circ$, $a \approx 50.6$, $b \approx 111.0$
 61. $c \approx 20.4$, $A \approx 40.0^\circ$, $B \approx 50.0^\circ$
 63. $c \approx 1.36$, $A \approx 9.3^\circ$, $B \approx 80.7^\circ$
 65. $b \approx 17.8$, $A \approx 44.9^\circ$, $B \approx 45.1^\circ$
 67. $a \approx 98.4$, $A \approx 62.5^\circ$, $B \approx 27.5^\circ$
 69. $b \approx 5.0$, $A \approx 67.4^\circ$, $B \approx 22.6^\circ$
 71. $a \approx 32.6$, $A \approx 21.2^\circ$, $B \approx 68.8^\circ$
 73. 9.44 ohms, 24.2° 75. $w \approx 194$ feet
 77. 9.51 inches
 79. $S \approx 158$ mph, $\theta \approx 11^\circ$
 81. 199,800 feet (37.8 miles)
 83. 830 feet 85. 215.9 cm
 87. $a = 4$, $c = 4\sqrt{2}$, $B = 45^\circ$

Solutions to skill and review problems

1. Graph the rational function

$$f(x) = \frac{2}{x^2 - 9}$$

Additional points:

$$f(x) = \frac{2}{(x-3)(x+3)}$$

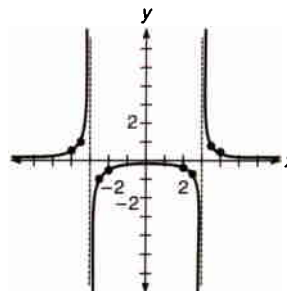
Vertical asymptotes: $x = 3$ and $x = -3$

y-intercept: $f(0) = \frac{2}{0-9} = -\frac{2}{9}$

x-intercepts: $0 = \frac{2}{x^2 - 9}$

No solution, so no x-intercepts.

x	-4	-3.5	-2.5	-2	2	2.5	3.5	4
y	0.3	0.6	-0.7	-0.4	-0.4	-0.7	0.6	0.3



2. Solve for y: $3x - 2y = 5$

$$-2y = -3x + 5$$

$$2y = 3x - 5$$

$$y = \frac{3}{2}x - \frac{5}{2}$$

$m =$ coefficient of $x = 1\frac{1}{2}$.

3. Solve equality to find critical points:

$$x^2 - 2x = 3$$

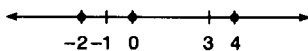
$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x-3 = 0 \text{ or } x+1 = 0$$

$$x = 3 \text{ or } x = -1$$

Choose test points on each interval.



Use -2, 0, 4: $x^2 - x > 3$

$$x = -2: (-2)^2 - (-2) > 3$$

$$6 > 3; \text{ True}$$

$$x = 0: 0 - 0 > 3; \text{ False}$$

$$x = 4: 4^2 - 4 > 3$$

$$12 > 3; \text{ True}$$

The result is those intervals where the test points make $x^2 - 2x > 3$ true.

Set-builder notation

$$\{x | x < -1 \text{ or } x > 3\}$$

$$(-\infty, -1) \text{ or } (3, \infty)$$

4. $c^2 = 16.8^2 + 9.0^2$

$$c = \sqrt{16.8^2 + 9.0^2} \approx 19.1$$

Solutions to trial exercise problems

6. $\csc 5.15^\circ \approx 11.1404$

$$5.15 \text{ [sin] } [1/x]$$

$$\text{TI-81: } ([) \text{ [SIN] } 5.15 \text{ [)] } [x^{-1}]$$

[ENTER]

21. $\cot 13^\circ 3' \approx 4.3143$

$$13 \text{ [°] } 3 \text{ ['] } [÷] 60 \text{ [=]}$$

$$\text{[tan] } [1/x]$$

$$\text{TI-81: } ([) \text{ [TAN] } ([) 13 \text{ [°] } 3 \text{ ['] }$$

$$[÷] 60 \text{ [)] } [x^{-1}]$$

[ENTER]

$$42. \cos \theta = \frac{8.25}{12.5}, \theta \approx 48.7^\circ$$

$$8.25 \text{ [÷] } 12.5 \text{ [=] } \text{[shift] [cos]}$$

$$\text{TI-81: } [2\text{nd}] \text{ [COS] } ([) 8.25 \text{ [÷] }$$

$$12.5 \text{ [)] } \text{[ENTER]}$$

$$44. \csc \theta = 1.1243, \text{ so } \sin \theta = \frac{1}{1.1243}$$

$$\theta \approx 62.8^\circ; 1.1243$$

$$[1/x] \text{ [shift] [sin]}$$

$$\text{TI-81: } [2\text{nd}] \text{ [SIN] } 1.1243 \text{ [x}^{-1}]$$

[ENTER]

$$49. a = 15.2, B = 38.3^\circ$$

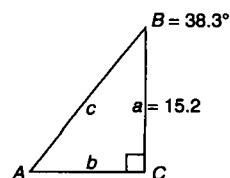
$$A = 90^\circ - 38.3^\circ = 51.7^\circ$$

$$\cos 38.3^\circ = \frac{15.2}{c};$$

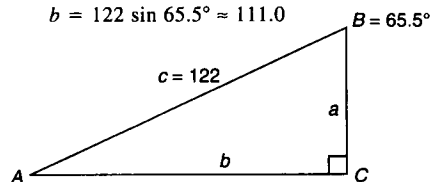
$$c = \frac{15.2}{\cos 38.3^\circ} \approx 19.4$$

$$\tan 38.3^\circ = \frac{b}{15.2};$$

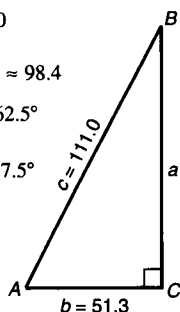
$$b = 15.2 \tan 38.3^\circ \approx 12.0$$



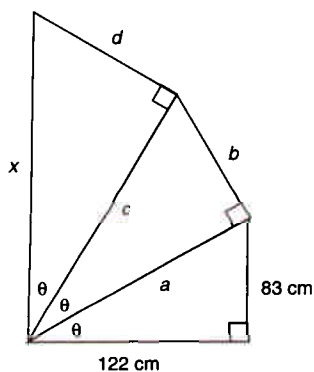
59. $c = 122$, $B = 65.5^\circ$
 $A = 90^\circ - 65.5^\circ = 24.5^\circ$
 $\cos 65.5^\circ = \frac{a}{122}$;
 $a = 122 \cos 65.5^\circ \approx 50.6$
 $\sin 65.5^\circ = \frac{b}{122}$;
 $b = 122 \sin 65.5^\circ \approx 111.0$



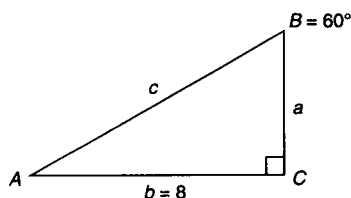
67. $b = 51.3$, $c = 111.0$
 $a^2 + 51.3^2 = 111^2$
 $a = \sqrt{111^2 - 51.3^2} \approx 98.4$
 $\cos A = \frac{51.3}{111}$, $A \approx 62.5^\circ$
 $\sin B = \frac{51.3}{111}$, $B \approx 27.5^\circ$



85. $a = \sqrt{122^2 + 83^2} \approx 147.5567687$ cm
 $\tan \theta = \frac{83}{122}$; $\theta \approx 34.22854584^\circ$
 $\cos \theta = \frac{a}{c}$;
 $c = \frac{a}{\cos \theta} = \frac{147.5567687}{\cos 34.22854584^\circ}$
 ≈ 178.4672131 cm
 $\cos \theta = \frac{c}{x}$;
 $x = \frac{c}{\cos \theta} = \frac{178.4672131}{\cos 34.22854584^\circ}$
 ≈ 215.8528302 cm
 Thus $x \approx 215.9$ cm.

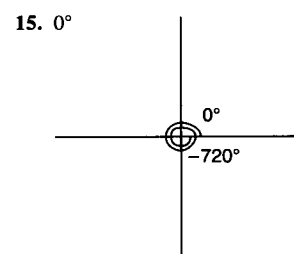
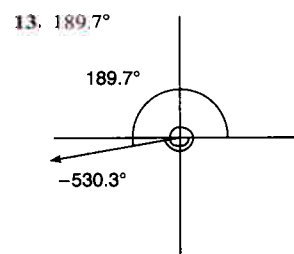
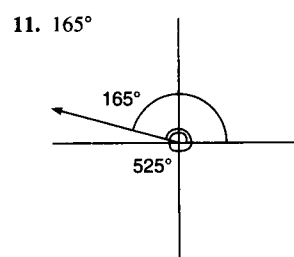
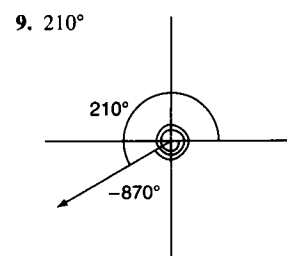
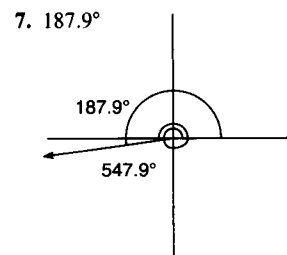
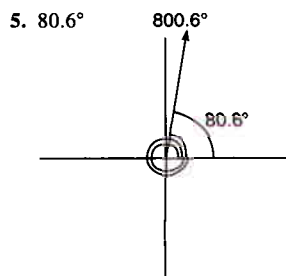
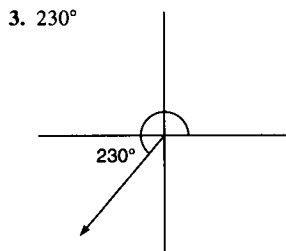
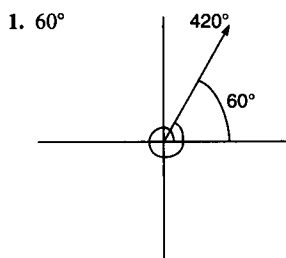


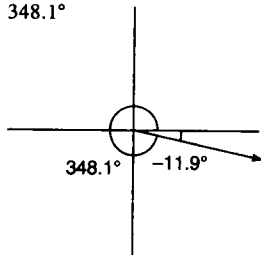
88. $A = 90^\circ - 60^\circ = 30^\circ$
 $\sin 60^\circ = \frac{8}{c}$, $\frac{\sqrt{3}}{2} = \frac{8}{c}$, $\sqrt{3}c = 16$,
 $c = \frac{16}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$;
 $\tan 60^\circ = \frac{8}{a}$, $\frac{\sin 60^\circ}{\cos 60^\circ} = \frac{8}{a}$, $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{8}{a}$,
 $\sqrt{3} = \frac{8}{a}$, $a = \frac{8}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{3}$.



Exercise 5-3

Answers to odd-numbered problems



17. 348.1° 19. 347° 21. 353.9°

In problems 23 through 37 answers are in the order sin, csc, cos, sec, tan, cot.

23. $\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{5}, \sqrt{5}, \frac{6}{3} = 2, \frac{1}{2}$

25. $\frac{8\sqrt{89}}{89}, \frac{\sqrt{89}}{8}, \frac{-5\sqrt{89}}{89}, -\frac{\sqrt{89}}{5}, -\frac{8}{5}, -\frac{5}{8}$

27. $-\frac{\sqrt{2}}{2}, -\sqrt{2}, \frac{\sqrt{2}}{2}, \sqrt{2}, -1, -1$

29. $\frac{4\sqrt{17}}{17}, \frac{\sqrt{17}}{4}, -\frac{\sqrt{17}}{17}, -\sqrt{17}, -4, -\frac{1}{4}$

31. $\frac{3\sqrt{13}}{13}, \frac{-\sqrt{13}}{3}, \frac{2\sqrt{13}}{13}, -\frac{\sqrt{13}}{2}, 1\frac{1}{2}, \frac{2}{3}$

33. $\frac{3\sqrt{38}}{19}, \frac{\sqrt{38}}{6}, \frac{-\sqrt{19}}{19}, -\sqrt{19}, -3\sqrt{2}, \frac{-\sqrt{2}}{6}$

35. $-\frac{\sqrt{10}}{5}, \frac{-\sqrt{10}}{2}, \frac{-\sqrt{15}}{5}, \frac{-\sqrt{15}}{3}, \frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{2}$

37. $-\frac{\sqrt{10}}{4}, \frac{2\sqrt{10}}{5}, \frac{\sqrt{6}}{4}, \frac{2\sqrt{6}}{3}, \frac{-\sqrt{15}}{3}, \frac{-\sqrt{15}}{5}$

$$\begin{aligned}
 39. \cot \theta &= \frac{x}{y} \quad \text{Definition of } \cot \theta \\
 &= \frac{1}{\frac{y}{x}} \\
 &= \frac{1}{\tan \theta}
 \end{aligned}
 \qquad
 \begin{aligned}
 41. \cos \theta &= \frac{x}{r} \\
 &= \frac{1}{\frac{r}{x}} \\
 &= \frac{1}{\sec \theta}
 \end{aligned}$$

43. Using (x_1, mx_1) we obtain

$$\begin{aligned}
 r_1 &= \sqrt{x_1^2 + m^2 x_1^2} = \sqrt{x_1^2(1 + m^2)} \\
 &= |x_1| \sqrt{1 + m^2}
 \end{aligned}$$

Using (x_2, mx_2) we obtain

$$\begin{aligned}
 r_2 &= \sqrt{x_2^2 + m^2 x_2^2} = \sqrt{x_2^2(1 + m^2)} \\
 &= |x_2| \sqrt{1 + m^2}
 \end{aligned}$$

For the first point:

$$\begin{aligned}
 \sin \theta &= \frac{y}{r} = \frac{mx_1}{|x_1| \sqrt{1 + m^2}} \\
 &= \pm \frac{m}{\sqrt{1 + m^2}}
 \end{aligned}$$

For the second point:

$$\begin{aligned}
 \sin \theta &= \frac{y}{r} = \frac{mx_2}{|x_2| \sqrt{1 + m^2}} \\
 &= \pm \frac{m}{\sqrt{1 + m^2}}
 \end{aligned}$$

Thus $\sin \theta$ has the same absolutevalue, $\frac{m}{\sqrt{1 + m^2}}$ in either case. Thesign of x_1 is the same as the sign of x_2 , so both values of $\sin \theta$ will have the same sign also.

Solutions to skill and review problems

1. Solve the triangle.

$$c^2 = 9^2 + 16.8^2$$

$$c = \sqrt{9^2 + 16.8^2} \approx 19.1$$

$$\tan A = \frac{9}{16.8}, A \approx 28.2^\circ$$

$$\tan B = \frac{16.8}{9}, B \approx 61.8^\circ$$

2. Use the graph of $y = \sqrt{x}$ to graph the function $f(x) = \sqrt{x} - 4 - 2$. The graph of $f(x)$ is the graph of $y = \sqrt{x}$ but shifted to the right 4 units and down 2 units. Thus the graph of $y = \sqrt{x}$ at the origin shifts to a new origin at $(4, -2)$.

x-intercept (let $y = 0$):

$$0 = \sqrt{x - 4} - 2$$

$$2 = \sqrt{x - 4}$$

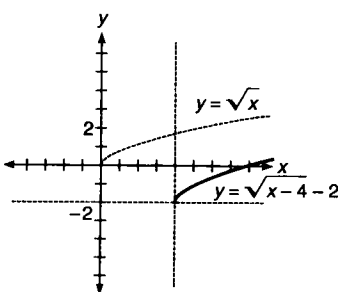
Square each side

$$4 = x - 4$$

$$8 = x$$

y-intercept (let $x = 0$):

$$y = \sqrt{0 - 4} - 2$$

Since $\sqrt{-4}$ is imaginary there is no y-intercept.

$$\begin{aligned}
 3. 8x^3 - 27 &= ((2x)^3 - 3^3) \\
 &= (2x - 3)((2x)^2 + (2x)(3) + 3^2) \\
 &= (2x - 3)(4x^2 + 6x + 9)
 \end{aligned}$$

4. $\frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{6}}{6} = \frac{\sqrt{6}}{2}$

5. $\frac{5 - 3^2}{8} - 2 = \frac{5 - 9}{8} - 2 = \frac{-4}{8} - 2$

$$= -\frac{1}{2} - \frac{4}{2} = \frac{-1 - 4}{2} = -\frac{5}{2}$$

or $-2\frac{1}{2}$

6. $C = 2\pi(15) \approx 2(3.14)(15) \approx 94.2$ inches

Solutions to trial exercise problems

35. $(-\sqrt{3}, -\sqrt{2})$

$$r = \sqrt{(-\sqrt{3})^2 + (-\sqrt{2})^2} = \sqrt{5}$$

$$\sin \theta = \frac{-\sqrt{2}}{\sqrt{5}} = -\frac{\sqrt{10}}{5}$$

$$\csc \theta = \frac{1}{\sin \theta} = -\frac{\sqrt{5}}{\sqrt{2}} = -\frac{\sqrt{10}}{2}$$

$$\cos \theta = \frac{-\sqrt{3}}{\sqrt{5}} = -\frac{\sqrt{15}}{5}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\frac{\sqrt{5}}{\sqrt{3}} = -\frac{\sqrt{15}}{3}$$

$$\tan \theta = \frac{-\sqrt{2}}{-\sqrt{3}} = \frac{\sqrt{6}}{3}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{6}}{2}$$

40. $\cot \theta = \frac{x}{y}$ if $y \neq 0$, definition of $\cot \theta$

$$= \frac{r}{\frac{y}{r}} \quad \text{Ok since } r \neq 0$$

$$= \frac{\cos \theta}{\sin \theta} \quad \text{definition of } \cos \theta, \sin \theta$$

If $y = 0$, then $\sin \theta = 0$ (since it is $\frac{y}{r}$). Thus if we restrict \sin θ so $\sin \theta \neq 0$, then $y \neq 0$, and thefraction $\frac{x}{y}$ is defined.

Exercise 5-4

Answers to odd-numbered problems

1. II 3. I 5. IV 7. II 9. IV

11. III 13. 15.8° 15. 67.1° 17. 75.3° 19. 49.3° 21. 1.0° 23. 80.5° 25. 72° 27. $\frac{\sqrt{2}}{2}$ 29. $-\frac{1}{2}$ 31. $-\sqrt{3}$ 33. $-\frac{\sqrt{3}}{2}$

35. $\frac{1}{2}$ 37. $-\frac{\sqrt{3}}{3}$ 39. 0
 41. 1 43. $\frac{1}{2}$ 45. $-\frac{2\sqrt{3}}{3}$
 47. 0.9178 49. 0.6899 51. 0.7813
 53. 1.0263 55. 0.9967 57. -1.0367
 59. -0.6845 61. 14.5° 63. 120°
 65. -58.0° 67. -36.2°
 69. a. 110.31 b. 156 c. 89.48
 d. -65.93 e. 132.73 f. 0
 71. a. 200 lb b. 181.3 lb c. 128.6 lb

Solutions to skill and review problems

1. Let θ be 30° .
 $\sin(2\theta) = 2 \sin \theta$
 $\sin 60^\circ = 2 \sin 30^\circ$
 $\frac{\sqrt{3}}{2} = 1$; Since these values are not equal, the statement $\sin(2\theta) = 2 \sin \theta$ is not necessarily true.
 2. Let θ be 60° .
 $\sin \frac{\theta}{2} = \frac{\sin \theta}{2}$
 $\sin 30^\circ = \frac{\sin 60^\circ}{2}$
 $\frac{1}{2} = \frac{\sqrt{3}}{4}$; Since these values are not equal, the statement $\sin \frac{\theta}{2} = \frac{\sin \theta}{2}$ is not necessarily true.
 3. Let $\alpha = 30^\circ$, $\beta = 60^\circ$.
 $\sin(\alpha + \beta) = \sin \alpha + \sin \beta$
 $\sin(30^\circ + 60^\circ) = \sin 30^\circ + \sin 60^\circ$
 $\sin 90^\circ = \sin 30^\circ + \sin 60^\circ$
 $1 = \frac{1 + \sqrt{3}}{2}$; Since these values are not equal the statement $\sin(\alpha + \beta) = \sin \alpha + \sin \beta$ is not necessarily true.

Solutions to trial exercise problems

6. $\sec \theta > 0$, $\csc \theta < 0$
 $\cos \theta > 0$, $\sin \theta < 0$
 I, IV; III, IV
 IV
 19. 130.7° ; $\theta = 130.7^\circ$ in quadrant II, so $\theta' = 180^\circ - \theta = 180^\circ - 130.7^\circ = 49.3^\circ$
 31. $\tan 300^\circ$; $\theta' = 60^\circ$; $\tan 60^\circ = \sqrt{3}$. In quadrant IV so $\tan 300^\circ < 0$: $\tan 300^\circ = -\sqrt{3}$.
 43. $\sin(-690^\circ)$; -690° coterminal with 30° , $\sin(-690^\circ) = \sin 30^\circ = \frac{1}{2}$.
 52. $\tan 527.2^\circ$, -0.2272
 527.2 \tan
 TI-81: \tan 527.2 ENTER

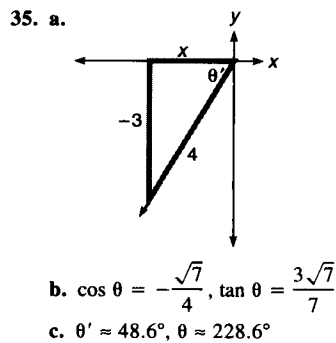
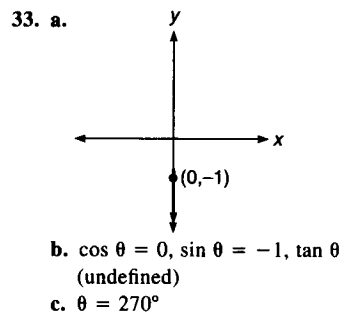
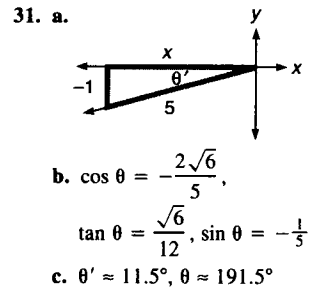
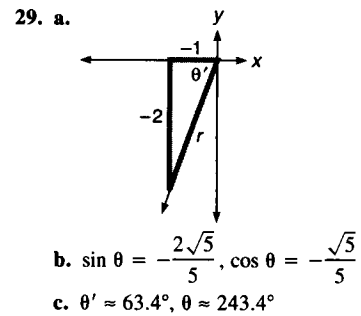
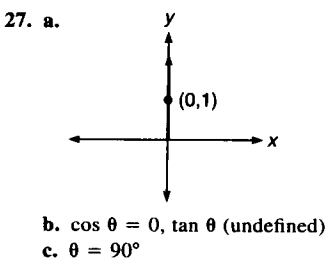
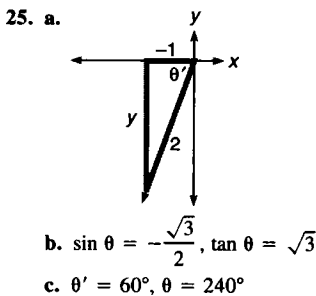
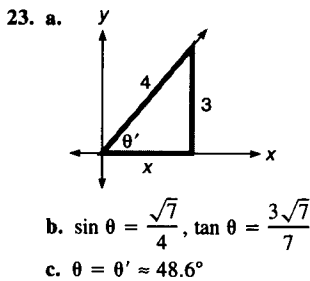
65. $\tan \theta = -\frac{8}{5}$; $\theta = \tan^{-1}(-\frac{8}{5})$
 $\approx -57.995^\circ \approx -58.0^\circ$
 8 \div 5 $=$ \pm SHIFT \tan
 TI-81: 2nd \tan $($ $(-)$ 8 $)$
 \div 5 $)$ ENTER

Exercise 5-5

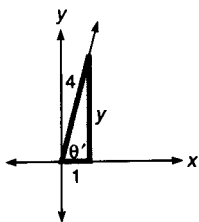
Answers to odd-numbered problems

1. 55.6° 3. 33.3° 5. 200.0° 7. 358°
 9. 22.0° 11. 168.7° 13. 224.4°
 In problems 15 through 21 part a answers are in the order \sin , \csc , \cos , \sec , \tan , \cot .

15. a. $\frac{4}{5}, \frac{5}{4}, -\frac{3}{5}, -\frac{5}{3}, -\frac{4}{3}, -\frac{3}{4}$
 b. $\theta \approx 126.9^\circ$
 17. a. $-\frac{5}{13}, -\frac{13}{5}, \frac{12}{13}, \frac{13}{12}, -\frac{5}{12}, -\frac{12}{5}$
 b. $\theta \approx 337.4^\circ$
 19. a. $-\frac{5\sqrt{41}}{41}, -\frac{\sqrt{41}}{5}, -\frac{4\sqrt{41}}{41}, -\frac{\sqrt{41}}{4}, \frac{5}{4}, \frac{4}{5}$
 b. $\theta \approx 231.3^\circ$
 21. a. $\frac{3\sqrt{13}}{13}, \frac{\sqrt{13}}{3}, \frac{2\sqrt{13}}{13}, \frac{\sqrt{13}}{2}, \frac{3}{2}, \frac{2}{3}$
 b. $\theta \approx 56.3^\circ$

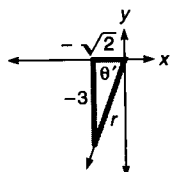


37. a.



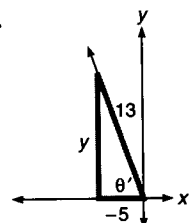
b. $\sin \theta = \frac{\sqrt{15}}{4}$,
 $\tan \theta = \frac{\sqrt{15}}{4}$, $\cos \theta = \frac{1}{4}$
 c. $\theta = \theta' \approx 75.5^\circ$

39. a.



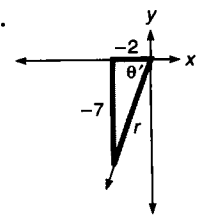
b. $\sin \theta = -\frac{3\sqrt{11}}{11}$,
 $\cos \theta = -\frac{\sqrt{22}}{11}$, $\tan \theta = \frac{3\sqrt{2}}{2}$
 c. $\theta' \approx 64.8^\circ$, $\theta \approx 244.8^\circ$

41. a.



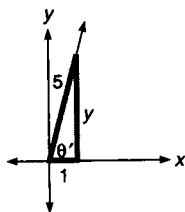
b. $\sin \theta = \frac{12}{13}$, $\tan \theta = -\frac{12}{5}$
 c. $\theta' \approx 67.4^\circ$, $\theta \approx 112.6^\circ$

43. a.



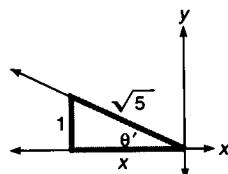
b. $\sin \theta = -\frac{7\sqrt{53}}{53}$, $\cos \theta = -\frac{2\sqrt{53}}{53}$
 c. $\theta' \approx 74.1^\circ$, $\theta \approx 254.1^\circ$

45. a.



b. $\sin \theta = \frac{2\sqrt{6}}{5}$, $\tan \theta = 2\sqrt{6}$,
 $\cos \theta = \frac{1}{5}$
 c. $\theta = \theta' \approx 78.5^\circ$

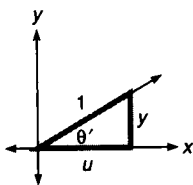
47. a.



b. $\cos \theta = -\frac{2\sqrt{5}}{5}$, $\tan \theta = -\frac{1}{2}$
 c. $\theta' \approx 26.6^\circ$, $\theta \approx 153.4^\circ$

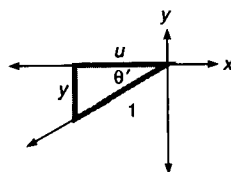
49. $\sec \theta = \frac{1}{u}$, $\sin \theta = \sqrt{1-u^2}$,

$\csc \theta = \frac{1}{\sqrt{1-u^2}}$, $\tan \theta = \frac{\sqrt{1-u^2}}{u}$,
 $\cot \theta = \frac{u}{\sqrt{1-u^2}}$

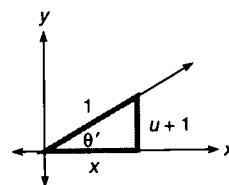


51. $\sec \theta = \frac{1}{u}$, $\sin \theta = -\sqrt{1-u^2}$,

$\csc \theta = -\frac{1}{\sqrt{1-u^2}}$,
 $\tan \theta = \frac{-\sqrt{1-u^2}}{u}$,
 $\cot \theta = \frac{-u}{\sqrt{1-u^2}}$



53. $\csc \theta = \frac{1}{u+1}$,
 $\cos \theta = \sqrt{-u^2-2u}$,
 $\sec \theta = \frac{1}{\sqrt{-u^2-2u}}$,
 $\tan \theta = \frac{u+1}{\sqrt{-u^2-2u}}$,
 $\cot \theta = \frac{\sqrt{-u^2-2u}}{u+1}$



55. $y \approx 4.77$ mm, $x \approx -4.85$ mm

57. $y \approx -5.89$ cm, $x \approx -5.77$ cm

59. The x -coordinates are ± 8.8 .

The angles are 60.6° , 119.4° , 240.6° , 299.4° .

61. $x \approx -1'10.0''$, $y \approx -1'1.5''$

63. 663.4 ft

Solutions to skill and review problems

1. -250° coterminal with $-250^\circ + 360^\circ = 110^\circ$. 110° is in quadrant II. Thus, $\theta' = 180^\circ - \theta = 180^\circ - 110^\circ = 70^\circ$.

2. $r = \sqrt{x^2 + y^2} = \sqrt{3^2 + (-4)^2} = 5$

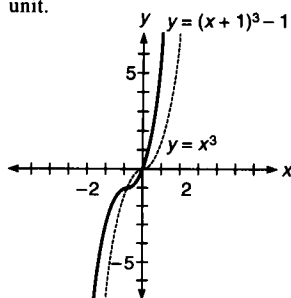
$\cos \theta = \frac{x}{r} = \frac{3}{5}$

3. $B = 90^\circ - 36.2^\circ = 53.8^\circ$

$\sin 36.2^\circ = \frac{a}{10}$, $a = 10 \sin 36.2^\circ \approx 5.9$

$\cos 36.2^\circ = \frac{b}{10}$, $b = 10 \cos 36.2^\circ \approx 8.1$

4. $y = (x + 1)^3 - 1$ is the graph of $y = x^3$ shifted down 1 unit and to the left 1 unit.



x-intercept:

$$(\text{set } y = 0) 0 = (x + 1)^3 - 1$$

$$1 = (x + 1)^3$$

$$1 = x + 1$$

$$0 = x$$

y-intercept:

$$(\text{set } x = 0) y = (0 + 1)^3 - 1$$

$$y = 0$$

Additional points:

x	-2	-1	0	1
y	-2	-1	0	7

5. $\frac{3x-5}{12} = 2(x-3) - 8x$

$$\frac{3x-5}{12} = -6x-6$$

$$\frac{3x-5}{12} \cdot 12 = 12(-6x-6)$$

$$3x-5 = -72x-72$$

$$75x-5 = -72$$

$$75x = -67$$

$$x = -\frac{67}{75}$$

6. $\frac{3x-9}{x-2} \leq 0$

This is a nonlinear inequality. We use the critical point/test point method.

Critical points:

$$\text{Solve the equality: } \frac{3x-9}{x-2} = 0$$

$$3x-9 = 0$$

$$3x = 9$$

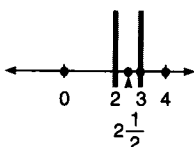
$$x = 3$$

$$\text{Find zeros of denominators: } x-2 = 0$$

$$x = 2$$

Critical points are 2 and 3.

Use test points 0, $2\frac{1}{2}$, 4.



$$\frac{3x-9}{x-2} \leq 0$$

$$x = 0: \frac{9}{2} \leq 0, \text{ false}$$

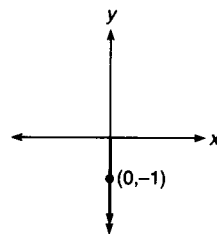
$$x = 2.5: -3 \leq 0, \text{ true}$$

$$x = 4: \frac{3}{2} \leq 0, \text{ false}$$

The solution is the interval between 2 and 3, along with the point $x = 3$.

$\{x \mid -2 < x \leq 3\}$ (set-builder notation)
 $(2, 3]$ (interval notation)

33. a.



b, c. $\csc \theta = -1$, $\sin \theta = \frac{1}{\csc \theta} = \frac{1}{-1} = -1$

θ is 270° ; pick a point, say $(0, -1)$ on the terminal side of the angle.

$$r = \sqrt{x^2 + y^2} = \sqrt{0^2 + (-1)^2} = 1.$$

$$\cos \theta = \frac{x}{r} = \frac{0}{1} = 0;$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{0} \text{ (undefined)}$$

51. $\cos \theta = u$ and θ terminates in quadrant III.

$$y = -\sqrt{1^2 - u^2} = -\sqrt{1 - u^2}$$

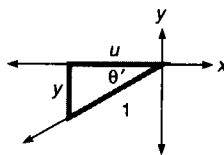
$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{u}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y = -\sqrt{1 - u^2}$$

$$\csc \theta = \frac{1}{\sin \theta} = -\frac{1}{\sqrt{1 - u^2}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{u} = \frac{-\sqrt{1 - u^2}}{u}$$

$$\cot \theta = \frac{-u}{\sqrt{1 - u^2}}$$



59. $x^2 + 15.5^2 = 17.8^2$, so $x = 8.8$.

Thus the x-coordinates are ± 8.8 .

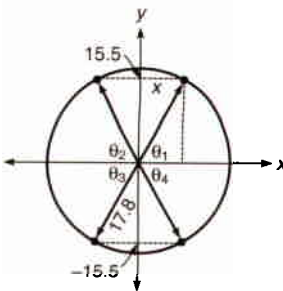
$$\sin \theta_1 = \frac{15.5}{17.8}, \text{ so } \theta_1 \approx 60.6^\circ.$$

Thus $\theta_1 \approx 60.6^\circ$

$$\theta_2 \approx 180^\circ - 60.6^\circ \approx 119.4^\circ$$

$$\theta_3 \approx 180^\circ + 60.6^\circ \approx 240.6^\circ$$

$$\theta_4 \approx 360^\circ - 60.6^\circ \approx 299.4^\circ$$



Solutions to trial exercise problems

13. $\cos \theta = -\frac{5}{7}$, $\tan \theta > 0$

$$\cos \theta' = \frac{5}{7}, \text{ so } \theta' \approx 44.4^\circ.$$

$\cos \theta < 0$, $\tan \theta > 0$ so θ is in quadrant III

$$\theta = 180^\circ + \theta' \approx 180^\circ + 44.4^\circ \approx 224.4^\circ$$

16. $(-5, -12)$; $r = \sqrt{(-5)^2 + (-12)^2} = 13$

a. $\sin \theta = \frac{y}{r} = -\frac{12}{13}$, $\csc \theta = \frac{1}{\sin \theta} = -\frac{13}{12}$

$$\cos \theta = \frac{x}{r} = -\frac{5}{13}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\frac{13}{5}$$

$$\tan \theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{5}{12}$$

$$\sin \theta' = \frac{12}{13} \text{ so } \theta' \approx 67.4^\circ$$

b. $(-5, -12)$ is in quadrant III, so θ terminates in quadrant III. Thus $\theta = 180^\circ + \theta' \approx 180^\circ + 67.4^\circ \approx 247.4^\circ$.

$$61. 4'3.5'' = 4\frac{3.5}{12} \approx 4.292'; r \approx \frac{4.292}{2} \text{ ft} \approx 2.146 \text{ ft}; \theta = 211.5^\circ.$$

$$\sin \theta = \frac{y}{r}; y = r \sin \theta; y \approx 2.146 \sin 211.5^\circ \approx -1.121 \text{ ft}.$$

$$0.121 \text{ ft} \times 12''/\text{ft} \approx 1.5'', \text{ so } y \approx -1'1.5''$$

$$\cos \theta = \frac{x}{r}, x = r \cos \theta, x \approx 2.146 \cos 211.5^\circ \approx -1.830 \text{ ft}$$

$$0.830 \text{ ft} \times 12''/\text{ft} \approx 10.0'', \text{ so } x \approx -1'10.0''$$

$$63. \sin p = \frac{AB \sin b}{AP} = \frac{512.4 \cdot \sin 28.3^\circ}{322.6} \approx 0.75302, p \approx 48.852^\circ$$

$$a = 180^\circ - (b + p) \approx 102.848^\circ$$

$$BP = \frac{AP \sin a}{\sin b} \approx \frac{322.6 \cdot \sin 102.848^\circ}{\sin 28.3^\circ} \approx 663.4 \text{ ft}$$

Exercise 5-6

Answers to odd-numbered problems

$$1. \tan \theta \cot \theta$$

$$\tan \theta \cdot \frac{1}{\tan \theta}$$

$$1$$

$$3. \cos \theta(1 - \sec \theta)$$

$$\cos \theta - \cos \theta \cdot \sec \theta$$

$$\cos \theta - \cos \theta \cdot \frac{1}{\cos \theta}$$

$$\cos \theta - 1$$

$$5. \sec \theta(\cot \theta + \cos \theta - 1)$$

$$\sec \theta \cdot \cot \theta + \sec \theta \cdot \cos \theta - \sec \theta$$

$$\frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin \theta} + \frac{1}{\cos \theta} \cdot \cos \theta - \sec \theta$$

$$\frac{1}{\sin \theta} + 1 - \sec \theta$$

$$\csc \theta - \sec \theta + 1$$

$$7. \frac{\cos \alpha - \sin \alpha}{\cos \alpha}$$

$$\frac{\cos \alpha}{\cos \alpha} - \frac{\sin \alpha}{\cos \alpha}$$

$$\frac{\cos \alpha}{\cos \alpha} - \tan \alpha$$

$$1 - \tan \alpha$$

$$9. 1 - \cos^2 \theta$$

$$(\sin^2 \theta + \cos^2 \theta) - \cos^2 \theta$$

$$\sin^2 \theta$$

$$11. \cos \beta(\sec \beta - \cos \beta)$$

$$\cos \beta \cdot \sec \beta - \cos^2 \beta$$

$$\cos \beta \cdot \frac{1}{\cos \beta} - \cos^2 \beta$$

$$1 - \cos^2 \beta$$

$$\sin^2 \beta$$

$$(\text{See problem 9.})$$

$$13. (\cos \theta + \sin \theta)(\cos \theta - \sin \theta)$$

$$+ 2 \sin^2 \theta$$

$$\cos^2 \theta - \cos \theta \sin \theta + \sin \theta \cos \theta$$

$$- \sin^2 \theta + 2 \sin^2 \theta$$

$$\cos^2 \theta + \sin^2 \theta$$

$$1$$

$$15. \csc \alpha(\cos \alpha - \sin \alpha)$$

$$\csc \alpha \cdot \cos \alpha - \csc \alpha \cdot \sin \alpha$$

$$\frac{1}{\sin \alpha} \cdot \cos \alpha - \frac{1}{\sin \alpha} \cdot \sin \alpha$$

$$\frac{\cos \alpha}{\sin \alpha} - 1$$

$$\cot \alpha - 1$$

$$17. \frac{\sin x - \cos x}{\sin x}$$

$$\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}$$

$$1 - \cot x$$

$$19. \tan \beta(\cot \beta - \cos \beta)$$

$$\tan \beta \cdot \cot \beta - \tan \beta \cdot \cos \beta$$

$$\tan \beta \cdot \frac{1}{\tan \beta} - \frac{\sin \beta}{\cos \beta} \cdot \cos \beta$$

$$1 - \sin \beta$$

$$21. \text{ a. } \sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 16^\circ 50' + \cos^2 16^\circ 50' = 1$$

$$(0.28959)^2 + (0.95715)^2 = 1$$

$$1 = 1$$

$$\text{ b. } \sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 50^\circ + \cos^2 50^\circ = 1$$

$$(0.76604)^2 + (0.64279)^2 = 1$$

$$1 = 1$$

The accuracy of these results depends on how much accuracy is used on a calculator.

$$23. \tan 32^\circ 40' \quad \left| \begin{array}{l} \sin 32^\circ 40' \\ \cos 32^\circ 40' \\ 0.53975 \\ 0.84182 \end{array} \right. \approx 0.64117$$

$$0.64117 \quad \left| \begin{array}{l} \sin 32^\circ 40' \\ \cos 32^\circ 40' \\ 0.53975 \\ 0.84182 \end{array} \right. \approx 0.64117$$

$$25. 60^\circ \quad 27. 60^\circ \quad 29. 11.5^\circ$$

$$31. 77.5^\circ \quad 33. 33.1^\circ \quad 35. 10^\circ$$

$$37. 30^\circ \quad 39. 6.5^\circ \quad 41. 30^\circ \quad 43. 34.7^\circ$$

$$45. 30^\circ \text{ or } 210^\circ \quad 47. 0^\circ \text{ or } 120^\circ$$

$$49. 0^\circ \text{ or } 180^\circ \quad 51. 153.4^\circ$$

Solutions to skill and review problems

$$1. \text{ a. } \cos \theta < 0 \text{ and } \tan \theta < 0, \text{ so } \theta \text{ is in quadrant II.}$$

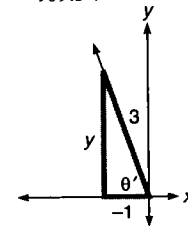
$$y = +\sqrt{3^2 - (-1)^2} = \sqrt{8} = 2\sqrt{2}.$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{y}{-1} = -2\sqrt{2}$$

$$\text{ b. } \cos \theta' = \frac{1}{3}, \text{ so } \theta' \approx 70.5^\circ$$

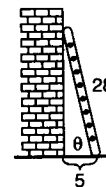
$$\theta = 180^\circ - \theta' \approx 180^\circ - 70.5^\circ$$

$$\approx 109.5^\circ.$$



$$2. \cos \theta = \frac{5}{28}$$

$$\theta = \cos^{-1} \frac{5}{28} \approx 79.7^\circ$$



$$3. C = 2\pi r$$

$$28.5 = 2\pi r$$

$$r = \frac{28.5}{2\pi} \approx 4.5 \text{ ft}$$

$$4. 7x^2 + 14x - 10 = 6x^2 + 12x + 5$$

$$x^2 + 2x - 15 = 0$$

$$(x + 5)(x - 3) = 0$$

$$x + 5 = 0 \quad \text{or} \quad x - 3 = 0$$

$$x = -5 \quad \text{or} \quad x = 3$$

$$x = -5 \text{ or } 3$$

Solutions to trial exercise problems

$$42. -3 \sin 2x = 0.75$$

$$\sin 2x = -0.25$$

$$(2x)' = \sin^{-1}(-0.25)$$

$$(2x)' \approx 14.18^\circ$$

$$2x \approx 194.48^\circ$$

$$(\text{Least positive solution in quadrant II.})$$

$$x \approx 97.2^\circ$$

$$46. 2 \sin^2 \theta + \sin \theta - 1 = 0$$

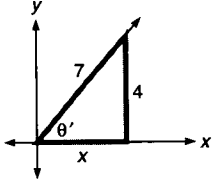
$$(2 \sin \theta - 1)(\sin \theta + 1) = 0$$

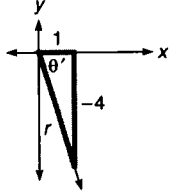
$$2 \sin \theta - 1 = 0 \quad \text{or} \quad \sin \theta + 1 = 0$$

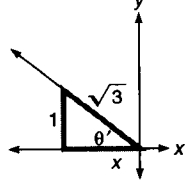
$$\sin \theta = \frac{1}{2} \quad \text{or} \quad \sin \theta = -1$$

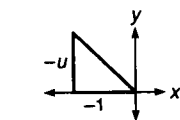
$$\theta = 30^\circ \text{ or } 270^\circ$$

Chapter 5 review

1. 165.783° 2. 37.3° 3. 78.2°
 4. $\sqrt{34} \approx 5.8$ 5. $10\sqrt{3} \approx 17.3$
 6. $2\sqrt{23} \approx 9.6$ 7. 13 8. 23.6 feet
 9. 121.0 knots 10. 0.7466
 11. 0.6080 12. 0.1823 13. 1.0033
 14. -4.18 15. 12.42° 16. 24.94°
 17. $A = 59.7^\circ$, $c \approx 14.0$, $b \approx 7.1$
 18. $B = 68.1^\circ$, $a \approx 4.7$, $b \approx 11.7$
 19. $c \approx 29.2$, $A \approx 59.1^\circ$, $B \approx 30.9^\circ$
 20. $a \approx 8.72$, $A \approx 57.0^\circ$, $B \approx 33.0^\circ$
 21. $R = 54.54$ ohms, $\theta \approx 24.6^\circ$
 22. 79 feet 23. 120° 24. 220°
 25. 176° 26. $331^\circ 15'$ 27. III
 28. IV 29. 27.4° 30. 7.7°
 31. 13.22° 32. 69.40° 33. 70°
 34. $\frac{\sqrt{2}}{2}$ 35. 0.6626 36. $-\sqrt{3}$
 37. $-\frac{2\sqrt{3}}{3}$ 38. -2.7852 39. 778.7 ft
 40. 58.3° 41. 234.4°
 Part a of 42, 43, and 44 is in this order: sin, csc, cos, sec, tan, cot.
 42. a. $-\frac{3\sqrt{10}}{10}$, $-\frac{\sqrt{10}}{3}$, $\frac{\sqrt{10}}{10}$, $\sqrt{10}$,
 -3 , $-\frac{1}{3}$
 b. $\theta \approx 288.4^\circ$
 43. a. $\frac{2\sqrt{5}}{5}$, $\frac{\sqrt{5}}{2}$, $-\frac{\sqrt{5}}{5}$, $-\sqrt{5}$, -2 , $-\frac{1}{2}$
 b. $\theta \approx 116.6^\circ$
 44. a. $-\frac{\sqrt{6}}{9}$, $-\frac{3\sqrt{6}}{2}$, $-\frac{5\sqrt{3}}{9}$, $-\frac{3\sqrt{3}}{5}$,
 $\frac{\sqrt{2}}{5}$, $\frac{5\sqrt{2}}{2}$
 b. $\theta \approx 195.8^\circ$
 45. a. 
 b. $\cos \theta = \frac{\sqrt{33}}{7}$, $\tan \theta = \frac{4\sqrt{33}}{33}$
 c. $\theta \approx 34.8^\circ$

46. a. 
 b. $\sin \theta = -\frac{4\sqrt{17}}{17}$, $\cos \theta = \frac{\sqrt{17}}{17}$
 c. $\theta \approx 284.0^\circ$

47. a. 
 b. $\sin \theta = \frac{1}{\sqrt{3}}$, $\cos \theta = \frac{\sqrt{6}}{3}$,
 $\tan \theta = \frac{\sqrt{2}}{2}$
 c. $\theta \approx 144.7^\circ$
 48. $\sin \theta = \frac{\sqrt{1-u^2}}{u}$, $\tan \theta = \frac{\sqrt{1-u^2}}{u}$
 49. $\sin \theta = \frac{-u}{\sqrt{u^2+1}}$, $\cos \theta = \frac{-1}{\sqrt{u^2+1}}$



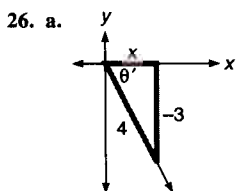
50. $\sin \theta \csc \theta$
 $\sin \theta \cdot \frac{1}{\sin \theta}$
 1
 51. $\sec \alpha (\cos \alpha - \cot \alpha)$
 $\sec \alpha \cdot \cos \alpha - \sec \alpha \cdot \cot \alpha$
 $\frac{1}{\cos \alpha} \cdot \cos \alpha - \frac{1}{\cos \alpha} \cdot \frac{\cos \alpha}{\sin \alpha}$
 $1 - \frac{1}{\sin \alpha}$
 $1 - \csc \alpha$
 52. $\frac{\sin \theta + 1}{\sin \theta}$
 $\frac{\sin \theta}{\sin \theta} + \frac{1}{\sin \theta}$
 $1 + \csc \theta$
 53. $\frac{\sin \beta - 1}{\cos \beta}$
 $\frac{\sin \beta}{\cos \beta} - \frac{1}{\cos \beta}$
 $\tan \beta - \sec \beta$

54. $\cos \theta (\sec \theta - \cos \theta)$
 $\cos \theta \cdot \sec \theta - \cos^2 \theta$
 $\cos \theta \cdot \frac{1}{\cos \theta} - \cos^2 \theta$
 $1 - \cos^2 \theta$
 $\sin^2 \theta + \cos^2 \theta - \cos^2 \theta$
 $\sin^2 \theta$
 55. $\cot x \left(\sec x - \tan x + \frac{1}{\cot^2 x} \right)$
 $\cot x \cdot \sec x - \cot x \cdot \tan x + \frac{\cot x}{\cot^2 x}$
 $\frac{\cot x}{\sin x} \cdot \frac{1}{\cos x} - 1 + \frac{1}{\cot x}$
 $\frac{1}{\sin x} - 1 + \tan x$
 $\csc x - 1 + \tan x$
 56. $(\sin \alpha - \cos \alpha)(\csc \alpha + \sec \alpha)$
 $\sin \alpha \cdot \csc \alpha + \sin \alpha \cdot \sec \alpha - \cos \alpha \cdot \csc \alpha - \cos \alpha \cdot \sec \alpha$
 $\sin \alpha \cdot \frac{1}{\sin \alpha} + \sin \alpha \cdot \frac{1}{\cos \alpha} - \cos \alpha$
 $\cdot \frac{1}{\sin \alpha} - \cos \alpha \cdot \frac{1}{\cos \alpha}$
 $1 + \frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha} - 1$
 $\tan \alpha - \cot \alpha$
 57. 30° 58. 30° 59. 59.0°
 60. 26.6° 61. 40° 62. 18.4°
 63. $\theta = 60^\circ$ or 180°

Chapter 5 test

1. 24.7° 2. $\sqrt{19}$ 3. 26 feet
 4. 262 knots 5. 0.4586 6. 0.1944
 7. 1.2723 8. 1.3380 9. -0.11
 10. 29.4° 11. $A = 70.7^\circ$, $c \approx 251.1$,
 $a \approx 237.0$ 12. $a \approx 93.5$, $A \approx 48.4^\circ$,
 $B \approx 41.6^\circ$ 13. 28.3° 14. 315°
 15. II 16. 31° 17. 12.1°
 18. -0.7683 19. $\sqrt{2}$ 20. -0.7813
 21. 1.5557 22. 342.7 meters
 23. 336.4°
 24. a. $\sin \theta = -\frac{\sqrt{5}}{5}$, $\csc \theta = -\sqrt{5}$,
 $\cos \theta = \frac{2\sqrt{5}}{5}$, $\sec \theta = \frac{\sqrt{5}}{2}$,
 $\tan \theta = -\frac{1}{2}$, $\cot \theta = -2$
 b. $\theta \approx 333.4^\circ$

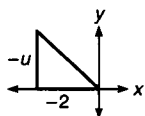
25. a. $\sin \theta = \frac{\sqrt{6}}{6}$, $\csc \theta = \sqrt{6}$,
 $\cos \theta = -\frac{\sqrt{30}}{6}$, $\sec \theta = -\frac{\sqrt{30}}{5}$,
 $\tan \theta = -\frac{\sqrt{5}}{5}$, $\cot \theta = -\sqrt{5}$
 b. $\theta \approx 155.9^\circ$



b. $\sin \theta = -\frac{3}{4}$, $\cos \theta = \frac{\sqrt{7}}{4}$,
 $\tan \theta = -\frac{3\sqrt{7}}{7}$

c. $\theta \approx 311.4^\circ$

27. $\sin \theta = -\frac{u}{\sqrt{u^2+4}}$, $\csc \theta = -\frac{\sqrt{u^2+4}}{u}$,
 $\cos \theta = -\frac{2}{\sqrt{u^2+4}}$, $\sec \theta = -\frac{\sqrt{u^2+4}}{2}$,
 $\tan \theta = \frac{u}{2}$, $\cot \theta = \frac{2}{u}$



28. $\tan \theta \cot \theta$

$\tan \theta \cdot \frac{1}{\tan \theta}$
 1

29. $\sec \theta (\cos \theta - \cos^3 \theta)$

$\sec \theta \cdot \cos \theta - \sec \theta \cdot \cos^3 \theta$

$\frac{1}{\cos \theta} \cdot \cos \theta - \frac{1}{\cos \theta} \cdot \cos^3 \theta$

$1 - \cos^2 \theta$
 $\sin^2 \theta + \cos^2 \theta - \cos^2 \theta$
 $\sin^2 \theta$

30. $(\sin \theta + \cos \theta)^2 - \sin \theta \cos \theta$

$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta - \sin \theta \cos \theta$

$\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - \sin \theta \cos \theta$

$\cos \theta$
 $1 + \sin \theta \cos \theta$

31. 45° 32. 121.0° 33. 25.8°

34. 30° or 150°

Chapter 6

Exercise 6-1

Answers to odd-numbered problems

1. $x^2 + y^2 = 1$ 3. $\frac{\pi}{4} \approx 0.79$

5. $\frac{5\pi}{9} \approx 1.75$ 7. $-\frac{5\pi}{3} \approx -5.24$

9. $\frac{3\pi}{2} \approx 4.71$ 11. $\frac{127\pi}{180} \approx 2.22$

13. $-\frac{61\pi}{36} \approx -5.32$ 15. 330° 17. 108°

19. 40° 21. -510° 23. $\frac{270^\circ}{\pi} \approx 85.94^\circ$

25. $-\frac{2,160^\circ}{17\pi} \approx -40.4^\circ$ 27. $\frac{360^\circ}{\pi} \approx 114.6^\circ$

29. $-\frac{900^\circ}{\pi} \approx -286.5^\circ$ 31. 0.7833

33. 0.5463 35. 1.5523 37. 0.7457

39. 1.4235 41. 1.6709 43. $\frac{\sqrt{3}}{2}$

45. $\frac{\sqrt{3}}{2}$ 47. $-\frac{1}{2}$ 49. $-\frac{\sqrt{2}}{2}$

51. $-\sqrt{3}$ 53. 2.7 radians 55. 6.5

inches 57. 24.0 in. 59. 4.30

61. a. 168.3 volts b. -195.5 volts

c. -132.7 volts d. -198.4 volts

63. a. -0.030 b. -0.366 65. Values

obtained with TI-81 calculator.

a. 0.6967025778 b. 0.5402777778

c. 0.2673002653 d. 0.984807753

67. $\frac{343}{18}\pi \approx 59.86 \text{ mm}^2$ 69. 30 in.^2

71. $\frac{64}{5}\pi \approx 40.21 \text{ in.}^2$ 73. $\frac{27}{8}\pi \approx 10.60 \text{ mm}^2$

75. 1.24 radians, 71.12°

Solutions to skill and review problems

1. $\frac{\csc x - 1}{\csc x}$

$\frac{\csc x}{\csc x} - \frac{1}{\csc x}$

$1 - \sin x$

2. $A = (n_1 - 1) + b(n_2 - n_1)$

$A = n_1 - 1 + bn_2 - bn_1$

$A - n_1 + 1 + bn_1 = bn_2$

$\frac{A + n_1(b - 1) + 1}{b} = n_2$

3. $A = (n_1 - 1) + b(n_2 - n_1)$

$A = n_1 - 1 + bn_2 - bn_1$

$A + 1 - bn_2 = n_1 - bn_1$

$A + 1 - bn_2 = n_1(1 - b)$

$\frac{A + 1 - bn_2}{1 - b} = n_1$

4. $\sin \theta = \frac{y}{r}$, so $\sin 30^\circ = \frac{b}{6}$;

$b = 6 \sin 30^\circ = 6 \cdot \frac{1}{2} = 3$

$\cos \theta = \frac{x}{r}$, so $\cos 30^\circ = \frac{a}{6}$;

$a = 6 \cos 30^\circ = 6 \cdot \frac{\sqrt{3}}{2} = 3\sqrt{3}$.

5. Solve the inequality $|\frac{1}{2}x + 10| > 8$.

$\frac{1}{2}x + 10 > 8$ or $\frac{1}{2}x + 10 < -8$

$\frac{1}{2}x > -2$ or $\frac{1}{2}x < -18$

$x > -4$ or $x < -36$

$\{x | x > -4 \text{ or } x < -36\}$

Solutions to trial exercise problems

12. $\frac{s}{\pi} = \frac{-422^\circ}{180^\circ}$; $s = \frac{-422^\circ \cdot \pi}{180^\circ} = -\frac{211\pi}{90}$
 ≈ -7.37

21. $\theta^\circ = \frac{180^\circ}{\pi} \cdot \left(-\frac{17\pi}{6}\right) = -510^\circ$

29. $\theta^\circ = \frac{180^\circ}{\pi} \cdot (-5) = -\frac{900}{\pi} \approx -286.5^\circ$

40. $\sec 5.2 = \frac{1}{\cos 5.2} = 2.1344$

44. $\frac{5\pi}{4}$ is in quadrant III, so $\tan \frac{5\pi}{4} > 0$ and

θ' is $\frac{5\pi}{4} - \pi = \frac{\pi}{4}$.

$\tan \frac{\pi}{4} = 1$, so $\tan \frac{5\pi}{4} = 1$.

45. $\frac{11\pi}{6}$ is in quadrant IV, so $\cos \frac{11\pi}{6} > 0$,

and $\theta' = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$.

$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$, so $\cos \frac{11\pi}{6} = +\frac{\sqrt{3}}{2}$.

48. $\frac{5\pi}{6}$ is in quadrant II so $\sin \frac{5\pi}{6} > 0$,

and $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

$\sin \frac{\pi}{6} = \frac{1}{2}$, so $\sin \frac{5\pi}{6} = +\frac{1}{2}$.

56. $L = 14.5 \text{ mm}$, $r = \frac{10.3}{2} = 5.15 \text{ mm}$

$L = rs$; $14.5 = 5.15s$; $s \approx 2.816$

radians

$\frac{\theta^\circ}{180^\circ} = \frac{s}{\pi}$; $\frac{\theta^\circ}{180^\circ} = \frac{2.816}{\pi}$;

$\theta^\circ = \frac{2.816 \cdot 180^\circ}{\pi} \approx 161.3^\circ$

57. Find L where $r = \frac{32.4}{2} = 16.2$ inches

and $\theta^\circ = 85^\circ$.

$$85^\circ = \frac{17}{36}\pi$$

$$L = rs$$

$$L = 16.2 \cdot \frac{17\pi}{36} \approx 24.0 \text{ in.}$$

73. $A_p = \frac{\theta^\circ(\pi r^2)}{360^\circ} = \frac{15^\circ \cdot \pi \cdot 9^2}{360^\circ} = \frac{27}{8}\pi$
 $\approx 10.60 \text{ mm}^2$

Exercise 6-2

Answers to odd-numbered problems

1. See figures 6-10, 6-13, and 6-16.

3. a. $\frac{\pi}{2} + 2k\pi$ b. $\frac{3\pi}{2} + 2k\pi$

c. $k\pi$ 5. $k\pi$

7. a. $\frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi$

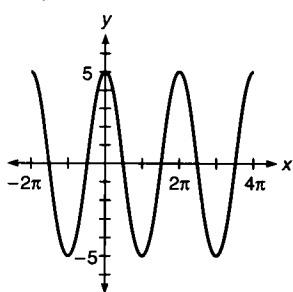
b. $\frac{5\pi}{6} + 2k\pi, \frac{7\pi}{6} + 2k\pi$

c. $\frac{3\pi}{4} + 2k\pi, \frac{5\pi}{4} + 2k\pi$

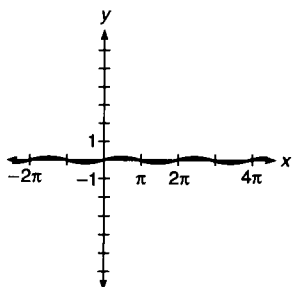
9. a. $-\frac{1}{2}$ b. $\frac{\sqrt{3}}{2}$ c. $-\frac{\sqrt{3}}{3}$

11. a. $-\frac{\sqrt{3}}{2}$ b. $-\frac{1}{2}$ c. $\sqrt{3}$

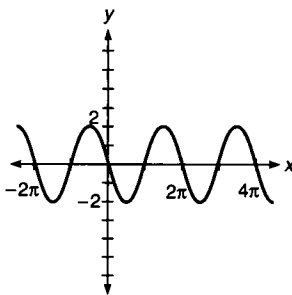
13. Amplitude is 5.



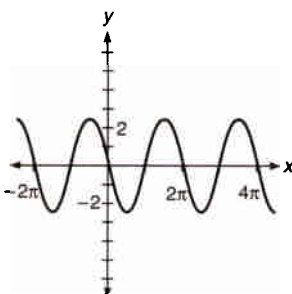
15.



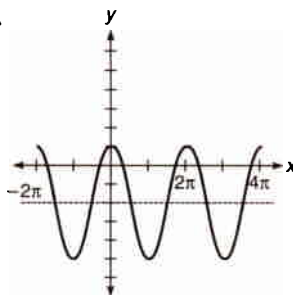
17.



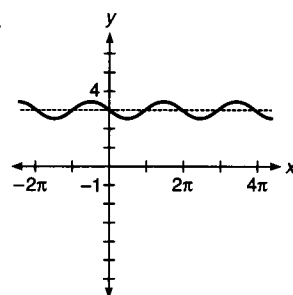
19.



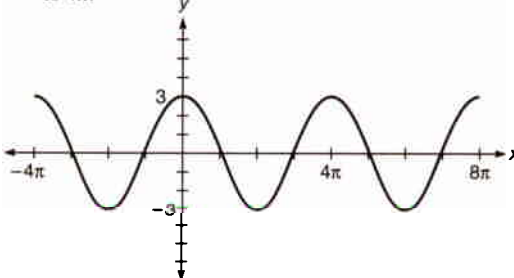
21.



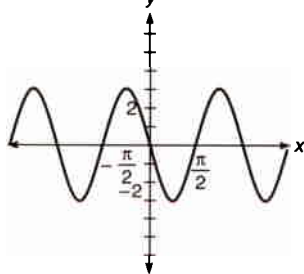
23.



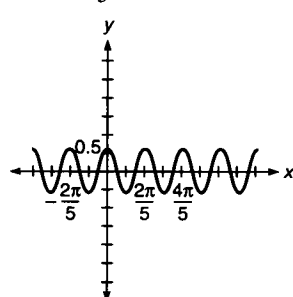
25. Amplitude is 3, phase shift is 0, period is 4π .



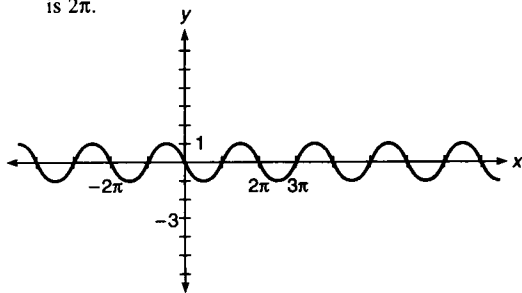
27. Amplitude is 3, phase shift is $-\frac{\pi}{2}$, period is π .



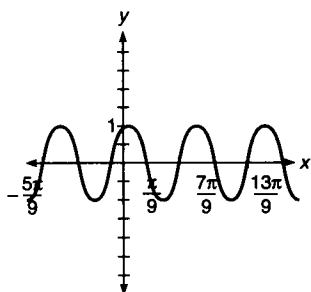
29. Amplitude is $\frac{5}{8}$, phase shift is 0, period is $\frac{2\pi}{5}$.



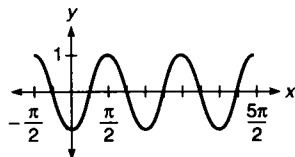
31. Amplitude is 1, phase shift is 0, period is 2π .



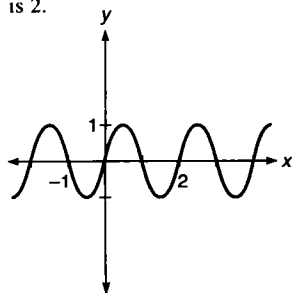
33. Amplitude is 1, phase shift is $\frac{\pi}{9}$, period is $\frac{2}{3}\pi$.



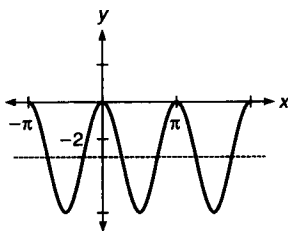
35. Amplitude is 1, phase shift is $\frac{\pi}{2}$, period is π .



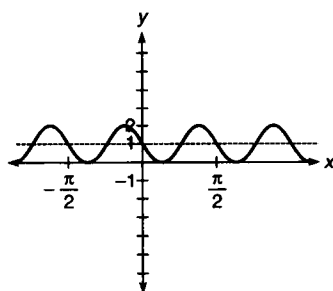
37. Amplitude is 1, phase shift is 0, period is 2.



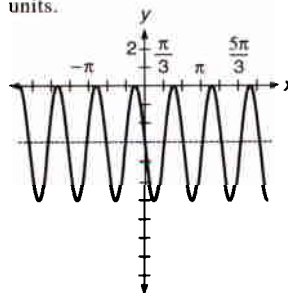
39. Amplitude is 3, phase shift is 0, period is π ; vertical shift 3 units downward.



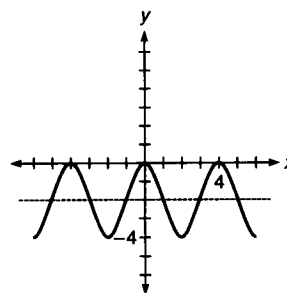
41. Amplitude is 1, phase shift is 0, period is $\frac{\pi}{2}$; vertical shift up 1 unit.



43. Amplitude is 3, phase shift is $-\frac{\pi}{3}$, period is $\frac{2\pi}{3}$; vertical shift downward 3 units.



45. Amplitude is 2; phase shift is 0, period is 4; vertical shift is 2 units downward.



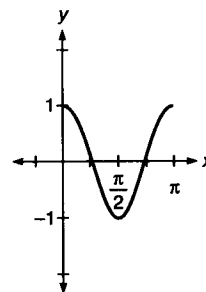
47. $y = \cos x$ 49. $y = \sin 5x$

51. $y = \cos(2x - 4)$

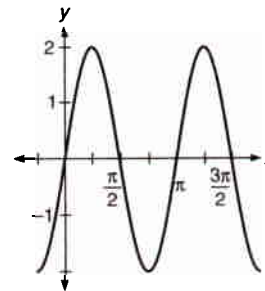
53. $y = \cos 2x + 4$

55. $y = -2 \sin\left(\frac{x}{3} + \pi\right)$

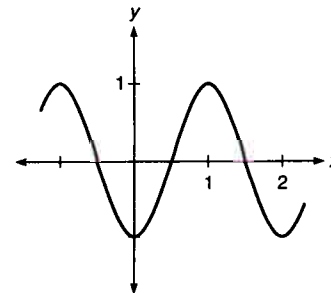
57. $y = \cos 2x$



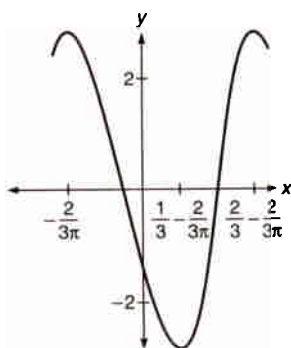
59. $y = -2 \sin(2x - \pi)$



61. $y = -\cos \pi x$



63. $y = 2 \cos(3\pi x + 2)$

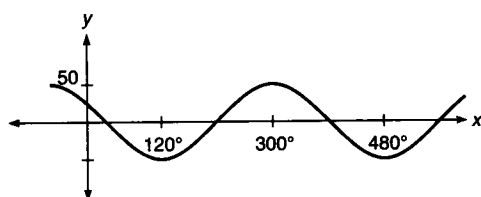


65. $y = 5 \sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

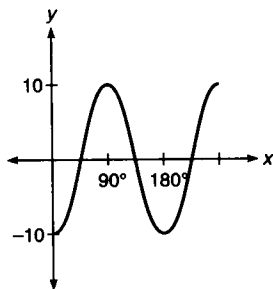
67. $y = 2 \sin \frac{2x}{3} + 3$ 69. $y = 5 \cos \frac{\pi x}{2}$

71. $y = 2 \cos\left(\frac{2}{3}x - \frac{\pi}{2}\right) + 3$

73.



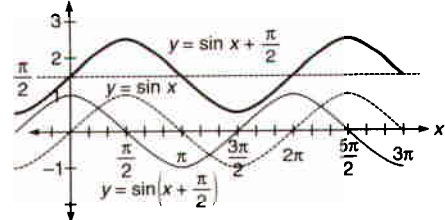
75.



77. $y = 6 \sin\left(\frac{x}{2} + 90^\circ\right)$

79. $y = \sin(11x - 990^\circ) + 2$

81.



83. odd; symmetry across the origin

85. even; symmetry about the y-axis

87. even; symmetry about the y-axis

89. odd; symmetry across the origin

91. even; symmetry about the y-axis

93. odd; symmetry across the origin

95. odd; symmetry across the origin

97. odd; symmetry across the origin

Solutions to skill and review problems

1. $\frac{\theta^\circ}{180^\circ} = \frac{s}{\pi}$

$$\theta^\circ = \frac{180^\circ}{\pi} s$$

$$\theta^\circ = \frac{180^\circ}{\pi} \cdot \frac{5\pi}{8}$$

$$\theta^\circ = 112.5^\circ$$

2. $\frac{23}{12}\pi \approx 6.02$ radians

3. $140^\circ = \frac{7}{9}\pi$

$$L = rs$$

$$L = 16.4 \cdot \frac{7}{9}\pi \approx 40.1 \text{ mm}$$

4. Using a reference triangle we learn that

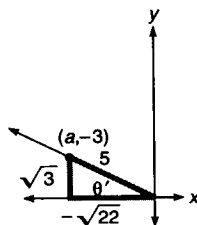
$$\tan \theta' = -\frac{\sqrt{3}}{\sqrt{22}}. \text{ Also, for any point}$$

 (x, y) on the terminal side of an angle

$$\alpha, \tan \alpha = \frac{y}{x}. \text{ For } \theta' = \frac{b}{a} = \frac{-3}{a}.$$

$$\text{Thus, } \frac{-3}{a} = -\frac{\sqrt{3}}{\sqrt{22}}, \text{ so } a\sqrt{3} = 3\sqrt{22},$$

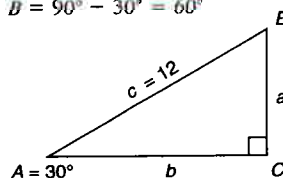
$$\text{so } a = \frac{3\sqrt{22}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{66}}{3} = \sqrt{66}.$$



5. $\sin 30^\circ = \frac{a}{12}, \frac{1}{2} = \frac{a}{12}, a = 6$

$$\cos 30^\circ = \frac{b}{12}, \frac{\sqrt{3}}{2} = \frac{b}{12}, b = 6\sqrt{3}$$

$$B = 90^\circ - 30^\circ = 60^\circ$$



Solutions to trial exercise problems

11. $\sin\left(-\frac{2\pi}{3}\right) = -\sin \frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

$$\cos\left(-\frac{2\pi}{3}\right) = \cos \frac{2\pi}{3} = -\frac{1}{2}$$

 $(\theta'$ is $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ is in quadrant II, where $\cos \theta < 0$).

$$\tan\left(-\frac{2\pi}{3}\right) = -\tan \frac{2\pi}{3}$$

$$= -(-\sqrt{3}) = \sqrt{3}$$

 $\tan \frac{2\pi}{3} < 0$ because $\frac{2\pi}{3}$ is in quadrantII, where $\tan \theta < 0$.

33. $y = -\sin\left(3x - \frac{\pi}{3}\right)$ Amplitude is 1.

Graph is reflected about the x-axis with respect to the graph of $y = \sin x$.

$$0 \leq 3x - \frac{\pi}{3} \leq 2\pi$$

$$\frac{\pi}{3} \leq 3x \leq 2\pi + \frac{\pi}{3}$$

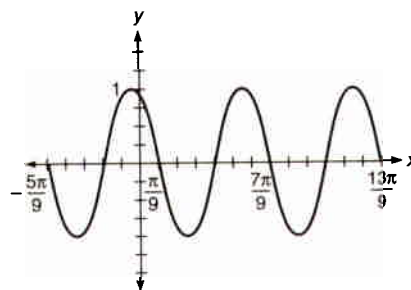
$$\frac{\pi}{3} \leq 3x \leq \frac{7\pi}{3}$$

$$\frac{1}{3} \cdot \frac{\pi}{3} \leq \frac{1}{3} \cdot 3x \leq \frac{1}{3} \cdot \frac{7\pi}{3}$$

$$\frac{\pi}{9} \leq x \leq \frac{7\pi}{9}; \text{ one basic sine cycle}$$

between $\frac{\pi}{9}$ and $\frac{7\pi}{9}$.Phase shift is $\frac{\pi}{9}$; period is $\frac{7\pi}{9}$

$$-\frac{\pi}{9} = \frac{2}{3}\pi.$$



55. $y = 2 \sin\left(-\frac{x}{3} - \pi\right)$

$$y = 2 \sin\left[-\left(\frac{x}{3} + \pi\right)\right]$$

$$y = -2 \sin\left(\frac{x}{3} + \pi\right)$$

65. Amplitude = $|A| = 5$; $A = 5$; $D = 0$, since there is no vertical translation.

A basic sine cycle runs from -1 to 3 .

To find B and C :

$$-1 \leq x \leq 3 \quad \text{Basic cycle}$$

Convert left member to 0.

$$0 \leq x + 1 \leq 4$$

Convert right member to 2π .

$$0 \leq \frac{x+1}{2} \leq 2 \quad \text{Divide by 2.}$$

$$0 \leq \frac{x+1}{2}\pi \leq 2\pi \quad \text{Multiply by } \pi.$$

$$0 \leq \frac{\pi}{2}x + \frac{\pi}{2} \leq 2\pi$$

$$\text{Thus } Bx + C = \frac{\pi}{2}x + \frac{\pi}{2},$$

$$\text{so } B = C = \frac{\pi}{2}.$$

$$\text{The equation is } y = 5 \sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right).$$

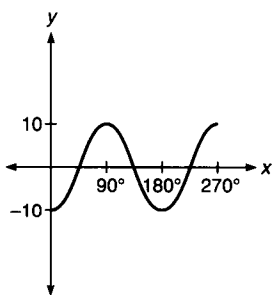
75. Amplitude is 10.

$$0 \leq 2x - 180^\circ \leq 360^\circ$$

$$180^\circ \leq 2x \leq 540^\circ$$

Add 180° to each member.

$$90^\circ \leq x \leq 270^\circ; \text{ one basic sine cycle.}$$



93. $f(x) = \frac{\cos x}{x}$

$$f(-x) = \frac{\cos(-x)}{-x} = \frac{\cos x}{-x} = -\frac{\cos x}{x}$$

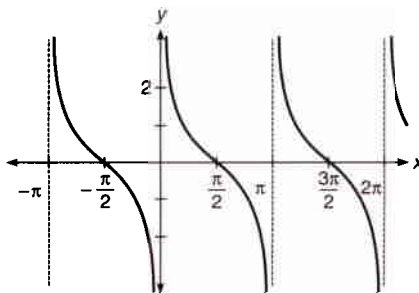
$$-f(x) = -\frac{\cos x}{x}$$

Thus $f(-x) = -f(x)$, so the function is odd. The symmetry would be across the origin.

Exercise 6-3

Answers to odd-numbered problems

1. Using figure 6-22 we obtain the following graph of the cotangent function.



Figures 6-23 and 6-24 are the graphs of the cosecant and secant functions.

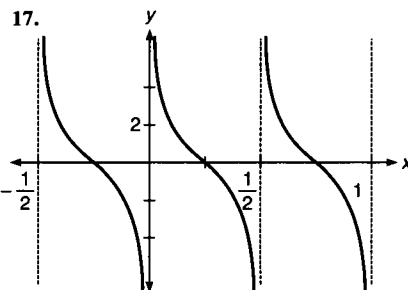
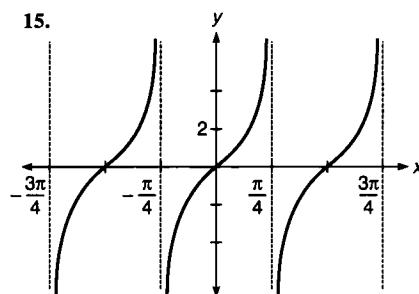
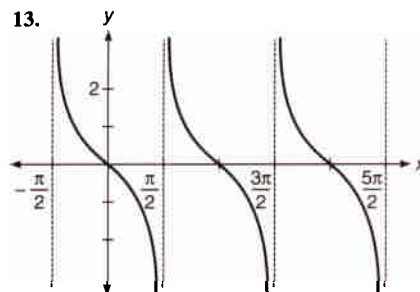
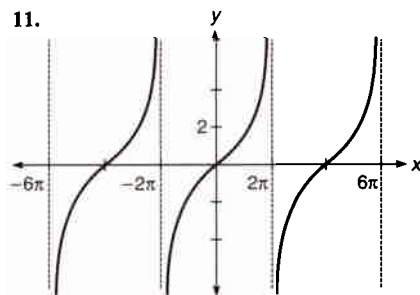
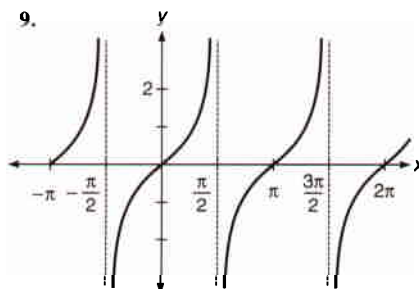
$$\begin{aligned} 3. \csc(-x) &= \frac{1}{\sin(-x)} = \frac{1}{-\sin x} \\ &= -\frac{1}{\sin x} = -\csc x \end{aligned}$$

Since $\csc(-x) = -\csc x$, cosecant is an odd function.

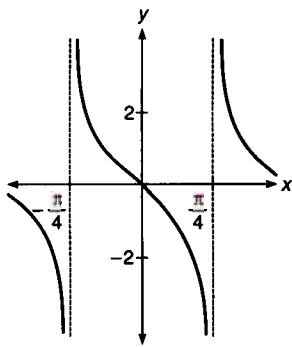
$$\begin{aligned} 5. \cot(-x) &= \frac{1}{\tan(-x)} = \frac{1}{-\tan x} \\ &= -\frac{1}{\tan x} = -\cot x \end{aligned}$$

$$\begin{aligned} 7. f(-x) &= \sin^2(-x) \tan(-x) \\ &= (-\sin x)^2 (-\tan x) \\ &= \sin^2 x (-\tan x) \\ &= -\sin^2 x \tan x \\ &= -f(x) \end{aligned}$$

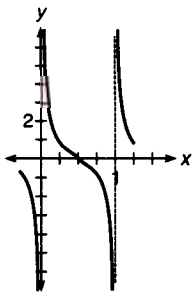
Thus f is an odd function.



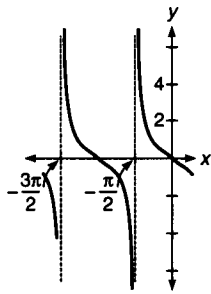
19. $-y = -\tan 2x$



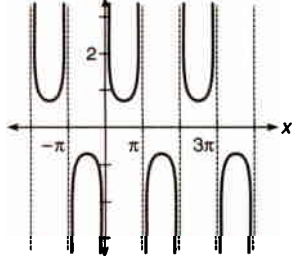
21. $y = \cot \pi x$



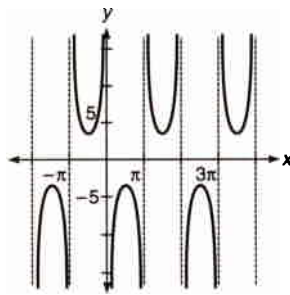
23. $y = -\tan(x + \pi)$



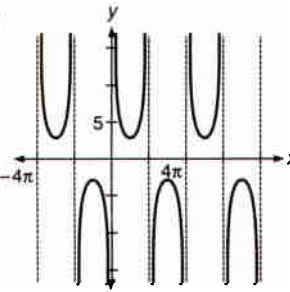
25.



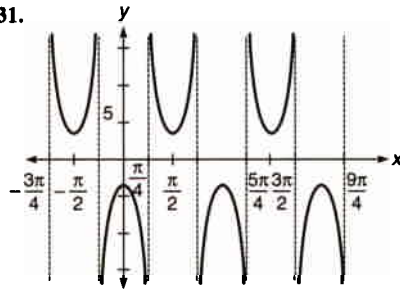
27.



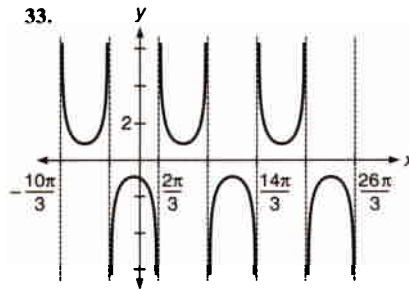
29.



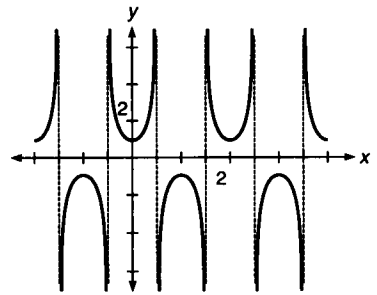
31.



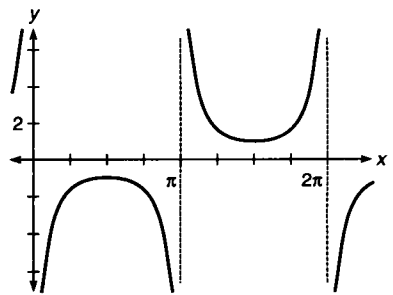
33.



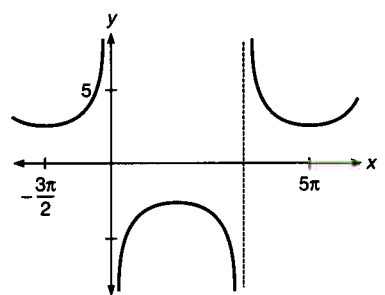
35.



37. $y = -\csc x$



39. $y = 2 \sec\left(\frac{x}{3} + \frac{\pi}{2}\right)$



Solutions to skill and review problems

1. Graph the function $f(x) = x^4 - x^3 - 7x^2 + x + 6$.

Recall that the zeros of the right member are the x -intercepts, and that the rational zero theorem and synthetic division can be used to help find these zeros. Possible rational zeros are the factors of $\frac{6}{1} = 6$. These are $\pm 1, \pm 2, \pm 3, \pm 6$.

Synthetic division with 1 shows the remainder is 0, so $x - 1$ is a factor of $f(x)$.

	1	-1	-7	1	6
		1	0	-7	-6
1	1	0	-7	-6	0

Thus, $f(x) = (x - 1)(x^3 - 7x - 6)$.

Synthetic division with -1:

	1	0	-7	-6
		-1	1	6
-1	1	-1	-6	0

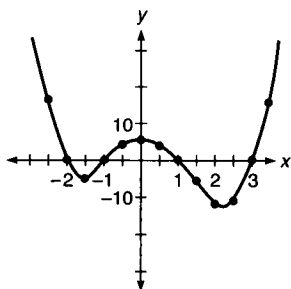
Thus, $f(x) = (x - 1)(x + 1)(x^2 - x - 6)$
 $= (x - 1)(x + 1)(x - 3)(x + 2)$

The x -intercepts of f are the zeros above, which are $-2, -1, 1, 3$.

The y -intercept is at $f(0) = 6$.

Additional points:

x	-2.5	-1.5	-0.5	0.5	1.5	2	2.5	3.3
y	14.4	-2.8	3.9	4.7	-6.6	-12	-11.8	15.7



$$\begin{aligned} 2. f(x) &= x^2 + 6x - 4 \\ &= x^2 + 6x + 3^2 - 4 - 3^2 \\ &= (x + 3)^2 - 13 \\ &= (x - (-3))^2 - 13 \end{aligned}$$

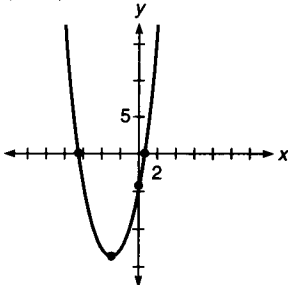
Vertex at $(h, k) = (-3, -13)$.

y -intercept: $f(0) = -4$; $(0, -4)$

x -intercepts: $0 = x^2 + 6x - 4$

$$x = \frac{-6 \pm \sqrt{36 - 4(-4)}}{2} = \frac{-6 \pm \sqrt{52}}{2} \approx -6.6, 0.6; (-6.6, 0),$$

$(0.6, 0)$

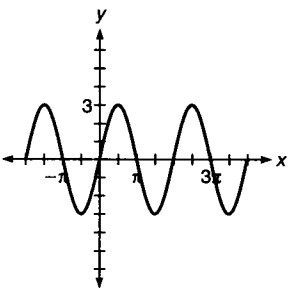


$$\begin{aligned} 3. \frac{\sqrt{3}}{\sqrt{3}-6} \cdot \frac{\sqrt{3}+6}{\sqrt{3}+6} &= \frac{3+6\sqrt{3}}{3-36} \\ &= \frac{3(1+2\sqrt{3})}{-33} = -\frac{1+2\sqrt{3}}{11} \end{aligned}$$

$$\begin{aligned} 4. (3-7i)(2+3i) \\ 6+9i-14i-21i^2 \\ 6-5i+21; i^2 = -1 \\ 27-5i \end{aligned}$$

$$\begin{aligned} 5. f(x) &= x^2 - \cos x \\ f(-x) &= (-x)^2 - \cos(-x) \\ &= x^2 - \cos x = f(x) \\ (-x)^2 &= x^2; \cos(-x) = \cos x \\ \text{Since } f(-x) &= f(x) \text{ the function is even.} \\ \text{It would have } &y\text{-axis symmetry.} \end{aligned}$$

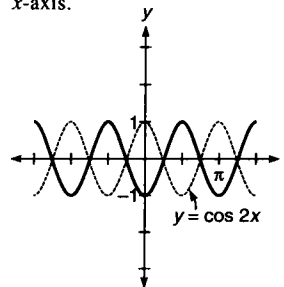
6. The graph of $f(x) = 3 \sin x$ is the graph of $y = \sin x$, but with an amplitude of 3.



7. $0 \leq 2x \leq 2\pi$

$$0 \leq x \leq \pi$$

The graph of $f(x) = -\cos 2x$ is the graph of $y = \cos x$ but with one complete cycle from 0 to π , and flipped over about the x -axis.



8. $0 \leq x + \frac{\pi}{5} \leq 2\pi$

$$-\frac{\pi}{5} \leq x \leq 2\pi - \frac{\pi}{5}$$

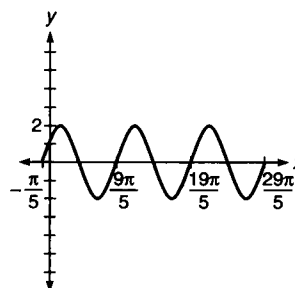
$$-\frac{\pi}{5} \leq x \leq \frac{9\pi}{5}$$

One complete sine cycle.

$$\text{Period is } \frac{9\pi}{5} - \left(-\frac{\pi}{5}\right) = 2\pi.$$

Other cycles start at $\frac{9\pi}{5}$

$$\text{and at } \frac{9\pi}{5} + 2\pi = \frac{19\pi}{5}.$$



Solutions to trial exercise problems

$$\begin{aligned} 6. f(-x) &= \sin^2(-x) \cos(-x) \\ &= [\sin(-x)]^2 \cos(-x) \\ &= [-\sin x]^2 \cos x \\ &= [\sin x]^2 \cos x \\ &= \sin^2 x \cos x \\ &= f(x) \end{aligned}$$

Thus f is an even function.

$$15. y = -\cot\left(2x + \frac{\pi}{2}\right)$$

$$0 \leq 2x + \frac{\pi}{2} \leq \pi$$

$$-\frac{\pi}{2} \leq 2x \leq \frac{\pi}{2}$$

$$-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

One basic cotangent cycle between

$$-\frac{\pi}{4} \text{ and } \frac{\pi}{4}.$$

See the answer to problem 15 for the graph.

$$23. y = \tan(-x - \pi) \\ = \tan[-(x + \pi)] \\ = -\tan(x + \pi)$$

This is the graph of $y = \tan(x + \pi)$, flipped about the x -axis.

$$-\frac{\pi}{2} \leq x + \pi \leq \frac{\pi}{2}$$

$$-\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2}$$

One basic tangent cycle between

$$-\frac{3\pi}{2} \text{ and } -\frac{\pi}{2}.$$

See the answer to problem 23 for the graph.

$$31. y = 3 \sec(2x + \pi) \\ \text{Graph three cycles of} \\ y = 3 \cos(2x + \pi).$$

$$0 \leq 2x + \pi < 2\pi$$

$$-\pi \leq 2x \leq \pi$$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

One basic secant cycle between

$$-\frac{\pi}{2} \text{ and } \frac{\pi}{2}.$$

Graph one basic cosine cycle between

$$-\frac{\pi}{2} \text{ and } \frac{\pi}{2}, \text{ then use it to sketch the}$$

related secant function. See the answer to problem 31 for the graph.

$$39. y = 2 \sec\left(-\frac{x}{3} - \frac{\pi}{2}\right)$$

$$= 2 \sec\left[-\left(\frac{x}{3} + \frac{\pi}{2}\right)\right]$$

$$= 2 \sec\left(\frac{x}{3} + \frac{\pi}{2}\right) \sec(-\theta) = \sec \theta$$

$$0 \leq \frac{x}{3} + \frac{\pi}{2} \leq 2\pi$$

$$-\frac{\pi}{2} \leq \frac{x}{3} \leq \frac{3\pi}{2}$$

$$-\frac{3\pi}{2} \leq x \leq \frac{9\pi}{2}$$

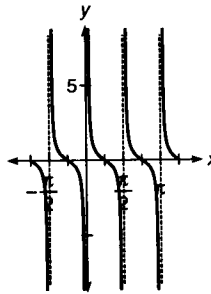
Graph a cosine cycle from $-\frac{3\pi}{2}$ to $\frac{9\pi}{2}$,

then "flip over" the graph. See the answer to problem 39 for the graph.

$$40. 0 < 2x < \pi$$

$$0 < x < \frac{\pi}{2}$$

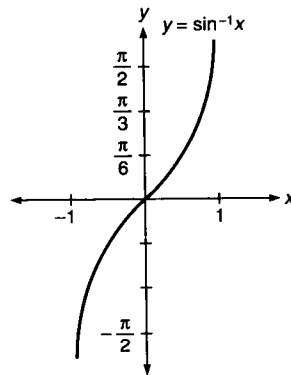
Period is $\frac{\pi}{2}$.



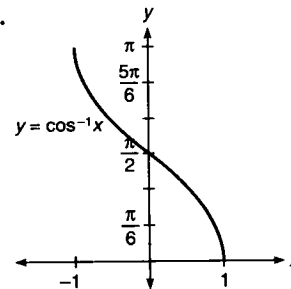
Exercise 6-4

Answers to odd-numbered problems

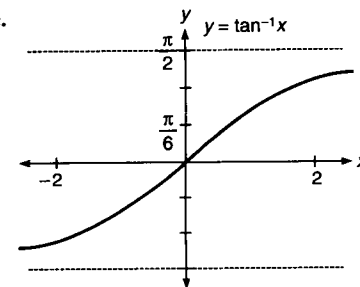
1. a.



b.



c.



$$3. \frac{\pi}{4}, 45^\circ \quad 5. \frac{\pi}{3}, 60^\circ \quad 7. \frac{\pi}{2}, 90^\circ$$

$$9. \frac{\pi}{6}, 30^\circ \quad 11. \frac{3\pi}{4}, 135^\circ \quad 13. \frac{\pi}{4}, 45^\circ$$

$$15. \pi, 180^\circ \quad 17. 0, 0^\circ \quad 19. 0, 0^\circ$$

$$21. -0.25, -14.3^\circ \quad 23. 1.48, 84.8^\circ$$

$$25. 0.75, 43.0^\circ \quad 27. 0.70, 40.1^\circ$$

$$29. -1.50, -86.0^\circ \quad 31. -1.29, -74.0^\circ$$

$$33. 2.13, 122.0^\circ \quad 35. \frac{4}{5} \quad 37. \frac{\sqrt{91}}{10}$$

$$39. \frac{10\sqrt{109}}{109} \quad 41. \frac{5\sqrt{34}}{34} \quad 43. \frac{\sqrt{39}}{8}$$

$$45. -\frac{\sqrt{5}}{2} \quad 47. \frac{\sqrt{30}}{6} \quad 49. \frac{\sqrt{66}}{3}$$

$$51. \frac{3\sqrt{5}}{5} \quad 53. \sqrt{1-z^2}$$

$$55. \frac{1+z}{\sqrt{-2z-z^2}} \quad 57. \frac{1}{\sqrt{1-2z}}$$

$$59. \sqrt{1-z^2} \quad 61. \frac{\sqrt{1-z^2}}{z}$$

$$63. \frac{1}{\sqrt{1+z^2}} \quad 65. \sqrt{1-9z^2}$$

$$67. \sqrt{z^2+2z+2} \quad 69. \frac{1}{\sqrt{1+2z}}$$

$$71. \frac{\pi}{6} \quad 73. -\frac{\pi}{6} \quad 75. \frac{\pi}{2} \quad 77. \frac{\pi}{4}$$

$$79. \frac{\pi}{6} \quad 81. 0 \quad 83. \sin^{-1} \frac{m}{r}$$

$$85. \tan^{-1} \frac{k}{h} \quad 87. \tan^{-1} \frac{2}{x} \quad 89. \sin^{-1} \frac{3,500}{z}$$

$$91. \cos^{-1}(-0.8) \quad 93. \tan^{-1} 4.1$$

$$95. \tan^{-1} \frac{5}{3} \quad 97. \tan^{-1} 50 \quad 99. \frac{1}{3} \tan^{-1} 9$$

$$101. \frac{2}{3} \sin^{-1}(-0.56) \quad 103. \frac{1}{4} \sin^{-1} 0.75$$

$$105. \frac{1}{5} \sin^{-1} \frac{25}{39} \quad 107. \frac{1}{B} \left(\tan^{-1} \frac{D}{A} - C \right)$$

$$109. \frac{1}{2} (\sin^{-1} 0.6 - 3)$$

$$111. \arcsin 1 = \frac{\pi}{2}$$

$$\arcsin(-1) = -\arcsin 1 = -\frac{\pi}{2}$$

Thus, the first and third parts of the identity are correct.

If $x = 0$, the formula $\arcsin(x)$

$$= \arctan\left(\frac{x}{\sqrt{1-x^2}}\right) \text{ is correct, since}$$

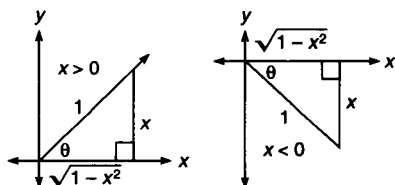
$$\arcsin 0 = \arctan\left(\frac{0}{\sqrt{1-0^2}}\right) = 0$$

The figures show the two remaining

$$\text{cases for } \theta = \arctan\left(\frac{x}{\sqrt{1-x^2}}\right),$$

when $1 > x > 0$ and when $-1 < x < 0$. In each case $r = 1$.

Examining the reference triangles shows that in each case $\sin \theta = \frac{x}{1} = x$, so that in each case $\theta = \arcsin(x)$.



Solutions to skill and review problems

1. $\cos x(\cos x + \sin x \tan x - \sec x)$

$$\cos x \cdot \cos x + \cos x \cdot \sin x \cdot \tan x - \cos x \cdot \sec x$$

$$\cos^2 x + \cos x \cdot \sin x \cdot \frac{\sin x}{\cos x} - \cos x \cdot \frac{1}{\cos x}$$

$$\frac{1}{\cos x} \cos^2 x + \sin^2 x - 1$$

$$1 - 1 = 0$$

2. $a = \sqrt{9.2^2 - 5^2} \approx 7.7$

$$\sin B = \frac{5}{9.2}; B = \sin^{-1} \frac{5}{9.2} \approx 32.9^\circ$$

$$\cos A = \frac{5}{9.2}; A = \cos^{-1} \frac{5}{9.2} \approx 57.1^\circ$$

3. $\frac{7\pi}{6}$ terminates in quadrant III,

so $\sin \frac{7\pi}{6}$ is negative, and

$$\theta' = \theta - \pi = \frac{7\pi}{6} - \frac{6\pi}{6} = \frac{\pi}{6}$$

$$\sin \frac{\pi}{6} = \frac{1}{2}, \text{ so } \sin \frac{7\pi}{6} = -\frac{1}{2}$$

4. Find a positive angle coterminal with

$$-\frac{11\pi}{3}. \text{ Add } 4\pi \text{ to get } -\frac{11\pi}{3} + 4\pi =$$

$$-\frac{11\pi}{3} + \frac{12\pi}{3} = \frac{\pi}{3}. \text{ Thus, } \tan\left(-\frac{11\pi}{3}\right) =$$

$$\tan \frac{\pi}{3} = \sqrt{3}.$$

5. $L = rs$

$$15 = 12s$$

$$L = 15, r = 12$$

$$s = \frac{5}{4} \text{ (radians)}$$

$$\frac{\theta^\circ}{180^\circ} = \frac{s}{\pi}$$

$$\theta^\circ = \frac{180^\circ}{\pi} \left(\frac{5}{4} \right) \approx 71.6^\circ$$

6. $f(x) = \frac{2x}{x^2 - 4}$

$$= \frac{2x}{(x-2)(x+2)}$$

Vertical asymptotes at -2 and 2 .

x-intercepts: (solve $f(x) = 0$)

$$0 = \frac{2x}{x^2 - 4}, \text{ so } x = 0.$$

Intercept at $(0,0)$.

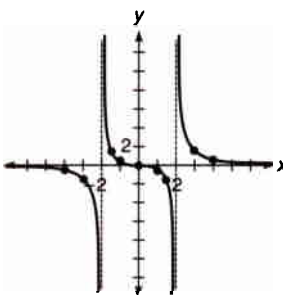
y-intercept: (compute $f(0)$)

$$f(0) = \frac{2(0)}{0 - 4} = 0.$$

Intercept at $(0,0)$.

Additional points:

x	-4	-3	-1.5	-1	1	1.5	3	4
y	-0.7	-1.2	1.7	0.7	-0.7	-1.7	1.2	0.7



Solutions to trial exercise problems

20. $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\arcsin \frac{\sqrt{3}}{2} = -\frac{\pi}{3}, -60^\circ$

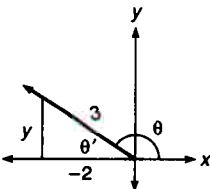
31. $\tan^{-1}(-3.4776)$

TI-81: $\boxed{3.4776} \boxed{\pm} \boxed{\text{INV}} \boxed{\text{TAN}} \boxed{\text{2nd}} \boxed{\text{TAN}} \boxed{(-)} \boxed{3.4776} \boxed{\text{ENTER}}$
 $-1.29, -74.0^\circ$

45. $\tan[\cos^{-1}(-\frac{2}{3})]$

The figure shows an angle θ whose cosine is $-\frac{2}{3}$.

$$y = \sqrt{5}; \tan \theta = \frac{y}{-2} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}.$$

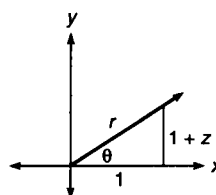


67. $\sec[\tan^{-1}(1+z)], z > 0$

$$r = \sqrt{1^2 + (1+z)^2} = \sqrt{z^2 + 2z + 2}$$

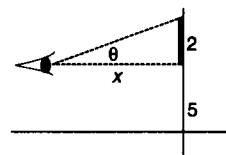
$$\cos \theta = \frac{1}{r}; \sec \theta = \frac{1}{\cos \theta}$$

$$= r = \sqrt{z^2 + 2z + 2}$$



79. $\cos^{-1}\left(\cos \frac{11\pi}{6}\right) = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}$

87. $\tan \theta = \frac{2}{x}$, so $\theta = \tan^{-1} \frac{2}{x}$.



105. $\frac{6 \sin 50}{5} = \frac{10}{13}$

$$6 \sin 50 = \frac{50}{13}$$

$$\sin 50 = \frac{1}{6} \cdot \frac{50}{13}$$

$$\sin 50 = \frac{25}{39}$$

$$50 = \sin^{-1} \frac{25}{39}$$

$$\theta = \frac{1}{3} \sin^{-1} \frac{25}{39}$$

109. $\sin(2x + 3) = 0.6$

$$2x + 3 = \sin^{-1} 0.6$$

$$2x = \sin^{-1} 0.6 - 3$$

$$x = \frac{1}{2}(\sin^{-1} 0.6 - 3)$$

Exercise 6-5

Answers to odd-numbered problems

1. $\frac{\pi}{6}$ 3. $\frac{\pi}{4}$ 5. $\frac{2\pi}{3}$ 7. $\frac{\pi}{3}$ 9. $\frac{\pi}{2}$

11. 0.30, 17.3° 13. 1.91, 109.5°

15. 0.19, 10.9° 17. 1.66, 95.2°

19. 0.32, 18.3° 21. $\sin(\csc^{-1} 3)$

$$= \sin(\sin^{-1} \frac{1}{3}) = \frac{1}{3}$$

23. $\frac{\sqrt{15}}{15}$

25. $\sqrt{26}$ 27. $\frac{3}{5}$ 29. $-\frac{\sqrt{11}}{5}$ 31. $\frac{1}{z}$
 33. $\frac{\sqrt{z^2+1}}{z}$ 35. $\frac{1}{2z}$ 37. $\sqrt{z^2+2z}$
 39. $\frac{3}{z}$ 41. 0.216

Solutions to skill and review problems

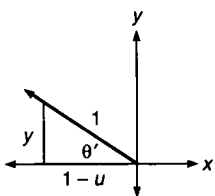
1. $\frac{2x}{x-3} - \frac{x}{x+5}$

$$\frac{2x(x+5) - x(x-3)}{(x-3)(x+5)}$$

$$\frac{x^2 + 13x}{x^2 + 2x - 15}$$

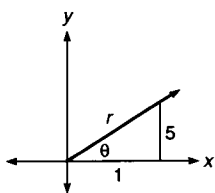
2. $\sin x' = \frac{1}{2}$, so $x' = \frac{\pi}{6}$. Since x terminates in quadrant III, $x = \pi + x'$
 $= \pi + \frac{\pi}{6} = \frac{7\pi}{6}$.

3. $y = \sqrt{1^2 - (1-u)^2} = \sqrt{2u - u^2}$;
 $\sin \theta = \frac{y}{1} = y = \sqrt{2u - u^2}$.

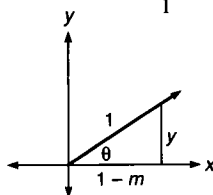


4. $3 \sin x = -2$
 $\sin x = -\frac{2}{3}$
 $\sin x' = \frac{2}{3}$, so $x' \approx 41.8^\circ$.
 Since $\sin x < 0$, x terminates in quadrants III or IV. The least nonnegative value is in quadrant III.
 Thus, $x = 180^\circ + x' \approx 221.8^\circ$.

5. $\cos(\tan^{-1}5)$
 $r = \sqrt{26}$; $\cos \theta = \frac{1}{r} = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$



6. $y = \sqrt{1^2 - (1-m)^2} = \sqrt{2m - m^2}$;
 $\csc \theta = \frac{1}{\sin \theta} = \frac{1}{\frac{y}{1}} = \frac{1}{y} = \frac{1}{\sqrt{2m - m^2}}$



7. $\csc\left(\frac{19\pi}{6}\right)$; $\frac{19\pi}{6} - 2\pi = \frac{7\pi}{6}$
 $-\frac{12\pi}{6} = \frac{7\pi}{6}$. Thus, $\csc\left(\frac{19\pi}{6}\right) =$
 $\csc\left(\frac{7\pi}{6}\right) = \frac{1}{\sin\left(\frac{7\pi}{6}\right)}$.

$\frac{7\pi}{6}$ terminates in quadrant III, so $\sin \frac{7\pi}{6}$
 < 0 and $\theta' = \frac{7\pi}{6} - \pi = \frac{\pi}{6}$.

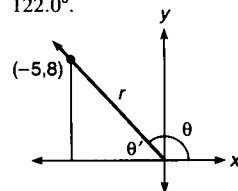
$\sin \frac{\pi}{6} = \frac{1}{2}$, so $\sin \frac{7\pi}{6} = -\frac{1}{2}$.

Thus, $\csc\left(\frac{19\pi}{6}\right) = \frac{1}{-\frac{1}{2}} = -2$.

8. a. $r = \sqrt{89}$; $\sin \theta = \frac{y}{r} = \frac{8}{\sqrt{89}}$

b. $\theta' = \tan^{-1} \left| \frac{y}{x} \right| = \tan^{-1} \frac{8}{5} \approx 58.0^\circ$

Since θ terminates in quadrant II,
 $\theta = 180^\circ - \theta' \approx 180^\circ - 58.0^\circ \approx$
 122.0° .



Solutions to trial exercise problems

7. $\operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right) = \sin^{-1}\left(\frac{1}{\frac{2\sqrt{3}}{3}}\right)$
 $= \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

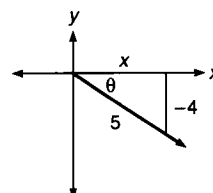
17. $\sec^{-1}(-11.1261) = \cos^{-1}\left(-\frac{1}{11.1261}\right)$
 ≈ 1.66 (calculator in radian mode)
 $\approx 95.2^\circ$ (calculator in degree mode)

11.1261 $\left[\frac{1}{x}\right]$ $\left[\pm\right]$ $\left[\text{INV}\right]$ $\left[\text{COS}\right]$

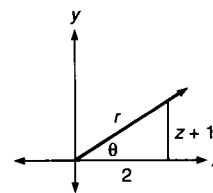
TI-81: $\left[\text{2nd}\right]$ $\left[\text{COS}\right]$ $\left[\left(-\right)\right]$

11.1261 $\left[x^{-1}\right]$ $\left[\text{ENTER}\right]$

27. $\cos[\operatorname{arccsc}(-\frac{5}{4})] = \cos[\sin^{-1}(-\frac{4}{5})]$
 $x = 3$; $\cos \theta = \frac{x}{5} = \frac{3}{5}$



40. $\sec\left(\cot^{-1}\left(\frac{2}{z+1}\right)\right)$, $z+1 > 0$
 Since $\cot \theta = \frac{2}{z+1}$, $\tan \theta = \frac{z+1}{2}$
 $r = \sqrt{(z+1)^2 + 2^2} = \sqrt{z^2 + 2z + 5}$;
 $\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{2}{r}} = \frac{r}{2}$
 $= \frac{1}{2}\sqrt{z^2 + 2z + 5}$



Chapter 6 review

1. $\frac{7}{4}\pi \approx 5.50$ 2. $-\frac{4}{3}\pi \approx -4.19$

3. $-\frac{37}{45}\pi \approx -2.58$ 4. $\frac{16}{9}\pi \approx 5.59$

5. 140° 6. 247.5° 7. $\frac{720}{\pi}$

$\approx -229.18^\circ$ 8. $\frac{135}{2\pi} \approx 21.49^\circ$

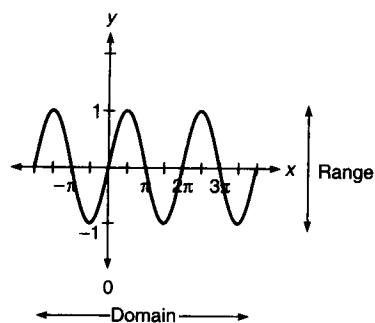
9. 30.4 inches 10. 602.1 millimeters

11. $3\frac{1}{5}$ (radians) $\approx 183.3^\circ$ 12. $2\frac{6}{7}$ radians

13. -0.7568 14. -0.8130 15. 1.0747

16. 0.6421 17. $-\frac{\sqrt{3}}{2}$ 18. $-\frac{\sqrt{3}}{3}$

19. domain: R ; range: $-1 \leq y \leq 1$; period: 2π

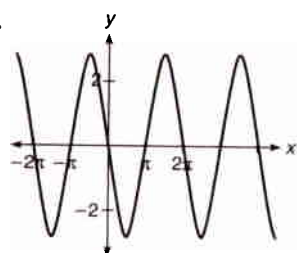


20. Even multiples of π : $\dots, -2\pi, 0, 2\pi, 4\pi, \dots$. Thus $x = 2k\pi$, k an integer.

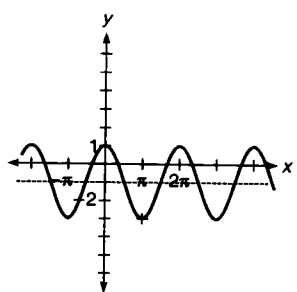
21. $-\frac{\sqrt{3}}{2}$ 22. $-\sqrt{3}$

23. even function, since $f(-x) = -f(x)$; symmetry about the y -axis 24. odd function; symmetry about the origin

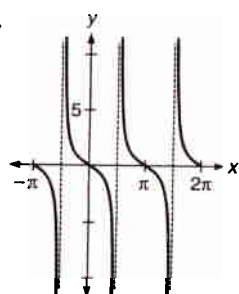
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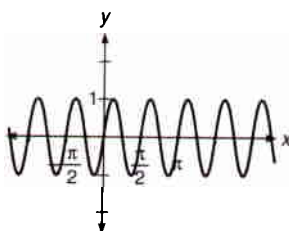
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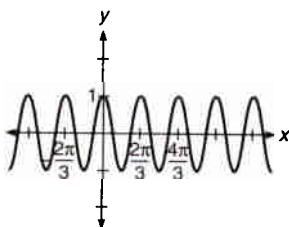
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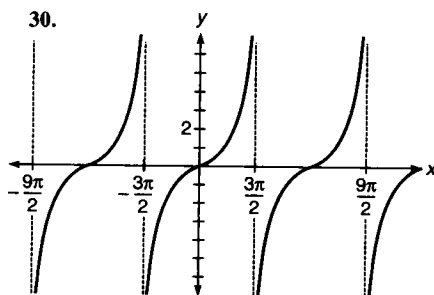
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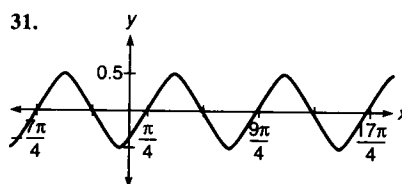
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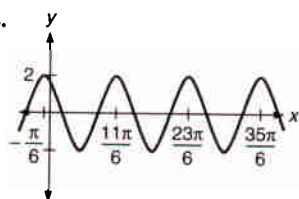
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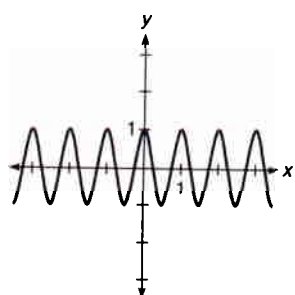
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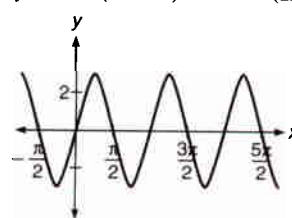
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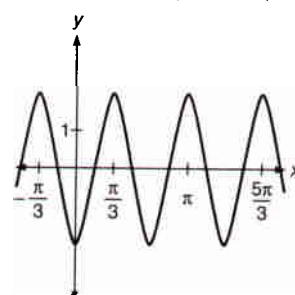
33.



34. $y = 3 \sin(\pi - 2x) = -3 \sin(2x - \pi)$



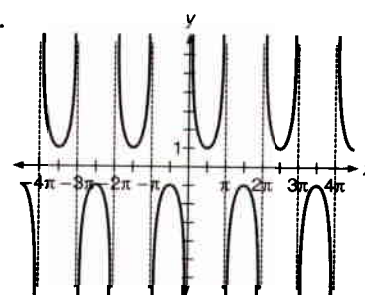
35. $y = 2 \cos(\pi - 3x) = 2 \cos(3x - \pi)$



36. $A = \frac{1}{2}$, $B = 1$, $C = -\frac{\pi}{4}$,

$D = 0$, $y = \frac{1}{2} \sin\left(x - \frac{\pi}{4}\right)$

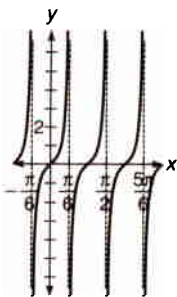
37.



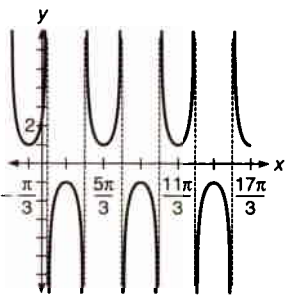
38. $f(x) = \sec x \cdot \sin^2 x + x^4$
 $f(-x) = \sec(-x)[\sin(-x)]^2 + (-x)^4$
 $= \sec x[-\sin x]^2 + x^4$
 $= \sec x[\sin x]^2 + x^4$
 $= \sec x \cdot \sin^2 x + x^4$
 $= f(x)$

Since $f(-x) = f(x)$, f is an even function.

39.



40.



41. $y = \tan(-x) = -\tan x$

The graph of $y = -\tan x$ is given in problem 27. 42. domain: $|x| \leq 1$; range: $0 \leq y \leq \pi$.

43. $\frac{\pi}{6}, 30^\circ$ 44. $\frac{\pi}{4}, 45^\circ$ 45. $\frac{\pi}{3}, 60^\circ$

46. $-\frac{\pi}{3}, -60^\circ$ 47. $-1.30, -74.3^\circ$

48. 2.01, 115.4° 49. 1.19, 68.2°

50. $\frac{\sqrt{55}}{8}$ 51. $-\frac{2\sqrt{5}}{5}$ 52. $\frac{\sqrt{3}}{2}$

53. $\frac{2\sqrt{14}}{7}$ 54. $\frac{2\sqrt{2}}{3}$ 55. $-\frac{5\sqrt{26}}{26}$

56. $\sqrt{1-4z^2}$ 57. $\frac{1-z}{\sqrt{2z-z^2}}$

58. $\sqrt{-z}$ 59. \sqrt{z} 60. $-\frac{\pi}{6}$

61. 0 62. $\frac{\pi}{3}$ 63. $\sin^{-1}\frac{6,000}{z}$

64. $3 \cos^{-1}\frac{1}{3}$ 65. $\frac{1}{2} \sin^{-1}\frac{1}{4}$

66. $\frac{1}{k} \tan^{-1}\frac{b}{a}$ 67. $\frac{\pi}{6}$ 68. $\frac{\pi}{4}$

69. 0.32, 18.4° 70. 1.96, 112.1°

71. $\frac{\sqrt{2}}{4}$ 72. $\sqrt{1+z^2}$

Chapter 6 test

1. $-\frac{25}{18}\pi, -4.36$ 2. 252° 3. 25 inches

4. $\frac{19}{14}, 77.8^\circ$ 5. 181° 6. -1.0002

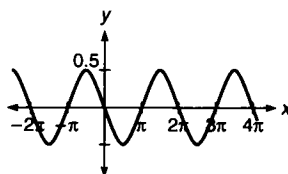
7. $\frac{\sqrt{3}}{2}$ 8. domain: all reals (\mathbb{R}); range: $-1 \leq y \leq 1$; period: 2π ; figure 6-14 shows the sketch 9. $x = \frac{3\pi}{2} + 2k\pi, k$ an integer

10. $\sqrt{3}$

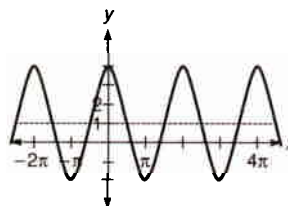
11. $f(x) = x + \sin x$
 $f(-x) = (-x) + \sin(-x)$
 $= (-x) + (-\sin x)$
 $= -(x + \sin x)$
 $= -f(x)$

Since $f(-x) = -f(x)$, f is an odd function. Its graph would have symmetry about the origin.

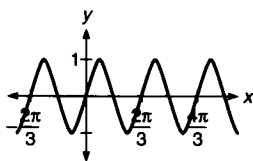
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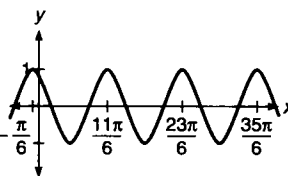
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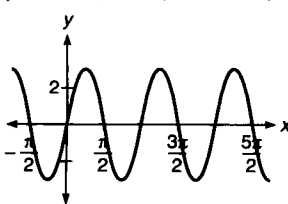
14.



15.



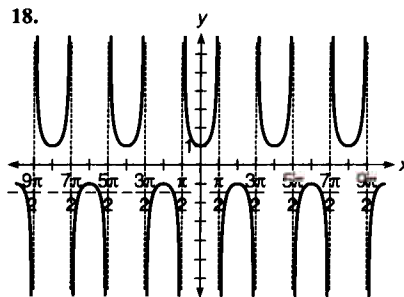
16. $y = 3 \sin(\pi - 2x) = -3 \sin(2x - \pi)$



17. $A = +2, D = 0, B = 1, C = \frac{\pi}{3}$;

$$y = 2 \sin\left(x + \frac{\pi}{3}\right)$$

18.



19. $f(x) = \sec x \cdot \sin x + x^3$

$$f(-x) = \sec(-x) \cdot \sin(-x) + (-x)^3$$

$$= \sec x \cdot (-\sin x) + (-x^3)$$

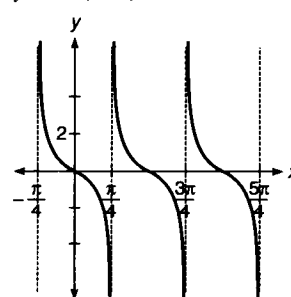
$$= -\sec x \cdot \sin x - x^3$$

$$= -(\sec x \cdot \sin x + x^3)$$

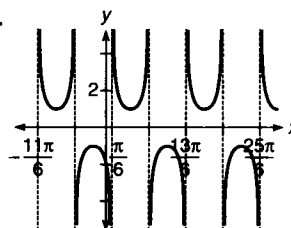
$$= -f(x)$$

Thus, $f(x)$ is an odd function.

20. $y = \tan(-2x) = -\tan 2x$



21.



22. domain: $-1 \leq x \leq 1$; range:

$0 \leq y \leq \pi$ 23. $-\frac{\pi}{3}, -60^\circ$

24. 2.50, 143.1° 25. $\frac{3\sqrt{7}}{7}$ 26. $-2\sqrt{2}$

27. $\frac{4\sqrt{3}}{3}$ 28. $\frac{1}{\sqrt{1+4z^2}}$ 29. $\frac{5\pi}{6}$

30. $\frac{\pi}{3}$ 31. $\cos^{-1}\frac{z}{5}$ 32. $\frac{1}{3} \cos^{-1}\frac{1}{6}$

33. 1.18, 67.8°

Chapter 7

Exercise 7-1

Answers to odd-numbered problems

1. $\frac{\sin \theta}{\tan \theta}$
 $\frac{\sin \theta}{\sin \theta}$
 $\frac{\sin \theta}{\cos \theta}$
 $\sin \theta \cdot \frac{\cos \theta}{\sin \theta}$
 $\cos \theta$
3. $\cot \theta \sec \theta$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta}$
 $\frac{1}{\sin \theta}$
 $\csc \theta$
5. $\cot^2 \theta \sin^2 \theta$
 $\frac{\cos^2 \theta}{\sin^2 \theta} \cdot \sin^2 \theta$
 $\cos^2 \theta$
7. $(\tan^2 \theta + 1)(1 - \sin^2 \theta)$
 $\sec^2 \theta (\sin^2 \theta + \cos^2 \theta - \sin^2 \theta)$
 $\sec^2 \theta \cos^2 \theta$
 $\frac{1}{\cos^2 \theta} \cos^2 \theta$
 1
9. $\frac{(\sec \theta - 1)(\sec \theta + 1)}{\sin^2 \theta}$
 $\frac{\sec^2 \theta - 1}{\sin^2 \theta}$
 $\frac{\tan^2 \theta}{\sin^2 \theta}$
 $\tan^2 \theta \cdot \frac{1}{\sin^2 \theta}$
 $\frac{\sin^2 \theta}{\cos^2 \theta} \cdot \frac{1}{\sin^2 \theta}$
 $\frac{1}{\cos^2 \theta}$
 $\sec^2 \theta$
11. $(\csc x + \cot x)(1 - \cos x)$
 $\csc x - \csc x \cos x + \cot x - \cot x \cos x$
 $\frac{1}{\sin x} - \frac{1}{\sin x} \cos x + \cot x - \frac{\cos x}{\sin x} \cos x$
 $\frac{1}{\sin x} - \cot x + \cot x - \frac{\cos^2 x}{\sin x}$
 $\frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$
 $\frac{1 - \cos^2 x}{\sin x}$
 $\frac{\sin^2 x}{\sin x}$
 $\sin x$
13. $\sec x - \tan x \sin x$
 $\sec x - \frac{\sin x}{\cos x} \cdot \sin x$
 $\frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$
 $\frac{1 - \sin^2 x}{\cos x}$
 $\frac{\cos^2 x}{\cos x}$
 $\cos x$
15. $\cot x \sec x$
 $\frac{\cos x}{\sin x} \cdot \frac{1}{\cos x}$
 $\frac{1}{\sin x}$
 $\csc x$
17. $\frac{\csc^2 \theta - 1}{\csc^2 \theta}$
 $\frac{\cot^2 \theta}{\csc^2 \theta}$
 $\frac{\cos^2 \theta}{\sin^2 \theta} \cdot \sin^2 \theta$
 $\cos^2 \theta$
19. $\sin x + \cos x \cot x$
 $\sin x + \cos x \frac{\cos x}{\sin x}$
 $\frac{\sin^2 x}{\sin x} + \frac{\cos^2 x}{\sin x}$
 $\frac{\sin^2 x + \cos^2 x}{\sin x}$
 $\frac{1}{\sin x}$
 $\csc x$
21. $\frac{\csc \theta \sin \theta}{\cot \theta}$
 $\csc \theta \sin \theta \tan \theta$
 $\frac{1}{\sin \theta} \cdot \sin \theta \cdot \tan \theta$
 $\tan \theta$
23. $\frac{\tan \theta \cot \theta}{\sin \theta}$
 $\tan \theta \cot \theta \csc \theta$
 $\tan \theta \cdot \frac{1}{\tan \theta} \cdot \csc \theta$
 $\csc \theta$
25. $\frac{\cos^2 \theta (1 + \cot^2 \theta)}{\cos^2 \theta \csc^2 \theta}$
 $\frac{\cos^2 \theta}{\sin^2 \theta}$
 $\cot^2 \theta$
27. $\sin^2 \theta (\csc^2 \theta - 1)$
 $\sin^2 \theta \csc^2 \theta - \sin^2 \theta$
 $1 - \sin^2 \theta$
 $\cos^2 \theta$
29. $\frac{\tan^2 \theta - \sec^2 \theta}{\tan^2 \theta - (\tan^2 \theta + 1)}$
 $\frac{-1}{\cot \theta \sec \theta}$
 $\frac{\csc \theta}{\cot \theta \sec \theta \sin \theta}$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \sin \theta$
 1
31. $\frac{\csc \theta}{\cot \theta \sec \theta \sin \theta}$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \sin \theta$
 1
33. $\frac{\csc \theta + \cot \theta}{\frac{1}{\sin \theta} + \frac{\cos \theta}{\sin \theta}}$
 $\frac{1 + \cos \theta}{\sin \theta}$
 $\frac{\csc \theta}{\sec \theta + \tan \theta}$
 $\frac{1}{\sin \theta}$
 $\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$
 $\frac{1}{\sin \theta} \cdot \frac{\cos \theta}{1 + \sin \theta}$
 $\frac{\cos \theta}{\sin \theta + \sin^2 \theta}$
 $\frac{1 + \csc \theta}{1 + \sec \theta}$
 $1 + \frac{1}{\sin \theta}$
 $1 + \frac{1}{\cos \theta}$
 $\frac{\sin \theta + 1}{\sin \theta}$
 $\frac{\cos \theta + 1}{\cos \theta}$
 $\frac{\sin \theta + 1}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta + 1}$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta + 1}{\cos \theta + 1}$
 $\cot \theta \left(\frac{1 + \sin \theta}{1 + \cos \theta} \right)$
35. $\frac{\csc \theta}{\sec \theta + \tan \theta}$
 $\frac{1}{\sin \theta}$
 $\frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$
 $\frac{1}{\sin \theta} \cdot \frac{\cos \theta}{1 + \sin \theta}$
 $\frac{\cos \theta}{\sin \theta + \sin^2 \theta}$
 $\frac{1 + \csc \theta}{1 + \sec \theta}$
 $1 + \frac{1}{\sin \theta}$
 $1 + \frac{1}{\cos \theta}$
 $\frac{\sin \theta + 1}{\sin \theta}$
 $\frac{\cos \theta + 1}{\cos \theta}$
 $\frac{\sin \theta + 1}{\sin \theta} \cdot \frac{\cos \theta}{\cos \theta + 1}$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{\sin \theta + 1}{\cos \theta + 1}$
 $\cot \theta \left(\frac{1 + \sin \theta}{1 + \cos \theta} \right)$
37. $\frac{\tan^2 \theta + \sec^2 \theta}{\sec^2 \theta}$
39. $\frac{\tan^2 \theta + \sec^2 \theta}{\sec^2 \theta}$
41. $\frac{\tan^2 \theta}{\tan^2 \theta}$
 $\frac{1}{\tan^2 \theta} + \frac{\cot^2 \theta}{\tan^2 \theta}$
 $\cot^2 \theta + \cot^2 \theta \cot^2 \theta$
 $\cot^2 \theta (1 + \cot^2 \theta)$
 $\cot^2 \theta \csc^2 \theta$
43. $\frac{1}{\sec - \cos \theta}$
 $\frac{1}{\cos \theta - \cos \theta}$
 $\frac{1}{1 - \cos^2 \theta}$
 $\frac{1}{\cos \theta}$
 $\frac{1}{\sin^2 \theta}$
 $\frac{\cos \theta}{\sin^2 \theta}$
 $\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta}$
 $\cot \theta \csc \theta$
45. $\frac{\csc \theta + 1}{\csc^2 \theta - 1}$
 $\frac{\csc \theta + 1}{\csc \theta + 1}$
 $(\csc \theta - 1)(\csc \theta + 1)$
 $\csc \theta - 1$
47. $\frac{\tan^2 \theta}{\sec \theta - 1}$
 $\frac{\sec^2 \theta - 1}{\sec \theta - 1}$
 $(\sec \theta - 1)(\sec \theta + 1)$
 $\sec \theta - 1$
 $\sec \theta + 1$
49. $2 \cos^2 x - 1$
 $2 \cos^2 x - (\sin^2 x + \cos^2 x)$
 $\cos^2 x - \sin^2 x$

$$51. \frac{1 + \sin y}{\cos y} \cdot \frac{1 - \sin y}{1 - \sin y} = \frac{1 - \sin^2 y}{\cos y(1 - \sin y)} = \frac{\cos^2 y}{\cos y(1 - \sin y)} = \frac{\cos y}{1 - \sin y}$$

$$53. \frac{\cot x + 1}{\cot x - 1} = \frac{\frac{\cos x}{\sin x} + 1}{\frac{\cos x}{\sin x} - 1} = \frac{\frac{\cos x + \sin x}{\sin x}}{\frac{\cos x - \sin x}{\sin x}} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$55. \frac{\sec x - \tan x}{\cos x} = \frac{1}{\cos x} - \frac{\sin x}{\cos x} = \frac{1 - \sin x}{\cos x} = \frac{\cos x}{1 - \sin x} \cdot \frac{\cos x}{\cos x} = \frac{\cos^2 x}{1 - \sin x}$$

$$57. \frac{1}{1 - \sin x} + \frac{1}{1 + \sin x} = \frac{(1 + \sin x) + (1 - \sin x)}{1 - \sin^2 x} = \frac{2}{\cos^2 x} = 2 \sec^2 x$$

$$59. \sin^4 x - \cos^4 x = (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) = (\sin^2 x - \cos^2 x)(1) = \sin^2 x - \cos^2 x$$

$$61. \csc^2 y + \sec^2 y = \frac{1}{\sin^2 y} + \frac{1}{\cos^2 y} = \frac{\cos^2 y + \sin^2 y}{\sin^2 y \cos^2 y} = \frac{1}{\sin^2 y \cos^2 y}$$

$$63. \frac{\cot^2 x - 1}{\cot^2 x + 1} = \frac{\frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}}{\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x}} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} = \frac{\cos^2 x - \sin^2 x}{1} = \cos^2 x - \sin^2 x$$

$$65. \sec^4 x - \sec^2 x = \sec^2 x(\sec^2 x - 1) = (\tan^2 x + 1)[(\tan^2 x + 1) - 1] = (\tan^2 x + 1)(\tan^2 x) = \tan^4 x + \tan^2 x$$

In each of problems 67-75, let $\theta = \frac{\pi}{3}$;

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}; \cos \frac{\pi}{3} = \frac{1}{2}; \tan \frac{\pi}{3} = \sqrt{3}.$$

$$67. \sin \theta = 1 - \cos \theta$$

$$\sin \frac{\pi}{6} = 1 - \cos \frac{\pi}{6}$$

$$\frac{1}{2} \neq 1 - \frac{\sqrt{3}}{2}$$

$$69. \sec \theta = \frac{1}{\csc \theta}$$

$$2 = \frac{1}{\frac{2}{\sqrt{3}}}$$

$$2 \neq \frac{\sqrt{3}}{2}$$

$$71. \sin^2 \theta - 2 \cos \theta \sin \theta + \cos^2 \theta = 2$$

$$\left(\frac{\sqrt{3}}{2}\right)^2 - 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)^2 = 2$$

$$\frac{3}{4} - \frac{\sqrt{3}}{2} + \frac{1}{4} = 2$$

$$1 - \frac{\sqrt{3}}{2} \neq 2$$

$$73. \csc \theta + \sec \theta \cot \theta = 2$$

$$\frac{2}{\sqrt{3}} + 2 \cdot \frac{1}{\sqrt{3}} = 2$$

$$\frac{4}{\sqrt{3}} \neq 2$$

$$75. \frac{1 - \cos \theta}{1 + \cos \theta} = \sin^2 \theta$$

$$\frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = \left(\frac{\sqrt{3}}{2}\right)^2$$

$$\frac{1}{3} \neq \frac{3}{4}$$

$$77. \text{a. } \theta = \frac{\pi}{6}:$$

$$(\csc^2 \theta - 1)(\sec^2 \theta - 1) = 1$$

$$\left(\csc^2 \frac{\pi}{6} - 1\right)\left(\sec^2 \frac{\pi}{6} - 1\right) = 1$$

$$(2^2 - 1)\left(\left(\frac{2}{\sqrt{3}}\right)^2 - 1\right)$$

$$(3)\left(\frac{4}{3} - 1\right)$$

$$1$$

$$\theta = \frac{\pi}{4}:$$

$$\left(\csc^2 \frac{\pi}{4} - 1\right)\left(\sec^2 \frac{\pi}{4} - 1\right) = 1$$

$$((\sqrt{2})^2 - 1)((\sqrt{2})^2 - 1)$$

$$(2 - 1)(2 - 1)$$

$$1$$

b. Yes:

$$(\csc^2 \theta - 1)(\sec^2 \theta - 1) = 1$$

$$\cot^2 \theta \tan^2 \theta$$

$$\frac{1}{\tan^2 \theta} \tan^2 \theta$$

$$1$$

$$79. \text{a. } 2 \sin^2 \theta + \sin \theta = 1$$

$$\theta = \frac{\pi}{6}:$$

$$2\left(\frac{1}{2}\right)^2 + \frac{1}{2} = 1$$

$$1 = 1$$

$$\theta = \frac{3\pi}{2}:$$

$$2(-1)^2 + (-1) = 1$$

$$1 = 1$$

b. No;

$$\text{let } \theta = 0:$$

$$2(0^2) + 0 = 1$$

$$0 \neq 1$$

Solutions to skill and review problems

$$1. r = \sqrt{x^2 + y^2} = \sqrt{40} = 2\sqrt{10}$$

$$\sin \theta = \frac{y}{r} = \frac{-2}{2\sqrt{10}} = -\frac{\sqrt{10}}{10};$$

$$\cos \theta = \frac{x}{r} = \frac{6}{2\sqrt{10}} = \frac{3\sqrt{10}}{10};$$

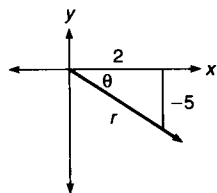
$$\tan \theta = \frac{y}{x} = -\frac{2}{6} = -\frac{1}{3}$$

$$2. \tan 15^\circ = \frac{x}{4}; x = 4 \tan 15^\circ \approx 1.07 \text{ feet}$$

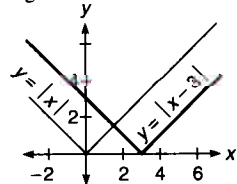
$$3. \frac{\theta^\circ}{180^\circ} = \frac{s}{\pi}; \theta^\circ = \frac{180^\circ}{\pi} s$$

$$= \frac{180^\circ}{\pi} \left(-\frac{6\pi}{5} \right) = -216^\circ$$

$$4. r = \sqrt{29}; \cos \theta = \frac{2}{r} = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$$



5. The graph of $y = |x - 3|$ is the graph of $y = |x|$, shifted three units to the right.



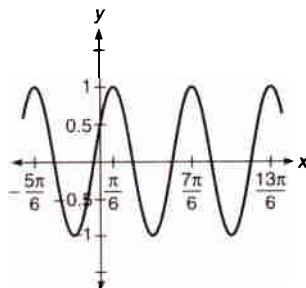
$$6. 0 \leq 2x - \frac{\pi}{3} \leq 2\pi$$

$$\frac{\pi}{3} \leq 2x \leq \frac{7\pi}{3}$$

$$\frac{\pi}{6} \leq x \leq \frac{7\pi}{6}; \text{ one complete cycle from}$$

$$\frac{\pi}{6} \text{ to } \frac{7\pi}{6}; \text{ period is } \frac{7\pi}{6} - \frac{\pi}{6} = \pi;$$

amplitude is 1



7. $4 \sin^2 x - 1 = 0$, $0 \leq x < 2\pi$ (This implies answers should be in radians measure.)

$$(2 \sin x - 1)(2 \sin x + 1) = 0$$

$$2 \sin x - 1 = 0 \text{ or } 2 \sin x + 1 = 0$$

$$2 \sin x = 1 \text{ or } 2 \sin x = -1$$

$$\sin x = \frac{1}{2} \text{ or } \sin x = -\frac{1}{2}; \text{ The reference}$$

angle for both cases is $x' = \frac{\pi}{6}$. $\sin x$

is positive in quadrants I and II.

$$\text{I: } x = x' = \frac{\pi}{6}$$

$$\text{II: } x = \pi - x' = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$\sin x$ negative in quadrants III and IV.

$$\text{III: } x = \pi + x' = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\text{IV: } x = 2\pi - x' = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

x is any one of $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, or $\frac{11\pi}{6}$.

Exercise 7-2

Answers to odd-numbered problems

$$1. \cos 72^\circ \quad 3. \cot 82^\circ \quad 5. \csc \frac{\pi}{6}$$

$$7. \sin\left(-\frac{\pi}{3}\right) \quad 9. \csc \frac{5\pi}{4} \quad 11. 1$$

$$13. 1 \quad 15. 1 \quad 17. -\frac{1}{4} \quad 19. 1 \quad 21. 1$$

$$23. 1 \quad 25. 1 \quad 27. \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$29. \frac{\sqrt{6} + \sqrt{2}}{4} \quad 31. \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$33. \frac{\sqrt{6} - \sqrt{2}}{4} \quad 35. \frac{\sqrt{2} - \sqrt{6}}{4}$$

$$37. 2 + \sqrt{3} \quad 39. \frac{2\sqrt{14} + 3}{12}$$

$$41. \frac{-15 - 12\sqrt{7}}{36 - 5\sqrt{7}} \quad 43. \frac{77}{85}$$

$$45. \frac{-\sqrt{5} - 4\sqrt{2}}{9} \quad 47. \frac{8 + \sqrt{5}}{51} \sqrt{17}$$

$$49. -\frac{119}{120} \quad 51. -\frac{\sqrt{5}}{5} \quad 53. \frac{-4\sqrt{6} + \sqrt{5}}{15}$$

$$55. 2 + \sqrt{3}$$

$$57. \sin(\pi - \theta) = \sin \pi \cos \theta - \cos \pi \sin \theta \\ = 0 \cos \theta - (-1) \sin \theta \\ = \sin \theta$$

$$59. \cos(\pi - \theta) = \cos \pi \cos \theta + \sin \pi \sin \theta \\ = (-1) \cos \theta + 0 \sin \theta \\ = -\cos \theta$$

$$61. \tan(\pi - \theta) = \frac{\tan \pi - \tan \theta}{1 + \tan \pi \tan \theta} \\ = \frac{0 - \tan \theta}{1 + 0 \tan \theta} \\ = -\tan \theta$$

$$63. \sin(\theta + 2\pi) = \sin \theta \cos 2\pi + \cos \theta \sin 2\pi \\ = \sin \theta(1) + \cos \theta(0) \\ = \sin \theta$$

$$65. \tan(\theta + \pi) = \frac{\tan \theta + \tan \pi}{1 - \tan \theta \tan \pi} \\ = \frac{\tan \theta + 0}{1 - \tan \theta(0)} \\ = \tan \theta$$

$$67. \frac{1}{2} [\sin(\alpha + \beta) - \sin(\alpha - \beta)] \\ \frac{1}{2} [\sin \alpha \cos \beta + \cos \alpha \sin \beta - (\sin \alpha \cos \beta - \cos \alpha \sin \beta)] \\ \frac{1}{2} [2 \cos \alpha \sin \beta] \\ \cos \alpha \sin \beta$$

$$69. \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \\ \frac{1}{2} [\cos \alpha \cos \beta + \sin \alpha \sin \beta - (\cos \alpha \cos \beta - \sin \alpha \sin \beta)] \\ \frac{1}{2} [2 \sin \alpha \sin \beta] \\ \sin \alpha \sin \beta$$

$$71. 3\frac{12}{29}$$

$$73. \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

This was shown true in the text.

$$\cos(\alpha + (-\beta)) = \cos \alpha \cos(-\beta) - \sin \alpha \sin(-\beta)$$

Replace β by $-\beta$. This is valid since the identity is true for all angles and α .

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta - \sin \alpha [-\sin \beta]$$

$$\alpha + (-\beta) = \alpha - \beta; \cos(-\theta) = \cos \theta; \sin(-\theta) = -\sin \theta.$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

This statement is true since the preceding statements are true.

$$75. \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta \quad \text{Proved true above.}$$

$$\text{Let } \alpha = \frac{\pi}{2} - \theta, \text{ then } \theta = \frac{\pi}{2} - \alpha.$$

$$\cos \alpha = \sin\left(\frac{\pi}{2} - \alpha\right)$$

Substitution of expression (section 1-3).

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$$

The variable name α or θ is unimportant.

$$77. \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha + (-\beta)) = \sin \alpha \cos(-\beta) + \cos \alpha \sin(-\beta)$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta + \cos \alpha [-\sin \beta]$$

Cosine is an even function, sine is odd.

$$= \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$79. \cot\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\tan\left(\frac{\pi}{2} - \theta\right)} = \frac{1}{\cot \theta} = \tan \theta$$

$$81. \csc\left(\frac{\pi}{2} - \theta\right) = \frac{1}{\sin\left(\frac{\pi}{2} - \theta\right)} = \frac{1}{\cos \theta} = \sec \theta$$

Solutions to skill and review problems

$$1. (x_1, y_1) = (-4, 3); (x_2, y_2) = (-8, 11)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 3}{-8 - (-4)} = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - (-4))$$

$$y - 3 = -2x - 8$$

$$y = -2x - 5$$

$$2. b = \sqrt{4^2 - 3^2} = \sqrt{7}$$

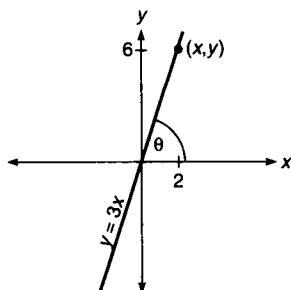
$$\cos B = \frac{3}{4}, \text{ so } B = \cos^{-1}\frac{3}{4} \approx 41.4^\circ$$

$$\sin A = \frac{3}{4}, \text{ so } A = \sin^{-1}\frac{3}{4} \approx 48.6^\circ$$

$$3. \text{ The point } (x, y) = (2, 6) \text{ lies on the}$$

terminal side of the angle. $\tan \theta = \frac{y}{x}$

$$= \frac{6}{2} = 3, \text{ so } \theta = \tan^{-1}3 \approx 71.6^\circ$$



4. Possible rational zeros of $3x^4 - 5x^3 - 14x^2 + 20x + 8$ have numerators that are factors of 8, and denominators that are factors of 3. Thus, the possible rational zeros are $\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3},$ and $\pm \frac{8}{3}$.

	3	-5	-14	20	8
		-6	22	-16	-8
-2	3	-11	8	4	0

-2 is a zero,

$$\text{so } 3x^4 - 5x^3 - 14x^2 + 20x + 8 = (x + 2)(3x^3 - 11x^2 + 8x + 4)$$

$$= (x + 2)(3x^3 - 11x^2 + 8x + 4)$$

	3	-11	8	4
		6	-10	-4
2	3	-5	-2	0

2 is a zero of $3x^3 - 11x^2 + 8x + 4$, so

$$3x^4 - 5x^3 - 14x^2 + 20x + 8 = (x + 2)(x - 2)(3x^2 - 5x - 2)$$

$$= (x + 2)(x - 2)(3x + 1)(x - 2)$$

$$= (x + 2)(x - 2)^2(3x + 1)$$

$$= (x + 2)(x - 2)^2(3x + 1)$$

Note that if the zero $-\frac{1}{3}$ were used in the synthetic division, the result would be $3(x + 2)(x - 2)^2(x + \frac{1}{3})$.

5. Using the circle with arc length L :

$$L = rs; 14.6 = 4s; 3.65 = s$$

Using the circle with arc length T :

$$T = rs; T = 6(3.65) = 21.9$$

$$6. \sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$$

$$7. \frac{\csc^2 x - 1}{\sin^2 x} = \frac{\cot^2 \theta}{\sin^2 \theta}$$

$$\cot^2 \theta + 1 = \csc^2 \theta, \text{ so } \csc^2 \theta - 1 = \cot^2 \theta$$

$$\cot^2 \theta \cdot \frac{1}{\sin^2 \theta}$$

$$\frac{\cos^2 \theta}{\sin^2 \theta} \cdot \csc^2 \theta$$

$$\cos^2 \theta \cdot \frac{1}{\sin^2 \theta} \cdot \csc^2 \theta$$

$$\cos^2 \theta \csc^2 \theta \csc^2 \theta$$

$$\cos^2 \theta \csc^4 \theta$$

Solutions to trial exercise problems

$$8. \sin\left(-\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{2} - \left(-\frac{\pi}{3}\right)\right) = \cos\frac{5\pi}{6}$$

$$17. \tan^2 8^\circ - \csc^2 82^\circ = \tan^2 8^\circ - \sec^2 8^\circ = -1$$

(Since $\tan^2 \theta + 1 = \sec^2 \theta$, $\tan^2 \theta - \sec^2 \theta = -1$.)

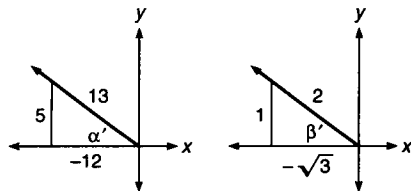
$$28. \tan \frac{\pi}{12} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan \frac{\pi}{4} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{4} \tan \frac{\pi}{6}}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 \cdot \left(\frac{\sqrt{3}}{3}\right)} \cdot \frac{3}{3} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}} \cdot \frac{3 - \sqrt{3}}{3 - \sqrt{3}}$$

$$= \frac{12 - 6\sqrt{3}}{6} = 2 - \sqrt{3}$$

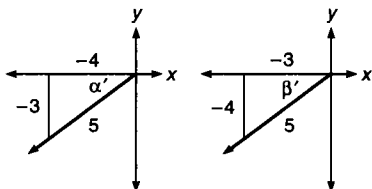
$$40. \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= -\frac{12}{13} \cdot \left(-\frac{\sqrt{3}}{2}\right) + \frac{5}{13} \cdot \frac{1}{2} = \frac{12\sqrt{3} + 5}{26}$$



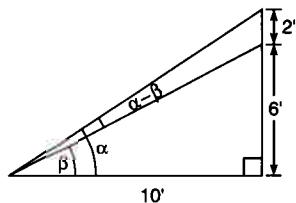
$$48. \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= -\frac{4}{5} \cdot \left(-\frac{3}{5}\right) + \left(-\frac{3}{5}\right) \cdot \left(-\frac{4}{5}\right) = \frac{24}{25}$$



$$70. \tan \alpha = \frac{2+6}{10} = \frac{4}{5}; \tan \beta = \frac{6}{10} = \frac{3}{5}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} = \frac{\frac{4}{5} - \frac{3}{5}}{1 + \frac{4}{5} \cdot \frac{3}{5}} = \frac{\frac{1}{5}}{1 + \frac{12}{25}} = \frac{\frac{1}{5}}{\frac{25}{25} + \frac{12}{25}} = \frac{1}{25 + 12} = \frac{1}{37}$$



$$76. \sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$$

This is identity [6], which we know is true.

$$\sin(\alpha + \beta) = \cos\left[\frac{\pi}{2} - (\alpha + \beta)\right]$$

Replace θ by $\alpha + \beta$. This is substitution of expression (section 1-3).

$$\sin(\alpha + \beta) = \cos\left[\left(\frac{\pi}{2} - \alpha\right) - \beta\right]$$

$$\text{Regroup } \frac{\pi}{2} - \alpha - \beta.$$

$$= \cos\left(\frac{\pi}{2} - \alpha\right) \cos \beta + \sin\left(\frac{\pi}{2} - \alpha\right) \sin \beta$$

Using identity [2].

$$= \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

Using identities [6] and [5].

Thus, identity [3] is true.

Exercise 7-3

Answers to odd-numbered problems

$$1. \sin \frac{\pi}{2} \quad 3. \cos 6\pi \quad 5. \cos \frac{\pi}{5}$$

$$7. 3 \tan 20^\circ \quad 9. \sin 12\theta \quad 11. 3 \cos 10\theta$$

$$13. 5 \tan 6\theta \quad 15. 2 \cos 14\theta$$

$$17. 3 \cos 6\theta \quad 19. 70^\circ \quad 21. \frac{5\pi}{12}$$

$$23. 35^\circ \quad 25. 20^\circ \quad 27. \frac{\pi}{8} \quad 29. \frac{4\pi}{5}$$

$$31. \frac{24}{25}, \frac{7}{25}, \frac{24}{7} \quad 33. -\frac{24}{25}, \frac{7}{25}, -\frac{24}{7}$$

$$35. \frac{10\sqrt{39}}{64}, \frac{7}{32}, \frac{5\sqrt{39}}{7}$$

$$37. \frac{\sqrt{70}}{10}, -\frac{\sqrt{30}}{10}, -\frac{\sqrt{21}}{3}$$

$$39. \frac{\sqrt{50-20\sqrt{5}}}{10}, -\frac{\sqrt{50+20\sqrt{5}}}{10}, 2 - \sqrt{5}$$

$$41. \text{a. } \frac{\sqrt{2-\sqrt{3}}}{2} \quad \text{b. } \frac{\sqrt{2+\sqrt{3}}}{2} \quad \text{c. } 2 - \sqrt{3}$$

$$43. \frac{\sqrt{6+3\sqrt{3}} + \sqrt{2-\sqrt{3}}}{4}$$

$$45. \frac{\sqrt{4 + \sqrt{6+2\sqrt{3}+2\sqrt{2}}} - \sqrt{4 + \sqrt{6-2\sqrt{3}-2\sqrt{2}}}}{4}$$

$$47. \frac{\sqrt{2 - \sqrt{2 + \sqrt{3}}}}{2}$$

$$49. \sin 2\theta + 1$$

$$2 \sin \theta \cos \theta + 1$$

$$2 \sin \theta \cos \theta + \sin^2 \theta + \cos^2 \theta$$

$$\sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta$$

$$(\sin \theta + \cos \theta)^2$$

$$51. \cos^4 \theta - \sin^4 \theta$$

$$(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$$

$$\cos 2\theta(1)$$

$$\cos 2\theta$$

$$53. \frac{1 + \cos 2\theta}{1 - \cos 2\theta}$$

$$\frac{1 + (2 \cos^2 \theta - 1)}{1 - (1 - 2 \sin^2 \theta)}$$

$$\frac{2 \cos^2 \theta}{2 \sin^2 \theta}$$

$$\frac{\cos^2 \theta}{\sin^2 \theta}$$

$$\cot^2 \theta$$

$$55. \frac{2 \cos 2\theta}{\sin 2\theta}$$

$$\frac{2(\cos^2 \theta - \sin^2 \theta)}{2 \sin \theta \cos \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos^2 \theta}{\sin \theta \cos \theta} - \frac{\sin^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta - \tan \theta$$

$$57. \sin 2\theta - 4 \sin^3 \theta \cos \theta$$

$$2 \sin \theta \cos \theta - 4 \sin^3 \theta \cos \theta$$

$$2 \sin \theta \cos \theta (1 - 2 \sin^2 \theta)$$

$$\sin 2\theta \cos 2\theta$$

$$59. \frac{2}{1 - \cos 2\theta}$$

$$\frac{2}{1 - (2 \cos^2 \theta - 1)}$$

$$\frac{2}{2 - 2 \cos^2 \theta}$$

$$\frac{2}{2(1 - \cos^2 \theta)}$$

$$\frac{1}{\sin^2 \theta}$$

$$\csc^2 \theta$$

61. Left side:

$$\frac{\tan 2\theta}{2 \tan \theta}$$

$$\frac{1 - \tan^2 \theta}{1 - \tan^4 \theta}$$

Right side:

$$\frac{2(\tan \theta + \tan^3 \theta)}{1 - \tan^4 \theta}$$

$$\frac{2 \tan \theta (1 + \tan^2 \theta)}{(1 - \tan^2 \theta)(1 + \tan^2 \theta)}$$

$$\frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$63. \frac{2 \csc 2\theta \sin \theta \cos \theta}{2 \sin \theta \cos \theta \csc 2\theta}$$

$$\sin 2\theta \cdot \frac{1}{\sin 2\theta} = 1$$

$$65. \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$1 - \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$1 + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$$

$$\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta}$$

$$\frac{\cos 2\theta}{1}$$

$$\cos 2\theta$$

$$67. \sec^2 \frac{\theta}{2}$$

$$\frac{1}{\cos^2 \frac{\theta}{2}}$$

$$\frac{1}{\left(\pm \sqrt{\frac{1 + \cos \theta}{2}}\right)^2}$$

$$\frac{1}{\frac{1 + \cos \theta}{2}}$$

$$\frac{2}{1 + \cos \theta}$$

$$69. \cos^2 \frac{\theta}{2}$$

$$\left(\pm \sqrt{\frac{1 + \cos \theta}{2}}\right)^2$$

$$\frac{1 + \cos \theta}{2}$$

$$\frac{1 + \cos \theta}{2} \cdot \frac{1 - \cos \theta}{1 - \cos \theta}$$

$$\frac{1 - \cos^2 \theta}{2 - 2 \cos \theta}$$

$$71. 2 \cos^2 \frac{\theta}{2} - \cos \theta$$

$$2\left(\pm \sqrt{\frac{1 + \cos \theta}{2}}\right)^2 - \cos \theta$$

$$2 \frac{1 + \cos \theta}{2} - \cos \theta$$

$$1 + \cos \theta - \cos \theta$$

$$1$$

$$73. \sin^2 \frac{\theta}{2} - \cos^2 \frac{\theta}{2}$$

$$\frac{1 - \cos \theta}{2} - \frac{1 + \cos \theta}{2}$$

$$\frac{-2 \cos \theta}{2}$$

$$-\cos \theta$$

$$75. \text{Left side:}$$

$$\frac{\tan^2 \frac{\theta}{2}}{2}$$

$$\left(\sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}\right)^2$$

$$\frac{1 - \cos \theta}{1 + \cos \theta}$$

Right side:

$$\frac{2}{1 + \cos \theta} - 1$$

$$\frac{2}{1 + \cos \theta} - \frac{1 + \cos \theta}{1 + \cos \theta}$$

$$\frac{2 - (1 + \cos \theta)}{1 + \cos \theta}$$

$$\frac{1 - \cos \theta}{1 + \cos \theta}$$

$$77. 4 \sin^2 \frac{\theta}{2} \cos^2 \frac{\theta}{2}$$

$$4 \frac{1 - \cos \theta}{2} \cdot \frac{1 + \cos \theta}{2}$$

$$4 \frac{1 - \cos^2 \theta}{4}$$

$$\sin^2 \theta$$

$$79. \tan \frac{\theta}{2} + \cot \frac{\theta}{2}$$

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{1}{\frac{\sin \theta}{1 - \cos \theta}}$$

$$\frac{\sin \theta}{1 + \cos \theta} + \frac{\sin \theta}{1 - \cos \theta}$$

$$\frac{\sin \theta (1 - \cos \theta) + \sin \theta (1 + \cos \theta)}{1 - \cos^2 \theta}$$

$$\frac{2 \sin \theta}{\sin^2 \theta}$$

$$\frac{2}{\sin \theta}$$

$$81. \sin 3\theta$$

$$\sin(2\theta + \theta)$$

$$\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$$

$$(2 \sin \theta \cos \theta)(\cos \theta) + (1 - 2 \sin^2 \theta)(\sin \theta)$$

$$2 \sin \theta \cos^2 \theta + \sin \theta - 2 \sin^3 \theta$$

$$2 \sin \theta (1 - \sin^2 \theta) + \sin \theta - 2 \sin^3 \theta$$

$$2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$$

$$3 \sin \theta - 4 \sin^3 \theta$$

$$83. \text{a. } \sin 4\theta$$

$$\sin 2(2\theta)$$

$$2 \sin 2\theta \cos 2\theta$$

$$2(2 \sin \theta \cos \theta)(1 - 2 \sin^2 \theta)$$

$$4 \sin \theta \cos \theta - 8 \sin^3 \theta \cos \theta$$

Depending on how $\cos 2\theta$ is expanded, other possible answers are

$$\sin 4\theta = 8 \cos^3 \theta \sin \theta - 4 \cos \theta \sin \theta$$

$$\sin 4\theta = 4 \cos^3 \theta \sin \theta - 4 \sin^3 \theta \cos \theta$$

$$\text{b. } \cos 4\theta = \cos 2(2\theta)$$

$$= 2 \cos^2(2\theta) - 1$$

$$= 2[\cos 2\theta]^2 - 1$$

$$= 2[2 \cos^2 \theta - 1]^2 - 1$$

$$= 2[4 \cos^4 \theta - 4 \cos^2 \theta + 1] - 1$$

$$= 8 \cos^4 \theta - 8 \cos^2 \theta + 1$$

$$\begin{aligned}
 85. \quad & \frac{3}{16} \frac{1 - (\cos^2 \alpha - \sin^2 \alpha)}{1 - \cos \alpha} \\
 & \frac{3}{16} \frac{(\sin^2 \alpha + \cos^2 \alpha) - (\cos^2 \alpha - \sin^2 \alpha)}{1 - \cos \alpha} \\
 & \frac{3}{16} \frac{2 \sin^2 \alpha}{1 - \cos \alpha} = \frac{3}{8} \frac{1 - \cos^2 \alpha}{1 - \cos \alpha} \\
 & \frac{3}{8} \frac{(1 - \cos \alpha)(1 + \cos \alpha)}{1 - \cos \alpha} \\
 & \frac{3}{8} (1 + \cos \alpha)
 \end{aligned}$$

$$87. \tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha}; \text{ let } \frac{\alpha}{2} = \theta, \text{ so } \alpha = 2\theta.$$

Use substitution of expression:

$$\begin{aligned}
 \tan \theta &= \frac{\sin 2\theta}{1 + \cos 2\theta} \\
 &= \frac{2 \sin \theta \cos \theta}{1 + (2 \cos^2 \theta - 1)} \\
 &= \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} \\
 &= \frac{\sin \theta}{\cos \theta} = \tan \theta
 \end{aligned}$$

$$89. \text{ a. Problem 41 shows that } \tan 15^\circ = 2 - \sqrt{3}.$$

$$\text{ b. Problem 41 shows that } \sin 15^\circ = \frac{\sqrt{2 - \sqrt{3}}}{2}, \text{ and that}$$

$$\cos 15^\circ = \frac{\sqrt{2 + \sqrt{3}}}{2}, \text{ so } \tan 15^\circ = \frac{\frac{\sqrt{2 - \sqrt{3}}}{2}}{\frac{\sqrt{2 + \sqrt{3}}}{2}}.$$

$$\begin{aligned}
 \text{ c. } \frac{\frac{\sqrt{2 - \sqrt{3}}}{2}}{\frac{\sqrt{2 + \sqrt{3}}}{2}} &= \frac{\sqrt{2 - \sqrt{3}}}{2} \cdot \frac{2}{\sqrt{2 + \sqrt{3}}} = \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}} \\
 &= \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{2 + \sqrt{3}}} \cdot \frac{\sqrt{2 + \sqrt{3}}}{\sqrt{2 + \sqrt{3}}} = \frac{\sqrt{(2 - \sqrt{3})(2 + \sqrt{3})}}{2 + \sqrt{3}} \\
 &= \frac{1}{2 + \sqrt{3}} = \frac{1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \frac{2 - \sqrt{3}}{1} = 2 - \sqrt{3}.
 \end{aligned}$$

$$91. \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha + \alpha) = \cos \alpha \cos \alpha - \sin \alpha \sin \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= (1 - \sin^2 \alpha) - \sin^2 \alpha$$

$$\sin^2 \theta + \cos^2 \theta = 1, \text{ so } \cos^2 \theta = 1 - \sin^2 \theta$$

$$= 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$= \cos^2 \alpha - (1 - \cos^2 \alpha)$$

$$\sin^2 \theta + \cos^2 \theta = 1, \text{ so } \sin^2 \theta = 1 - \cos^2 \theta$$

$$= 2 \cos^2 \alpha - 1$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha + \alpha) = \frac{\tan \alpha + \tan \alpha}{1 - \tan \alpha \tan \alpha}$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$93. 10 \frac{34}{55}$$

95. Left side:

$$\sin 2\alpha - \sin 2\beta = 2 \sin \alpha \cos \alpha - 2 \sin \beta \cos \beta$$

Right side:

$$2 \sin(\alpha - \beta) \cdot \cos(\alpha + \beta)$$

$$2(\sin \alpha \cos \beta - \cos \alpha \sin \beta)(\cos \alpha \cos \beta - \sin \alpha \sin \beta)$$

$$2(\sin \alpha \cos^2 \beta \cos \alpha - \sin^2 \alpha \cos \beta \sin \beta - \cos^2 \alpha \sin \beta \cos \beta + \cos \alpha \sin^2 \beta \sin \alpha)$$

$$2(\sin \alpha \cos^2 \beta \cos \alpha + \cos \alpha \sin^2 \beta \sin \alpha - \sin^2 \alpha \cos \beta \sin \beta - \cos^2 \alpha \sin \beta \cos \beta)$$

$$2[\sin \alpha \cos \alpha (\cos^2 \beta + \sin^2 \beta) - \sin \beta \cos \beta (\sin^2 \alpha + \cos^2 \alpha)]$$

$$2[\sin \alpha \cos \alpha (1) - \sin \beta \cos \beta (1)]$$

$$2 \sin \alpha \cos \alpha - 2 \sin \beta \cos \beta$$

97. Left side:

$$\cos 2\alpha - \cos 2\beta = (2 \cos^2 \alpha - 1) - (2 \cos^2 \beta - 1) = 2 \cos^2 \alpha - 2 \cos^2 \beta$$

Right side:

$$-2 \sin(\alpha + \beta) \cdot \sin(\alpha - \beta)$$

$$-2(\sin \alpha \cos \beta + \cos \alpha \sin \beta)(\sin \alpha \cos \beta - \cos \alpha \sin \beta)$$

$$-2(\sin^2 \alpha \cos^2 \beta - \cos^2 \alpha \sin^2 \beta)$$

$$-2[(1 - \cos^2 \alpha)(\cos^2 \beta) - \cos^2 \alpha (1 - \cos^2 \beta)]$$

$$-2[\cos^2 \beta - \cos^2 \alpha \cos^2 \beta - \cos^2 \alpha + \cos^2 \alpha \cos^2 \beta]$$

$$-2(\cos^2 \beta - \cos^2 \alpha)$$

$$2 \cos^2 \alpha - 2 \cos^2 \beta$$

Solutions to skill and review problems

$$\begin{aligned}
 1. \cos \frac{\pi}{12} &= \cos \left(\frac{\pi}{4} - \frac{\pi}{6} \right) = \cos \frac{\pi}{4} \cos \frac{\pi}{6} + \sin \frac{\pi}{4} \sin \frac{\pi}{6} \\
 &= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} \\
 &= \frac{\sqrt{6} + \sqrt{2}}{2}.
 \end{aligned}$$

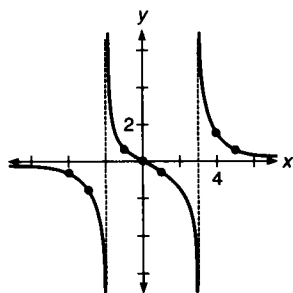
$$2. f(x) = \frac{2x}{x^2 - x - 6} = \frac{2x}{(x-3)(x+2)}$$

Vertical asymptotes at $x = -2$ and $x = 3$.

Intercepts are at the origin.

Additional points:

x	-4	-3	-1	1	2	4	5
y	-0.6	-1	0.5	-0.3	-1	1.3	0.7



$$3. 2x - y = 3$$

$$-y = -2x + 3$$

$$y = 2x - 3$$

The first equation is solved for y .

$$x + 3y = 5$$

$$x + 3(2x - 3) = 5$$

Substitute $2x - 3$ for y .

$$7x = 14$$

$$x = 2$$

Solve for y .

$$y = 2x - 3$$

$$y = 2(2) - 3 = 1$$

The point of intersection is $(x, y) = (2, 1)$.

$$4. \frac{1 - \cos \theta}{\sin \theta} \cdot \frac{\sin \theta}{\sin \theta} = \frac{\sin \theta - \cos \theta}{1 + \sin \theta}$$

$$5. -3 \sin x = 1$$

$$\sin x = -\frac{1}{3}$$

$$x' = \sin^{-1} \frac{1}{3} \approx 19.5^\circ$$

$\sin x < 0$ in quadrants III and IV. The least nonnegative solution is in quadrant III.

Therefore, $x = 180^\circ + x' = 199.5^\circ$.

Solutions to trial exercise problems

$$6. \frac{2 \tan \frac{\pi}{6}}{1 - \tan^2 \frac{\pi}{6}}$$

$$\frac{2 \tan \alpha}{1 - \tan^2 \alpha} = \tan 2\alpha; \text{ Let } \alpha = \frac{\pi}{6}, \text{ so } 2\alpha = \frac{\pi}{3},$$

$$\text{and } \tan 2\alpha = \tan \frac{\pi}{3}$$

$$8. 8 \cos^2 \frac{\pi}{2} - 4$$

$$2 \cos^2 \alpha - 1 = \cos 2\alpha$$

$$8 \cos^2 \alpha - 4 = 4 \cos 2\alpha$$

Multiply each member by 4.

$$\text{Let } \alpha = \frac{\pi}{2}, \text{ so } 2\alpha = \pi, \text{ and } 4 \cos 2\alpha = 4 \cos \pi.$$

$$11. 6 \cos^2 5\theta - 3$$

$$2 \cos^2 \alpha - 1 = \cos 2\alpha$$

$$6 \cos^2 \alpha - 3 = 3 \cos 2\alpha$$

Multiply each member by 3. Let $\alpha = 5\theta$, so 2α is 10θ , and $3 \cos 2\alpha$ represents $3 \cos 10\theta$.

$$25. \sin 10^\circ = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\frac{\alpha}{2} = 10^\circ, \text{ so } \alpha = \theta = 20^\circ$$

$$33. \cos \theta = -\frac{4}{5}, \frac{\pi}{2} < \theta < \pi$$

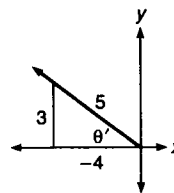
$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cdot \frac{3}{5} \left(-\frac{4}{5}\right) = -\frac{24}{25}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$$

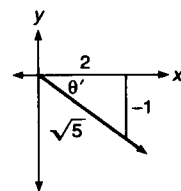
$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = -\frac{24}{7}$$



$$39. \cot \theta = -2, \frac{3\pi}{2} < \theta < 2\pi; \cos \theta = \frac{2\sqrt{5}}{5}$$

$$\frac{3\pi}{4} \leq \frac{\theta}{2} \leq \pi, \left(\frac{\theta}{2} \text{ in quadrant II}\right) \text{ so } \sin \frac{\theta}{2} > 0,$$

$$\cos \frac{\theta}{2} < 0, \tan \frac{\theta}{2} < 0.$$



$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{2\sqrt{5}}{5}}{2}} = \sqrt{\frac{1}{2} \left(\frac{5 - 2\sqrt{5}}{5}\right)}$$

$$= \sqrt{\frac{5 - 2\sqrt{5}}{10}} = \frac{\sqrt{5 - 2\sqrt{5}}}{\sqrt{10}} = \frac{\sqrt{50 - 20\sqrt{5}}}{10}$$

$$\cos \frac{\theta}{2} = -\sqrt{\frac{1 + \cos \theta}{2}} = -\frac{\sqrt{50 + 20\sqrt{5}}}{10}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{1 - \frac{2\sqrt{5}}{5}}{-\frac{1}{\sqrt{5}}} = -\sqrt{5} \left(1 - \frac{2}{\sqrt{5}}\right)$$

$$= -\sqrt{5} + 2 = 2 - \sqrt{5}$$

64. $\sec 2\theta$

$$\frac{1}{\cos 2\theta}$$

$$\frac{1}{1 - 2\sin^2\theta}$$

70. Left side:

$$\frac{\csc \theta - \cot \theta}{1 + \cos \theta}$$

$$\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta}$$

$$\frac{1 + \cos \theta}{1 - \cos \theta}$$

$$\frac{\sin \theta}{1 + \cos \theta}$$

$$\frac{1 - \cos \theta}{\sin \theta} \cdot \frac{1}{1 + \cos \theta}$$

$$\frac{1}{\sin \theta} \cdot \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$\csc \theta \frac{1 - \cos \theta}{1 + \cos \theta}$$

Right side:

$$\csc \theta \tan^2 \frac{\theta}{2}$$

$$\csc \theta \left(\pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \right)^2$$

$$\csc \theta \frac{1 - \cos \theta}{1 + \cos \theta}$$

b. $\cos 5\theta$

$$\cos(4\theta + \theta)$$

$$\cos 4\theta \cos \theta - \sin 4\theta \sin \theta$$

$$\cos \theta (8 \cos^4 \theta - 8 \cos^2 \theta + 1) - \sin \theta (4 \cos^3 \theta \sin \theta - 4 \sin^3 \theta \cos \theta)$$

$$8 \cos^5 \theta - 8 \cos^3 \theta + \cos \theta - 4 \cos^3 \theta \sin^2 \theta + 4 \sin^4 \theta \cos \theta$$

$$8 \cos^5 \theta - 8 \cos^3 \theta + \cos \theta - 4 \cos^3 \theta (1 - \cos^2 \theta)$$

$$+ 4(1 - \cos^2 \theta)^2 \cos \theta$$

$$16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta$$

$$93. \tan \frac{\theta}{2} = \tan \frac{\theta}{2} = \frac{3}{8}; \cos \theta = \frac{8}{x}$$

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}, \text{ so } \frac{3}{8} = \sqrt{\frac{1 - \frac{8}{x}}{1 + \frac{8}{x}}}$$

Square both members.

$$\frac{9}{64} = \frac{1 - \frac{8}{x}}{1 + \frac{8}{x}}$$

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc.$$

$$9 + \frac{72}{x} = 64 - \frac{512}{x}$$

Multiply each member by x .

$$9x + 72 = 64x - 512$$

$$584 = 55x$$

$$x = \frac{584}{55} = 10\frac{34}{55}$$

82. $\cos 3\theta = \cos(2\theta + \theta)$

$$= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta$$

$$= (2 \cos^2 \theta - 1)(\cos \theta) - (2 \sin \theta \cos \theta)(\sin \theta)$$

$$= 2 \cos^3 \theta - \cos \theta - 2 \sin^2 \theta \cos \theta$$

$$= 2 \cos^3 \theta - \cos \theta - 2(1 - \cos^2 \theta)(\cos \theta)$$

$$= 2 \cos^3 \theta - \cos \theta - 2 \cos \theta + 2 \cos^3 \theta$$

$$= 4 \cos^3 \theta - 3 \cos \theta$$

84. This problem uses the results of problem 83.

a. $\sin 5\theta$

$$\sin(4\theta + \theta)$$

$$\sin 4\theta \cos \theta + \cos 4\theta \sin \theta$$

$$\cos \theta (8 \cos^4 \theta - 8 \cos^2 \theta + 1) + \sin \theta (8 \cos^3 \theta \sin \theta - 4 \sin^3 \theta \cos \theta)$$

$$+ 8 \sin \theta \cos^4 \theta - 8 \sin^3 \theta \cos^2 \theta + 8 \sin \theta \cos^4 \theta$$

$$- 8 \sin \theta \cos^2 \theta + \sin \theta$$

We know $\cos^2 \theta = 1 - \sin^2 \theta$, so

$$\cos^4 \theta = (1 - \sin^2 \theta)^2 = 1 - 2 \sin^2 \theta + \sin^4 \theta$$

Replace these in the equation.

$$4 \sin \theta (1 - \sin^2 \theta) - 8 \sin^3 \theta (1 - \sin^2 \theta) + 8 \sin \theta (1 - 2 \sin^2 \theta + \sin^4 \theta)$$

$$- 8 \sin \theta (1 - \sin^2 \theta) + \sin \theta$$

$$16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$

Exercise 7-4**Answers to odd-numbered problems**

1. $\frac{3\pi}{4}$ (135°), $\frac{7\pi}{4}$ (315°)

3. $\frac{\pi}{3}$ (60°), $\frac{5\pi}{3}$ (300°)

5. $\frac{\pi}{6}$ (30°), $\frac{7\pi}{6}$ (210°)

7. $\frac{7\pi}{6}$ (210°), $\frac{11\pi}{6}$ (330°)

9. $\frac{\pi}{2}$ (90°), $\frac{3\pi}{2}$ (270°)

11. 0 (0°), π (180°) 13. 0 (0°), $\frac{3\pi}{2}$ (270°)

15. $\frac{\pi}{4}$ (45°), $\frac{3\pi}{4}$ (135°), $\frac{5\pi}{4}$ (225°), $\frac{7\pi}{4}$ (315°)

17. 0 (0°), π (180°), $\frac{\pi}{2}$ (90°)

19. 0 (0°), π (180°), $\frac{\pi}{3}$ (60°), $\frac{4\pi}{3}$ (240°)

21. $\frac{3\pi}{2}$ (270°), $\frac{\pi}{6}$ (30°), $\frac{5\pi}{6}$ (150°)

23. 0 (0°), π (180°), $\frac{\pi}{4}$ (45°),

$\frac{3\pi}{4}$ (135°), $\frac{5\pi}{4}$ (225°), $\frac{7\pi}{4}$ (315°)

25. 0 (0°), π (180°), $\frac{\pi}{3}$ (60°), $\frac{5\pi}{3}$ (300°)

27. $\frac{5\pi}{6}$ (150°), $\frac{11\pi}{6}$ (330°), $\frac{\pi}{2}$ (90°), $\frac{3\pi}{2}$ (270°)

29. 0 (0°), π (180°), $\frac{\pi}{2}$ (90°), $\frac{3\pi}{2}$ (270°)

31. $\frac{\pi}{6}$ (30°), $\frac{5\pi}{6}$ (150°), $\frac{3\pi}{2}$ (270°)

33. $\frac{3\pi}{4}$ (135°), $\frac{7\pi}{4}$ (315°)

35. $\frac{\pi}{3}$ (60°), $\frac{5\pi}{3}$ (300°), π (180°)

37. $\frac{\pi}{3}$ (60°), $\frac{2\pi}{3}$ (120°), $\frac{4\pi}{3}$ (240°), $\frac{5\pi}{3}$ (300°)

39. $\frac{\pi}{4}$ (45°), $\frac{3\pi}{4}$ (135°), $\frac{5\pi}{4}$ (225°), $\frac{7\pi}{4}$ (315°)

41. 0 (0°), π (180°), $\frac{\pi}{6}$ (30°), $\frac{5\pi}{6}$ (150°)

43. 0.65, 2.49, 3.42, 6.01, 37.4°, 142.6°, 195.9°, 344.1° 45. 0.27, 2.08, 3.42, 5.22, 15.7°, 119.3°, 195.7°, 299.3° 47. 1.26, 5.03, 2.51, 3.77, 72°, 288°, 144°, 216°

49. 5.08 (290.8°), 0.56 (32.3°)

51. $\frac{\pi}{3} + 2k\pi$ ($60^\circ + k \cdot 360^\circ$),

$\frac{5\pi}{3} + 2k\pi$ ($300^\circ + k \cdot 360^\circ$)

$$53. \frac{5\pi}{6} + k\pi \quad (150^\circ + k \cdot 180^\circ)$$

$$55. \frac{5\pi}{4} + 2k\pi \quad (225^\circ + k \cdot 360^\circ),$$

$$\frac{7\pi}{4} + 2k\pi \quad (315^\circ + k \cdot 360^\circ)$$

$$57. \frac{\pi}{4} + k\pi \quad (45^\circ + k \cdot 180^\circ)$$

$$59. \frac{\pi}{6} + 2k\pi \quad (30^\circ + k \cdot 360^\circ),$$

$$\frac{5\pi}{6} + 2k\pi \quad (150^\circ + k \cdot 360^\circ)$$

$$61. \frac{2\pi}{3} + 4k\pi \quad (120^\circ + k \cdot 720^\circ),$$

$$\frac{4\pi}{3} + 4k\pi \quad (240^\circ + k \cdot 720^\circ)$$

$$63. \frac{\pi}{3} + \frac{2k\pi}{3} \quad (60^\circ + k \cdot 120^\circ)$$

$$65. \frac{\pi}{6} + k \frac{\pi}{2} \quad (30^\circ + k \cdot 90^\circ)$$

$$67. \frac{\pi}{6} + k \frac{\pi}{2} \quad (30^\circ + k \cdot 90^\circ),$$

$$\frac{\pi}{3} + k \frac{\pi}{2} \quad (60^\circ + k \cdot 90^\circ)$$

$$69. \frac{\pi}{3} + k\pi \quad (60^\circ + k \cdot 180^\circ),$$

$$\frac{2\pi}{3} + k\pi \quad (120^\circ + k \cdot 180^\circ)$$

$$71. \frac{\pi}{12} + k \frac{\pi}{2} \quad (15^\circ + k \cdot 90^\circ)$$

$$73. \frac{\pi}{12} + k\pi \quad (15^\circ + k \cdot 180^\circ),$$

$$\frac{5\pi}{12} + k\pi \quad (75^\circ + k \cdot 180^\circ)$$

$$75. \frac{\pi}{9} + \frac{2k\pi}{3} \quad (20^\circ + k \cdot 120^\circ),$$

$$\frac{5\pi}{9} + \frac{2k\pi}{3} \quad (100^\circ + k \cdot 120^\circ)$$

$$77. \frac{2\pi}{3} + 4k\pi \quad (120^\circ + k \cdot 720^\circ)$$

$$79. \frac{7\pi}{6} \quad (210^\circ), \frac{11\pi}{6} \quad (330^\circ), \frac{\pi}{2} \quad (90^\circ)$$

$$81. 0 \quad (0^\circ), \pi \quad (180^\circ), \frac{2\pi}{3} \quad (120^\circ), \frac{4\pi}{3} \quad (240^\circ)$$

$$83. 0 \quad (0^\circ), \pi \quad (180^\circ), \frac{\pi}{6} \quad (30^\circ), \frac{5\pi}{6} \quad (150^\circ)$$

$$85. 0 \quad (0^\circ) \quad 87. \frac{\pi}{3} \quad (60^\circ) \text{ or } \frac{5\pi}{3} \quad (300^\circ)$$

$$89. 0 \quad (0^\circ), \frac{3\pi}{2} \quad (270^\circ) \quad 91. \frac{\pi}{2} \quad (90^\circ), \frac{3\pi}{2}$$

$$(270^\circ), \pi \quad (180^\circ), 0.93 \quad (53.1^\circ) \quad 93. \frac{\pi}{2},$$

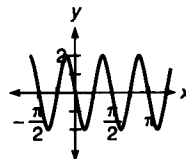
$$1.31 \quad 95. 0, 0.55, 0.69, 0.72, 0.66$$

Solutions to skill and review problems

1. The basic graph, $y = \sin x$, is reflected about the x -axis because of the coefficient -2 .

$$0 \leq 4x \leq 2\pi$$

$$0 \leq x \leq \frac{\pi}{2}$$



$$2. f(x) = x^2 + 4x - 8$$

$$f(x) = x^2 + 4x + 4 - 4 - 8$$

$$f(x) = x^2 + 4x + 4 - 12$$

$$f(x) = (x + 2)^2 - 12$$

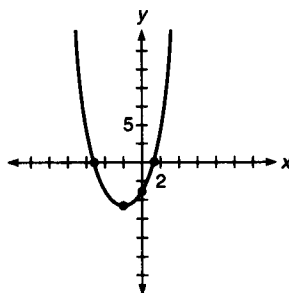
$$\text{Vertex: } (-2, -12)$$

Intercepts:

$$f(0) = -8: (0, -8)$$

$$0 = x^2 + 4x - 8$$

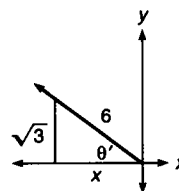
$$x = -2 \pm 2\sqrt{3} \approx (1.5, 0), (-5.5, 0)$$



$$3. x = -\sqrt{6^2 - (\sqrt{3})^2} = -\sqrt{33};$$

$$\tan \theta = \frac{\sqrt{3}}{x} = -\frac{\sqrt{3}}{\sqrt{33}} = -\sqrt{\frac{3}{33}}$$

$$= -\sqrt{\frac{1}{11}} = -\frac{\sqrt{11}}{11}$$



$$4. -5a^8 + 5a^2x^6$$

$$-5a^2(a^6 - x^6)$$

$$-5a^2(a^3 - x^3)(a^3 + x^3)$$

$$-5a^2(a - x)(a^2 + ax + x^2)(a + x)$$

$$(a^2 - ax + x^2)$$

5. The x -intercept is the point $(4, 0)$.

Let $(x_1, y_1) = (1, 3)$, and $(x_2, y_2) = (4, 0)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 3}{4 - 1} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1(x - 1)$$

$$y = -x + 4$$

$$6. \frac{3}{x^2 - 1} - \frac{1}{x - 1}$$

$$\frac{3}{(x - 1)(x + 1)} - \frac{1}{x - 1} \cdot \frac{x + 1}{x + 1}$$

$$\frac{3 - (x + 1)}{(x - 1)(x + 1)}$$

$$\frac{-x + 2}{x^2 - 1}$$

Solutions to trial exercise problems

$$4. 2 \cos \theta + 1 = 0$$

$$2 \cos \theta = -1$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta' = \cos^{-1} \frac{1}{2} = \frac{\pi}{3} \quad (60^\circ)$$

$\cos \theta < 0$ in quadrants II and III, so $\theta = \pi - \theta'$

$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3} \quad (180^\circ - 60^\circ = 120^\circ) \text{ and } \theta = \pi + \theta'$$

$$= \pi + \frac{\pi}{3} = \frac{4\pi}{3} \quad (180^\circ + 60^\circ = 240^\circ)$$

$$20. \cos^2 \theta - \frac{1}{2} \cos \theta = 0$$

$$\cos \theta (\cos \theta - \frac{1}{2}) = 0$$

$$\cos \theta = 0$$

$$\text{or } \cos \theta - \frac{1}{2} = 0$$

$$\frac{\pi}{2} \quad (90^\circ), \frac{3\pi}{2} \quad (270^\circ) \quad \cos \theta = \frac{1}{2}$$

$$\frac{\pi}{3} \quad (60^\circ), 2\pi - \theta' = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \quad (300^\circ)$$

$$27. \sqrt{3} \tan \theta \cot \theta + \cot \theta = 0$$

$$\cot \theta (\sqrt{3} \tan \theta + 1) = 0$$

$$\cot \theta = 0 \text{ or } \sqrt{3} \tan \theta + 1 = 0$$

$$\frac{\cos \theta}{\sin \theta} = 0 \text{ or } \tan \theta = -\frac{\sqrt{3}}{3}$$

$$\cos \theta = 0 \quad \frac{5\pi}{6} (150^\circ) \text{ and } \frac{11\pi}{6} (330^\circ)$$

$$\frac{\pi}{2} (90^\circ), \frac{3\pi}{2} (270^\circ)$$

$$34. 2 - \sin x - \csc x = 0$$

$$2 - \sin x - \frac{1}{\sin x} = 0$$

$$2 \sin x - \sin^2 x - 1 = 0$$

$$\sin^2 x - 2 \sin x + 1 = 0$$

$$(\sin x - 1)^2 = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

$$x = \frac{\pi}{2} (90^\circ)$$

$$41. 2 \tan^2 x \sin x = \tan^2 x$$

$$2 \tan^2 x \sin x - \tan^2 x = 0$$

$$\tan^2 x (2 \sin x - 1) = 0$$

$$\tan^2 x = 0 \text{ or } 2 \sin x - 1 = 0$$

$$\tan x = 0 \text{ or } \sin x = \frac{1}{2}$$

$$0 (0^\circ), \pi (180^\circ), \frac{\pi}{6} (30^\circ), \frac{5\pi}{6} (150^\circ)$$

$$46. \tan^2 x + 5 \tan x + 2 = 0$$

$$a = 1, b = 5, c = 2: \tan x = \frac{-5 \pm \sqrt{5^2 - 4(2)}}{2} = \frac{-5 \pm \sqrt{17}}{2}$$

$$\tan x = \frac{-5 + \sqrt{17}}{2} \approx -0.4384$$

$$x = \tan^{-1} \left| \frac{-5 + \sqrt{17}}{2} \right|$$

$$\approx 0.413 (23.7^\circ)$$

$$\tan x < 0 \text{ in quadrants II and IV.}$$

$$x = \pi - x' \approx \pi - 0.413 \approx 2.74$$

$$= 2\pi - x' \approx 2\pi - 0.413 \approx 5.87$$

$$x = 180^\circ - x' \approx 180^\circ - 23.7^\circ \approx 156.3^\circ$$

$$= 360^\circ - x' \approx 360^\circ - 23.7^\circ \approx 336.3^\circ$$

$$\tan x = \frac{-5 - \sqrt{17}}{2} \approx -4.5616$$

$$x = \tan^{-1} \left| \frac{-5 - \sqrt{17}}{2} \right|$$

$$\approx 1.355 (77.6^\circ)$$

$$\tan x < 0 \text{ in quadrants II and IV.}$$

$$x = \pi - x' \approx \pi - 1.355 \approx 1.79$$

$$= 2\pi - x' \approx 2\pi - 1.355 \approx 4.93$$

$$x = 180^\circ - x' \approx 180^\circ - 77.6^\circ \approx 102.4^\circ$$

$$= 360^\circ - x' \approx 360^\circ - 77.6^\circ \approx 282.4^\circ$$

$$57. \tan x = 1$$

$$x = \tan^{-1} 1 = \frac{\pi}{4} (45^\circ)$$

$$\text{Primary solutions are in quadrants I and III: } \frac{\pi}{4} (45^\circ) \text{ and } \frac{5\pi}{4}$$

$$(225^\circ). \text{ These differ by } \pi (180^\circ), \text{ so we can write all solutions}$$

$$\text{with one of them: } \frac{\pi}{4} + k\pi (45^\circ + k \cdot 180^\circ).$$

$$64. \sec \frac{x}{2} = 1; \cos \frac{x}{2} = 1$$

$$\text{Primary solutions: } \frac{x}{2} = \cos^{-1} 1 = 0 (0^\circ)$$

$$\text{All solutions: } \frac{x}{2} = 0 + 2k\pi (0^\circ + k \cdot 360^\circ);$$

$$x = 4k\pi (k \cdot 720^\circ)$$

$$74. \sin \frac{\theta}{3} = \frac{\sqrt{3}}{2}$$

$$\left(\frac{\theta}{3} \right)' = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3} (60^\circ)$$

$$\text{Primary solutions: } \frac{\theta}{3} = \frac{\pi}{3} (60^\circ), \frac{2\pi}{3} (120^\circ)$$

$$\text{All solutions: } \frac{\theta}{3} = \frac{\pi}{3} + 2k\pi (60^\circ + k \cdot 360^\circ), \frac{2\pi}{3} + 2k\pi$$

$$(120^\circ + k \cdot 360^\circ); \theta = \pi + 6k\pi (180^\circ + k \cdot 1080^\circ), 2\pi + 6k\pi$$

$$(360^\circ + k \cdot 1080^\circ).$$

$$81. \sin 2\theta + \sin \theta = 0$$

$$2 \sin \theta \cos \theta + \sin \theta = 0$$

$$\sin \theta (2 \cos \theta + 1) = 0$$

$$\sin \theta = 0$$

$$0 (0^\circ), \pi (180^\circ)$$

$$2 \cos \theta + 1 = 0$$

$$\cos \theta = -\frac{1}{2}$$

$$\frac{2\pi}{3} (120^\circ), \frac{4\pi}{3} (240^\circ)$$

$$88. \sin^2 \frac{\theta}{2} = \cos \theta$$

$$\left(\pm \sqrt{\frac{1 - \cos \theta}{2}} \right)^2 = \cos \theta$$

$$\frac{1 - \cos \theta}{2} = \cos \theta$$

$$1 - \cos \theta = 2 \cos \theta$$

$$1 = 3 \cos \theta$$

$$\frac{1}{3} = \cos \theta$$

$$\theta' = \cos^{-1} \frac{1}{3}$$

$$\theta = \cos^{-1} \frac{1}{3} \text{ or } 2\pi - \cos^{-1} \frac{1}{3}$$

$$\theta = 1.23 (70.5^\circ) \text{ or } 5.05 (289.5^\circ)$$

$$92. 0 = x \cos 0.855 \cos 1.052 - x^2 \cos 0.855 \sin 1.052 - x^3 \sin 0.855$$

$$0 = 0.32538x - 0.56987x^2 - 0.75457x^3$$

$$x(0.75457x^2 + 0.56987x - 0.32538) = 0$$

$$x = 0 \text{ or } 0.75457x^2 + 0.56987x - 0.32538 = 0$$

$$\text{Solve the quadratic equation with the quadratic formula.}$$

$$x \approx 0, -1.14, 0.38$$

$$\begin{aligned}
 93. \quad -8 &= 2 \cos A \cos 0.7 - 4 \cos A \sin 0.7 - 8 \sin A \\
 -8 &= (2 \cos 0.7) \cos A - (4 \sin 0.7) \cos A - 8 \sin A \\
 -8 &= 1.5297 \cos A - 2.5769 \cos A - 8 \sin A \\
 -8 &= -1.0472 \cos A - 8 \sin A \\
 8 \sin A - 8 &= -1.0472 \cos A \\
 \text{Divide each member by 8.} \\
 \sin A - 1 &= -0.1309 \cos A \\
 \sin A &= 1 - 0.1309 \cos A \\
 (\sin A)^2 &= (1 - 0.1309 \cos A)^2 \\
 \sin^2 A &= 1 - 0.2618 \cos A + 0.017134 \cos^2 A \\
 1 - \cos^2 A &= 1 - 0.2618 \cos A + 0.017134 \cos^2 A \\
 0 &= 1.0171 \cos^2 A - 0.2618 \cos A \\
 0 &= \cos A(1.0171 \cos A - 0.2618) \\
 \cos A &= 0 \text{ or } 1.0171 \cos A - 0.2618 = 0 \\
 A &= \cos^{-1} 0 \text{ or } 1.0171 \cos A = 0.2618 \\
 A &= \frac{\pi}{2} \text{ or } \cos A = 0.25739 \\
 A &\approx 1.310476103
 \end{aligned}$$

Thus, A is $\frac{\pi}{2}$ or 1.31

$$\begin{aligned}
 96. \quad \sin \theta \sqrt{1.44 - \sin^2 \theta} &= 0.5 \\
 (\sin \theta \sqrt{1.44 - \sin^2 \theta})^2 &= (0.5)^2 \\
 \sin^2 \theta (1.44 - \sin^2 \theta) &= 0.25 \\
 1.44 \sin^2 \theta - \sin^4 \theta &= 0.25 \\
 \sin^4 \theta - 1.44 \sin^2 \theta + 0.25 &= 0 \\
 \text{Let } u &= \sin^2 \theta. \\
 u^2 - 1.44u + 0.25 &= 0 \\
 u \approx 1.2381 \quad \text{or } u \approx 0.20193 \\
 \sin^2 \theta \approx 1.2381 \quad \text{or } \sin^2 \theta \approx 0.20193 \\
 \sin \theta \approx \pm \sqrt{1.2381} \quad \text{or } \sin \theta \approx \pm \sqrt{0.20193} \\
 \sin \theta \approx \pm 1.1127 \quad \text{or } \sin \theta \approx \pm 0.44936 \\
 \text{No solution} \quad \theta \approx \pm 26.7^\circ \\
 \text{Because } 0 \leq \theta \leq 90^\circ, \text{ we choose the positive value for } \theta. \\
 \theta \approx 26.7^\circ
 \end{aligned}$$

Chapter 7 review

$$\begin{aligned}
 1. \quad &\frac{\cot \theta}{\cos \theta} \\
 &\frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\cos \theta} \\
 &\frac{1}{\sin \theta} \\
 &\csc \theta
 \end{aligned}$$

$$\begin{aligned}
 2. \quad &\sec \theta \tan \theta \\
 &\frac{1}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta} \\
 &\frac{1}{\cos^2 \theta} \sin \theta \\
 &\sec^2 \theta \sin \theta
 \end{aligned}$$

$$\begin{aligned}
 3. \quad &\frac{\tan^2 \theta}{\sec^2 \theta - 1} = \sin^4 \theta \csc^4 \theta \\
 &\frac{\tan^2 \theta}{\tan^2 \theta} \cdot \frac{\sin^4 \theta}{\sin^4 \theta} \\
 &1 \cdot 1
 \end{aligned}$$

$$\begin{aligned}
 4. \quad &\frac{\csc^2 \theta - 1}{\sec^2 \theta - 1} \\
 &\frac{\cot^2 \theta}{\tan^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 &\frac{\cot^2 \theta}{\tan^2 \theta} \\
 &\frac{\cot^2 \theta \cot^2 \theta}{\cot^4 \theta} \\
 &\frac{\csc \theta \tan \theta}{\sin \theta} \\
 &\csc \theta \tan \theta \cdot \frac{1}{\sin \theta} \\
 &\frac{1}{\sin \theta} \cdot \frac{\sin \theta}{\cos \theta} \cdot \frac{1}{\sin \theta} \\
 &\frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} \\
 &\csc \theta \sec \theta
 \end{aligned}$$

$$\begin{aligned}
 6. \quad &\sin^2 \theta - \cos^2 \theta \\
 &\sin^2 \theta - (1 - \sin^2 \theta) \\
 &2 \sin^2 \theta - 1
 \end{aligned}$$

$$\begin{aligned}
 7. \quad &\frac{\cos \theta - \sin \theta}{\sin \theta \cos \theta}
 \end{aligned}$$

$$\begin{aligned}
 8. \quad &\frac{1}{\sin \theta \cos \theta}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad &\frac{\sin \theta}{\sin^2 \theta - \cos^2 \theta}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad &\frac{\sin \theta - \cos \theta}{\sin \theta + 1}
 \end{aligned}$$

$$11. \sin^2 \theta$$

$$12. \sin^2 \theta - \cos^2 \theta$$

$$\begin{aligned}
 13. \quad &\csc x - \tan x \cot x \\
 &\frac{1}{\sin x} - \tan x \frac{1}{\tan x} \\
 &\csc x - 1
 \end{aligned}$$

$$\begin{aligned}
 14. \quad &\sin^2 x + \sin^2 x \cot^2 x = 1 \\
 &\sin^2 x + \sin^2 x \frac{\cos^2 x}{\sin^2 x} \\
 &\sin^2 x + \cos^2 x \\
 &1
 \end{aligned}$$

$$\begin{aligned}
 15. \quad &\csc^2 x \sec^2 x (\cos^2 x - \sin^2 x) \\
 &\csc^2 x \sec^2 x \cos^2 x - \csc^2 x \sec^2 x \sin^2 x \\
 &\csc^2 x \frac{1}{\cos^2 x} \cos^2 x - \frac{1}{\sin^2 x} \sec^2 x \sin^2 x \\
 &\csc^2 x - \sec^2 x
 \end{aligned}$$

$$\begin{aligned}
 16. \quad &\frac{1}{\csc x - \cot x} \\
 &\frac{1}{\frac{1}{\sin x} - \frac{\cos x}{\sin x}} \\
 &\frac{1}{\frac{1 - \cos x}{\sin x}} \\
 &\frac{\sin x}{1 - \cos x}
 \end{aligned}$$

$$\begin{aligned}
 17. \quad &(\tan x - 1)(\csc^2 x - \cot^2 x) = \sec x (\sin x - \cos x) \\
 &(\tan x - 1)[(\cot^2 x + 1) - \cot^2 x] \quad \sec x \sin x - \sec x \cos x \\
 &(\tan x - 1)(1) \quad \frac{1}{\cos x} \sin x - \frac{1}{\cos x} \cos x \\
 &\tan x - 1 \quad \tan x - 1
 \end{aligned}$$

$$\begin{aligned}
 18. \quad &\frac{\csc^2 x - 1}{\sin^2 x} \\
 &\frac{\cot^2 x}{\sin^2 x}
 \end{aligned}$$

$$\begin{aligned}
 &\cot^2 x \cdot \frac{1}{\sin^2 x} \\
 &\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\sin^2 x} \\
 &\cos^2 x \cdot \frac{1}{\sin^4 x} \\
 &\cos^2 x \csc^4 x
 \end{aligned}$$

$$\begin{aligned}
 19. \quad &\frac{1}{1 + \csc x} + \frac{1}{1 - \csc x} \\
 &\frac{(1 - \csc x) + (1 + \csc x)}{(1 + \csc x)(1 - \csc x)} \\
 &\frac{2}{1 - \csc^2 x} \\
 &\frac{2}{-(\csc^2 x - 1)} \\
 &\frac{2}{-\cot^2 x} \\
 &-2 \tan^2 x
 \end{aligned}$$

$$\begin{aligned}
 20. \quad &\sin^2 x + \sin^2 x \cos^2 x \\
 &\sin^2 x (1 + \cos^2 x) \\
 &(1 - \cos^2 x)(1 + \cos^2 x) \\
 &1 - \cos^4 x
 \end{aligned}$$

$$\begin{aligned}
 21. \quad &\tan^4 x + \tan^2 x \\
 &\tan^2 x (\tan^2 x + 1) \\
 &\tan^2 x \cdot \sec^2 x \\
 &\frac{1}{\cot^2 x} \cdot \sec^2 x \\
 &\frac{\sec^2 x}{\cot^2 x}
 \end{aligned}$$

$$\begin{aligned}
 22. \quad &\frac{1 - \cot x}{1 + \csc x} \\
 &1 - \frac{\cos x}{\sin x} \cdot \frac{\sin x}{\sin x} \\
 &1 + \frac{1}{\sin x} \\
 &\frac{\sin x - \cos x}{\sin x + 1}
 \end{aligned}$$

23. $\frac{\sqrt{6}-\sqrt{2}}{4}$ 24. $-2+\sqrt{3}$
 25. $\frac{\sqrt{6}+\sqrt{2}}{4}$ 26. $\frac{\sqrt{6}+\sqrt{2}}{4}$
 27. $\frac{63}{65}$ 28. $-\frac{12\sqrt{5}+10}{39}$
 29. $\frac{270-169\sqrt{2}}{28}$
 30. $\cos\left(\frac{3\pi}{2}+\theta\right)=\cos\frac{3\pi}{2}\cos\theta-\sin\frac{3\pi}{2}\sin\theta$
 $=0\cos\theta-(-1)\sin\theta=\sin\theta$
 31. $\sin\left(\frac{\pi}{4}-\theta\right)=\sin\frac{\pi}{4}\cos\theta-\cos\frac{\pi}{4}\sin\theta$
 $=\frac{\sqrt{2}}{2}\cos\theta-\frac{\sqrt{2}}{2}\sin\theta=\frac{\sqrt{2}}{2}(\cos\theta-\sin\theta)$
 32. $\frac{\cos(\alpha+\beta)}{\sin\alpha\cos\beta}=\frac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\sin\alpha\cos\beta}$
 $=\frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta}-\frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta}=\frac{\cos\alpha}{\sin\alpha}-\frac{\sin\beta}{\cos\beta}$
 $=\cot\alpha-\tan\beta$
 33. $\sin(\theta+2\pi)=\sin\theta\cos 2\pi+\cos\theta\sin 2\pi$
 $=\sin\theta(1)+\cos\theta(0)$
 $=\sin\theta$
 34. $\frac{\sqrt{2}}{10}$ 35. 124° 36. 10π
 37. $\frac{7\pi}{6}$ 38. 12° 39. $a=3, b=1$
 40. a. $\frac{-5\sqrt{119}}{47}$ b. $\frac{5\sqrt{119}}{72}$
 41. a. $-\frac{7}{23}$ b. $\frac{24}{7}$
 42. $\sin 2x - \cos x$
 $2\sin x \cos x - \cos x$
 $\cos x(2\sin x - 1)$
 43. $1 + \cos 2x$
 $1 + (2\cos^2 x - 1)$
 $2\cos^2 x$
 44. $\frac{\sqrt{12-8\sqrt{2}}}{2}$ 45. $\frac{\sqrt{2+\sqrt{2}}}{2}$
 46. a. $\frac{\sqrt{26}}{26}$ b. $\frac{1}{5}$
 47. a. $\frac{\sqrt{18-6\sqrt{5}}}{6}$ b. $-\frac{3+\sqrt{5}}{2}$
 48. $\cot\frac{\theta}{2}=\frac{1}{\tan\frac{\theta}{2}}=\frac{1}{\frac{\sin\theta}{1+\cos\theta}}=\frac{1+\cos\theta}{\sin\theta}$
 49. $\sec^2\frac{\theta}{2}-\tan^2\frac{\theta}{2}$
 $\left(\tan^2\frac{\theta}{2}+1\right)-\tan^2\frac{\theta}{2}$
 1
 50. $\frac{\pi}{6}(30^\circ), \frac{5\pi}{6}(150^\circ)$
 51. $\frac{\pi}{3}(60^\circ), \frac{2\pi}{3}(120^\circ), \frac{4\pi}{3}(240^\circ), \frac{5\pi}{3}(300^\circ)$
 52. $\frac{\pi}{2}(90^\circ), \frac{\pi}{3}(60^\circ), \frac{5\pi}{3}(300^\circ)$
 53. $\frac{\pi}{6}(30^\circ), \frac{5\pi}{6}(150^\circ), \frac{7\pi}{6}(210^\circ),$
 $\frac{11\pi}{6}(330^\circ), \frac{\pi}{3}(60^\circ), \frac{5\pi}{3}(300^\circ)$
 54. $\frac{\pi}{2}(90^\circ), \frac{3\pi}{2}(270^\circ), \frac{\pi}{4}(45^\circ), \frac{5\pi}{4}(225^\circ)$
 55. $\frac{\pi}{3}(60^\circ), \frac{2\pi}{3}(120^\circ), \frac{4\pi}{3}(240^\circ), \frac{5\pi}{3}(300^\circ)$
 56. $\frac{2\pi}{3}(120^\circ), \frac{4\pi}{3}(240^\circ), 0(0^\circ)$
 57. $\frac{\pi}{6}(30^\circ), \frac{5\pi}{6}(150^\circ), \frac{3\pi}{2}(270^\circ)$
 58. $\frac{\pi}{2}(90^\circ), \frac{3\pi}{2}(270^\circ), \frac{\pi}{6}(30^\circ), \frac{5\pi}{6}(150^\circ)$
 59. $\frac{2\pi}{3}(120^\circ), \frac{4\pi}{3}(240^\circ), \pi(180^\circ)$
 60. $\frac{\pi}{24}, \frac{13\pi}{24}, \frac{25\pi}{24}, \frac{37\pi}{24}, \frac{5\pi}{24}, \frac{17\pi}{24}, \frac{29\pi}{24},$
 $\frac{41\pi}{24}$ 61. $\frac{\pi}{3}(60^\circ)$ 62. $\frac{5\pi}{3}(300^\circ)$
 63. $\pi(180^\circ)$ 64. $\frac{4\pi}{3}(240^\circ)$ 65. $\frac{\pi}{15}, \frac{7\pi}{15},$
 $\frac{13\pi}{15}, \frac{19\pi}{15}, \frac{5\pi}{3}, \frac{\pi}{3}, \frac{11\pi}{15}, \frac{17\pi}{15}, \frac{23\pi}{15}, \frac{29\pi}{15}$
 or $12^\circ, 84^\circ, 156^\circ, 228^\circ, 300^\circ, 60^\circ, 132^\circ,$
 $204^\circ, 276^\circ, 348^\circ$
 66. $\frac{\pi}{2}(90^\circ), \frac{3\pi}{2}(270^\circ),$
 $\frac{7\pi}{6}(210^\circ), \frac{11\pi}{6}(330^\circ)$ 67. 0.96, 2.19, 4.10, 5.33
 68. 3.52, 5.90 69. $\frac{\pi}{3}, \frac{5\pi}{3}, \pi$
 70. $\frac{2\pi}{3}, \frac{11\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}, \frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}$
 71. $\frac{\pi}{4}, \frac{5\pi}{4}, 1.94, 2.78, 5.08, 5.92$
- Chapter 7 test**
1. $\csc^2 x \sin x \cos x$
 $\frac{1}{\sin^2 x} \sin x \cos x$
 $\frac{\cos x}{\sin x}$
 $\cot x$
2. $\frac{\csc x - \sec x}{\tan x + \cot x}$
 $\frac{\frac{1}{\sin \theta} - \frac{1}{\cos \theta}}{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}$
 $\frac{\frac{1}{\sin \theta} - \frac{1}{\cos \theta}}{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}}$
 $\frac{\frac{1}{\sin \theta} - \frac{1}{\cos \theta}}{\frac{1}{\sin \theta \cos \theta}}$
 $\cos \theta - \sin \theta$
 3. $\cot x - 2 \tan x = 1$
 $\cot \frac{\pi}{4} - 2 \tan \frac{\pi}{4} = 1$
 $1 - 2(1) = 1$
 $-1 \neq 1$
 4. $a = 8, b = 4$ 5. $\frac{8\sqrt{3}-15}{34}$
 6. $-\frac{240}{289}$ 7. 3 8. $\sqrt{4-2\sqrt{2}}$
 9. $\frac{1+\cot \theta}{\csc \theta}$
 $\frac{1}{\csc \theta} + \frac{\cot \theta}{\csc \theta}$
 $\sin \theta + \cot \theta \frac{1}{\csc \theta}$
 $\sin \theta + \frac{\cos \theta}{\sin \theta} \cdot \sin \theta$
 $\sin \theta + \cos \theta$
 $\frac{\cos^2 x - 1}{\sin^2 x}$
 10. $\frac{\cos^2 x - 1}{\sin^2 x}$
 $\frac{-(1 - \cos^2 x)}{\sin^2 x}$
 $\frac{-\sin^2 x}{\sin^2 x}$
 -1
 11. $\tan^4 x + \tan^2 x$
 $\tan^2 x(\tan^2 x + 1)$
 $\tan^2 x \sec^2 x$
 $\frac{1}{\cot^2 x} \sec^2 x$
 $\frac{\sec^2 x}{\cot^2 x}$
 12. $\cos\left(\theta - \frac{3\pi}{2}\right)$
 $\cos \theta \cos \frac{3\pi}{2} + \sin \theta \sin \frac{3\pi}{2}$
 $\cos \theta (0) + \sin \theta (-1)$
 $-\sin \theta$

13. $\cos 2x - \sin 2x$
 $(2 \cos^2 x - 1) - (2 \sin x \cos x)$
 $2 \cos^2 x - 2 \sin x \cos x - 1$
 $2 \cos x (\cos x - \sin x) - 1$
14. $\frac{\pi}{3} (60^\circ), \frac{2\pi}{3} (120^\circ), \frac{4\pi}{3} (240^\circ), \frac{5\pi}{3} (300^\circ)$
15. $\frac{\pi}{6} (30^\circ), \frac{7\pi}{6} (210^\circ), \frac{2\pi}{3} (120^\circ), \frac{4\pi}{3} (240^\circ)$
16. $0.16, 2.97, \frac{3\pi}{2}$ 17. $\frac{\pi}{12}, \frac{3\pi}{4}, \frac{17\pi}{12}, \frac{\pi}{4}$,
 $\frac{11\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{21\pi}{12}, \frac{7\pi}{12}, \frac{5\pi}{4}, \frac{23\pi}{12}$

Chapter 8

Exercise 8-1

Answers to odd-numbered problems

1. $C = 96^\circ, b \approx 16.4, c \approx 21.7$
 3. $A = 34.4^\circ, b \approx 0.52, c \approx 1.64$
 5. $C = 33^\circ, c \approx 51.2^\circ, a \approx 68.7$
 7. $B = 20.6^\circ, b \approx 0.172, c \approx 0.489$
 9. $C = 35^\circ, a \approx 8.58, b \approx 6.16$
 11. $A \approx 45.6^\circ, C \approx 85.4^\circ, c \approx 17.4$
 13. $B \approx 18.0^\circ, C \approx 30.0^\circ, b \approx 1.77$
 15. $C \approx 47.4^\circ, A \approx 88.9^\circ, a \approx 133.9$;
 $C \approx 132.6^\circ, A \approx 3.7^\circ, a \approx 8.7$
 17. no solution
 19. $B \approx 85.0^\circ, A \approx 52.7^\circ, a \approx 5.07$;
 $B \approx 95.01^\circ, A \approx 42.7^\circ, a \approx 4.32$
 21. $C \approx 26.23^\circ, A \approx 108.77^\circ, a \approx 10.71$
 23. 32.3 miles
 25. 843,400 miles
 27. 15.8 knots
 29. 25 miles
 31. By the definitions of section 5-3, \tan
 $A = \frac{y}{x}$ in each figure. By the trigonometric ratios (for a right
 triangle) it can be seen in each case that $\tan C = \frac{y}{b-x}$.
 (Note that x is negative in the right figure, so that $b-x$ is
 larger than b itself.) Also, as noted in the problem, $y = h$.
 Putting these values in the expression for h we obtain

$$\frac{b \cdot \tan A \cdot \tan C}{\tan A + \tan C}$$

$$b \cdot \frac{\frac{y}{x} \cdot \frac{y}{b-x}}{\frac{y}{x} + \frac{y}{b-x}} = \frac{by^2}{x(b-x)} = y = h$$
33. Let (x, y) be the point at B . It is on the terminal side of angle
 A . Then \cos
 $A = \frac{x}{r}$, where r is the length of AB . But then $r = c$, so $\cos A$
 $= \frac{x}{c}$. Next, using right triangles we see that in each figure

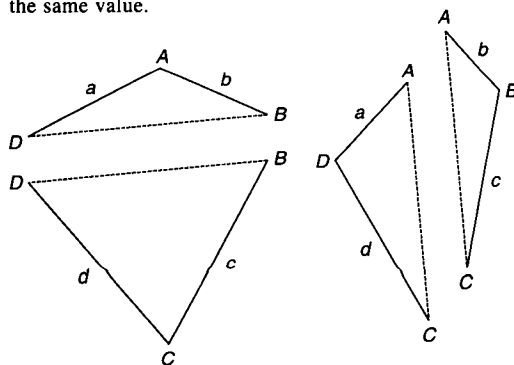
$\cos C = \frac{b-x}{a}$. Note that when A is obtuse (the right-hand
 figure) x is negative, so $b-x$ is the length of $|b| + |x|$.
 $\cos C = \frac{b-x}{a}$ $\cos A = \frac{x}{c}$
 $a \cos C = b-x$ $c \cos A = x$
 $b - a \cos C = x$
 $b - a \cos C = c \cos A$
 $b = c \cos A + a \cos C$
 Thus, [2] is true.

[1] and [3] can be shown true by putting angles B and C
 in standard position and proceeding in the same manner. In
 fact this is not really necessary, since the labeling in a
 triangle is arbitrary, and thus, for example, we could obtain
 [1] by changing the label B to A , C to B , and A to C , and
 labeling the sides appropriately.

35. Consider any triangle ABC ; place it as shown in the figure for
 problem 33, so angle A is in standard position. The figure
 covers the cases where A is acute, right, or obtuse. Then it
 can be seen that if h is the height of the triangle then $h = y$.

We know that $\sin A = \frac{y}{c} = \frac{h}{c}$, so $h = c \sin A$. The area is
 $\frac{1}{2}bh = \frac{1}{2}b(c \sin A) = \frac{1}{2}bc \sin A$.

37. a. It can be seen that the sum of the area of the four triangles
 shown in the figure is $\frac{1}{2}ab \sin A + \frac{1}{2}cd \sin$
 $C + \frac{1}{2}ad \sin D + \frac{1}{2}bc \sin B$. This total is twice as large as
 the total area of the four-sided figure, so the area of the
 four-sided figure is $\frac{1}{2}$ this sum, or $\frac{1}{4}(ab \sin A + ad$
 $\sin D + bc \sin B + cd \sin C)$.
 b. The difference between the Egyptian formula and the
 correct formula is the factors $\sin A$, $\sin B$, $\sin C$, and \sin
 D . The value of the sine of each angle is between 0 and 1.
 Thus, $ab \geq ab \sin A$, $ad \geq ad \sin D$, $bc \geq bc \sin B$, $cd \geq$
 $cd \sin C$, so $ab + ad + bc + cd \geq ab \sin A + ad \sin$
 $D + bc \sin B + cd \sin C$.
 $\frac{1}{4}(ab + ad + bc + cd) \geq$
 $\frac{1}{4}(ab \sin A + ad \sin D + bc \sin B + cd \sin C)$
 If the figure is a rectangle, $A = B = C = D = 90^\circ$, and \sin
 $A = \sin B = \sin C = \sin D = 1$, so both expressions give
 the same value.

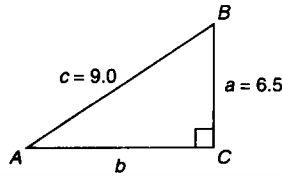


Solutions to skill and review problems

1. $b^2 = \sqrt{9^2 - 6.5^2} \approx 6.2$

$$\sin A = \frac{6.5}{9} \text{ so } A = \sin^{-1} \frac{6.5}{9} \approx 46.2^\circ;$$

$$\cos B = \frac{6.5}{9}, B = \cos^{-1} \frac{6.5}{9} \approx 43.8^\circ$$



4. Possible rational zeros of $2x^5 - x^4 - 10x^3 + 5x^2 + 8x - 4$ are $\pm 1, \pm 2, \pm 4$, and $\pm \frac{1}{2}$.

	2	-1	-10	5	8	-4
		2	1	-9	-4	4
1	2	1	-9	-4	4	0

1 is a zero

$$(x - 1)(2x^4 + x^3 - 9x^2 - 4x + 4).$$

	2	1	-9	-4	4
		-2	1	8	-4
-1	2	-1	-8	4	0

-1 is a zero

$$\begin{aligned} 2x^5 - x^4 - 10x^3 + 5x^2 + 8x - 4 &= (x - 1)(x + 1)(2x^3 - x^2 - 8x + 4) \\ 2x^3 - x^2 - 8x + 4 &= x^2(2x - 1) - 4(2x - 1) \\ &= (2x - 1)(x^2 - 4) \\ &= (2x - 1)(x - 2)(x + 2), \text{ so} \\ 2x^5 - x^4 - 10x^3 + 5x^2 + 8x - 4 &= (x - 1)(x + 1)(2x - 1)(x - 2)(x + 2) \end{aligned}$$

5. $\frac{\theta^\circ}{180^\circ} = \frac{s}{\pi}$, so $s = \frac{\pi}{180^\circ} \theta^\circ$,

$$\text{so } s = \frac{\pi}{180^\circ} \cdot 24^\circ = \frac{2\pi}{15}.$$

$$L = rs$$

$$L = 18 \left(\frac{2\pi}{15} \right) = \frac{12\pi}{5} \approx 7.5 \text{ meters}$$

Solutions to trial exercise problems

7. $a = 0.452, A = 67.6^\circ, C = 91.8^\circ$

$$B = 180^\circ - 67.6^\circ - 91.8^\circ = 20.6^\circ$$

$$\frac{\sin 67.6^\circ}{0.452} = \frac{\sin 20.6^\circ}{b} = \frac{\sin 91.8^\circ}{c}$$

$$\frac{\sin 67.6^\circ}{0.452} = \frac{\sin 20.6^\circ}{b} \quad \frac{\sin 67.6^\circ}{0.452} = \frac{\sin 91.8^\circ}{c}$$

$$b \approx 0.172 \quad c \approx 0.489$$

2. $\sec x = -\frac{2}{3}\sqrt{3}$

$$\cos x = -\frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{2}$$

$$x' = \cos^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{6}; \cos x < 0 \text{ so } x$$

terminates in quadrant II or III. $x = \pi$

$$-x' \text{ or } \pi + x' = \frac{5\pi}{6} \text{ or } \frac{7\pi}{6}$$

13. $a = 4.25, c = 2.86, A = 132^\circ$

$$\frac{\sin 132^\circ}{4.25} = \frac{\sin B}{b} = \frac{\sin C}{2.86}$$

$$\frac{\sin 132^\circ}{4.25} = \frac{\sin C}{2.86}$$

$$\text{so } \sin C = \frac{2.86 \sin 132^\circ}{4.25}$$

$$C' = \sin^{-1} \frac{2.86 \sin 132^\circ}{4.25} \approx 30.01^\circ$$

$$C = 30.01^\circ \text{ or } 180^\circ - 30.01^\circ \approx 149.99^\circ$$

Case 1: $C \approx 30.01^\circ$

$$B = 180^\circ - 132^\circ - 30.01^\circ \approx 17.99^\circ$$

$$\frac{\sin 132^\circ}{4.25} = \frac{\sin 17.99^\circ}{b}$$

$$b \approx 1.77$$

Case 2: $C \approx 149.99^\circ$

$$B = 180^\circ - 132^\circ - 149.99^\circ$$

$$\approx -101.99^\circ \text{ (no solution)}$$

Thus, the only solution is $B \approx 18.0^\circ$,

$$C \approx 30.0^\circ, b \approx 1.77.$$

15. $b = 92.5, c = 98.6, B = 43.7^\circ$

$$\frac{\sin A}{a} = \frac{\sin 43.7^\circ}{92.5} = \frac{\sin C}{98.6}$$

$$\frac{\sin 43.7^\circ}{92.5} = \frac{\sin C}{98.6};$$

$$\sin C = \frac{98.6 \sin 43.7^\circ}{92.5}$$

$$C' = \sin^{-1} \frac{98.6 \sin 43.7^\circ}{92.5} \approx 47.429^\circ$$

$$C = 47.429^\circ \text{ or } 180^\circ - 47.429^\circ$$

$$\approx 132.571^\circ$$

Case 1: $C \approx 47.429^\circ$

$$A = 180^\circ - 43.7^\circ - 47.429^\circ \approx 88.871^\circ$$

$$\frac{\sin 88.871^\circ}{a} = \frac{\sin 43.7^\circ}{92.5}$$

$$a \approx 133.86$$

Solution 1: $C \approx 47.4^\circ, A \approx 88.9^\circ$,

$$a \approx 133.9$$

3. $(\sin \theta - \sec \theta)(\csc \theta + \cos \theta)$

$$\sin \theta \csc \theta + \sin \theta \cos \theta - \sec \theta \csc \theta$$

$$- \sec \theta \cos \theta$$

$$1 + \sin \theta \cos \theta - \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta} - 1$$

$$\sin \theta \cos \theta - \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta}$$

$$\frac{\sin^2 \theta \cos^2 \theta}{\sin \theta \cos \theta} - \frac{1}{\sin \theta \cos \theta}$$

$$\frac{\sin^2 \theta \cos^2 \theta - 1}{\sin \theta \cos \theta}$$

Case 2: $C \approx 132.571^\circ$

$$A = 180^\circ - 43.7^\circ - 132.571^\circ \approx 3.729^\circ$$

$$\frac{\sin 3.729^\circ}{a} = \frac{\sin 43.7^\circ}{92.5}$$

$$a \approx 8.71$$

Solution 2: $C \approx 132.6^\circ, A \approx 3.7^\circ$,

$$a \approx 8.7$$

27. Angle $A = 90^\circ - 58^\circ = 32^\circ$;

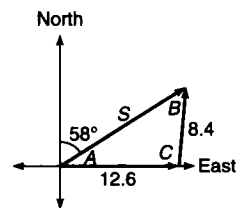
$$\frac{\sin 32^\circ}{8.4} = \frac{\sin B}{12.6};$$

$$\sin B = \frac{12.6 \sin 32^\circ}{8.4}, \text{ so } B' \approx 52.64^\circ.$$

$$C \approx 180^\circ - 32^\circ - 52.64^\circ \approx 95.36^\circ.$$

$$\frac{\sin 32^\circ}{8.4} = \frac{\sin 95.36^\circ}{S}; S \approx 15.78. \text{ Thus,}$$

$$S \approx 15.8 \text{ knots.}$$



Exercise 8-2

Answers to odd-numbered problems

1. $c \approx 4.0, A \approx 30.7^\circ, B \approx 109.9^\circ$

3. $a \approx 77.2, B \approx 41.1^\circ, C \approx 14.9^\circ$

5. $b \approx 38.3, A \approx 53.8^\circ, C \approx 25.9^\circ$

7. $C \approx 109.0^\circ, A \approx 39.4^\circ, B \approx 31.6^\circ$

9. $B \approx 105.3^\circ, A \approx 24.4^\circ, C \approx 50.3^\circ$

11. $c \approx 18.1, A \approx 28.3^\circ, B \approx 12.3^\circ$

13. $a \approx 28.1, C \approx 40.5^\circ, B \approx 115.0^\circ$

15. $b \approx 41.8, C \approx 26.3^\circ, A \approx 41.7^\circ$

17. $C \approx 110.9^\circ, A \approx 38.3^\circ, B \approx 30.8^\circ$

19. $c \approx 0.28, A \approx 1.12^\circ, B \approx 177.38^\circ$

21. 326.9 ft 23. $C \approx 82.4^\circ$

25. $B \approx 125.5^\circ$ 27. 102.9°

29. 39.9 miles 31. $A \approx 75.0^\circ$

33. Yes, the law of cosines can be used:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

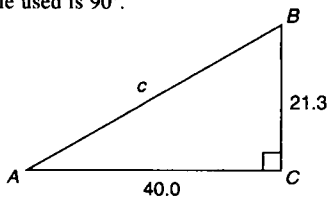
$$c^2 = 21.3^2 + 40^2 - 2(21.3)(40) \cos 90^\circ$$

$$c^2 = 21.3^2 + 40^2 - 2(21.3)(40)(0)$$

$$c^2 = 21.3^2 + 40^2$$

$$c \approx 45.3$$

Since $\cos 90^\circ = 0$, the law of cosines is the same as the Pythagorean theorem when the angle used is 90° .



Solutions to skill and review problems

$$1. \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{6.8} = \frac{\sin 43^\circ}{12} = \frac{\sin C}{c}$$

$$\sin A = \frac{\sin 43^\circ}{12} (6.8) \approx 0.3865, \text{ so}$$

$$A' \approx 22.73^\circ$$

$$A \approx 22.73^\circ \text{ or } 180^\circ - 22.73^\circ \approx 157.27^\circ$$

$$\text{Case 1: } A \approx 22.73^\circ$$

$$C \approx 180^\circ - 22.73^\circ - 43^\circ$$

$$\approx 114.27^\circ$$

$$c = \frac{12 \sin 114.27^\circ}{\sin 43^\circ} \approx 16.0$$

$$c \approx 16.0, A \approx 22.7^\circ, C \approx 114.3^\circ$$

$$\text{Case 2: } A \approx 157.27^\circ$$

$$C \approx 180^\circ - 157.27^\circ - 43^\circ$$

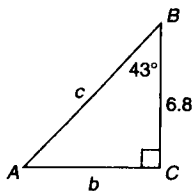
$$\approx -20.27^\circ$$

No solution.

$$2. \tan 43^\circ = \frac{b}{6.8}; b = 6.8 \tan 43^\circ \approx 6.3$$

$$A = 90^\circ - 43^\circ = 47^\circ$$

$$\cos 43^\circ = \frac{6.8}{c}; c = \frac{6.8}{\cos 43^\circ} \approx 9.3$$



$$3. f(x) = 2x^4 + 5x^3 - 8x^2 - 17x - 6$$

We look for zeros because these are x -intercepts. We factor the expression on the right at the same time. Possible rational zeros are $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2},$ and $\pm \frac{3}{2}$. By synthetic division we find -1 is a zero, so $f(x) = (x + 1)(2x^3 + 3x^2 - 11x - 6)$.

The value 2 is a zero, so $f(x)$

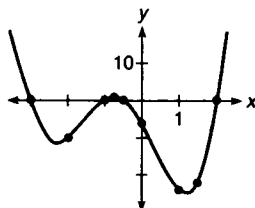
$$= (x + 1)(x - 2)(2x^2 + 7x + 3).$$

$$\text{Thus, } f(x) = (x + 1)(x - 2)(2x + 1)$$

$(x + 3)$, and the x -intercepts are $-1, 2, -\frac{1}{2}, -3$.

The y -intercept is $f(0) = -6$. We plot additional points between the x -intercepts:

x	-4	-2	$-\frac{3}{4}$	1	1.5	3
y	126	-12	0.8	-24	-22.5	168



$$4. f(x) = -(x - 3)^2 + 1$$

The graph of $f(x)$ is the graph of x^2 but flipped over and with vertex at $(3, 1)$.

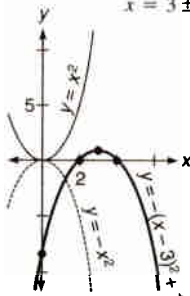
Intercepts: $f(0) = -8$ $(0, -8)$

$$0 = -(x - 3)^2 + 1$$

$$(x - 3)^2 = 1$$

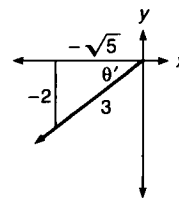
$$x - 3 = \pm 1$$

$$x = 3 \pm 1; (2, 0) \text{ and } (4, 0).$$



$$5. \sin \theta = -\frac{2}{5} \text{ and } \cos \theta < 0$$

$$\tan \theta = \frac{-2}{-\sqrt{5}} = \frac{2\sqrt{5}}{5}$$



Solutions to trial exercise problems

$$3. b = 61.3, c = 23.9, A = 124.0^\circ$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 61.3^2 + 23.9^2 - 2(61.3)(23.9) \cos 124^\circ \approx 5967.4$$

$$a \approx 77.249$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 124^\circ}{77.249} = \frac{\sin B}{61.3} = \frac{\sin C}{23.9}$$

Find angle C first; it is the smallest and therefore acute.

$$\sin C = \frac{23.9 \sin 124^\circ}{77.249}; C \approx 14.9^\circ$$

$$B \approx 180^\circ - 14.9^\circ - 124^\circ \approx 41.1^\circ$$

$$\text{Thus, } a \approx 77.2, B \approx 41.1^\circ, C \approx 14.9^\circ.$$

$$7. a = 23.5, b = 19.4, c = 35.0$$

$$c^2 = a^2 + b^2 - 2ab \cos C,$$

$$\text{so } \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{23.5^2 + 19.4^2 - 35^2}{2(23.5)(19.4)}$$

$$\text{so } C \approx 108.97^\circ \approx 109.0^\circ$$

$$23.5 \boxed{x^2} \boxed{+} 19.4 \boxed{x^2} \boxed{-} 35$$

$$\boxed{x^2} \boxed{=} \boxed{\div} 2 \boxed{\div} 23.5 \boxed{\div}$$

$$19.4 \boxed{=} \boxed{\text{SHIFT}} \boxed{\cos}$$

$$\text{TI-81: } \boxed{2\text{nd}} \boxed{\cos} \boxed{(} \boxed{(}$$

$$23.5 \boxed{x^2} \boxed{+} 19.4 \boxed{x^2} \boxed{-} 35$$

$$\boxed{x^2} \boxed{)} \boxed{\div} \boxed{(} 2 \boxed{\times} 23.5$$

$$\boxed{\times} 19.4 \boxed{)} \boxed{)} \boxed{\text{ENTER}}$$

Since C is the largest angle in the triangle, we know A and B are acute.

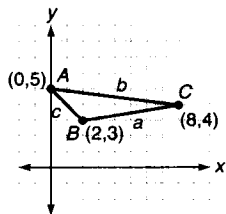
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{23.5} = \frac{\sin B}{19.4} = \frac{\sin 108.97^\circ}{35}$$

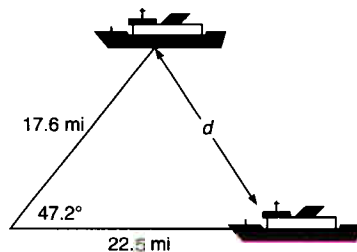
$$\sin A = \frac{\sin 108.97^\circ}{35} (23.5); A \approx 39.4^\circ$$

$$B \approx 180^\circ - 39.4^\circ - 109.0^\circ \approx 31.6^\circ$$

25. Use the distance formula, which states that for two points (x_1, y_1) , (x_2, y_2) , the distance d between them is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. We find that $a = \sqrt{37}$, $b = \sqrt{65}$, and $c = \sqrt{8}$. The largest angle is opposite the longest side, b . Thus, $b^2 = a^2 + c^2 - 2ac \cos B$, so $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{37 + 8 - 65}{2(\sqrt{37})(\sqrt{8})} \approx -0.5812$, $B \approx 125.5^\circ$.



26. $d^2 = 17.6^2 + 22.5^2 - 2(17.6)(22.5) \cos 47.2^\circ$; $d \approx 16.7$ miles



Exercise 8-3

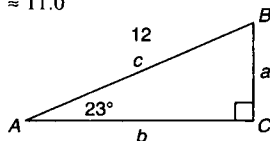
Answers to odd-numbered problems

1. $(20\sqrt{3}, 20)$ 3. $(-53.4, 84.5)$
 5. $(-9.4, -3.4)$ 7. $\left(12\frac{1}{2}, -\frac{25\sqrt{3}}{2}\right)$
 9. $(3\sqrt{2}, -3\sqrt{2})$ 11. $(-0.8, 7.8)$
 13. $(5, 53.1^\circ)$ 15. $(6.0, 120.0^\circ)$
 17. $(2.6, -49.1^\circ)$ 19. $(11.2, -63.4^\circ)$
 21. $(7.6, 153.4^\circ)$ 23. $(3.16, -116.6^\circ)$
 25. $(-1, 20)$ 27. $(8\sqrt{2}, 0)$
 29. $(45.2, 31.2^\circ)$ 31. $(36.5, -122.1^\circ)$
 33. $(9.0, -54.2^\circ)$ 35. $(47.1, 139.7^\circ)$
 37. $(6.3, 89.3^\circ)$ 39. $(20.3, 83.9^\circ)$
 41. $(12.9, 21.0^\circ)$ 43. west at 136 knots and north at 63 knots 45. east at 193 knots and north at 52 knots 47. west: 228 knots; north: 395 knots
 49. a. 24 nm b. 38 nm 51. No. The horizontal component of the force is 1,887 pounds. This is not enough to move the sled. 53. a. 966, 259 b. 1932, 518; yes c. 866, 500; no

55. 40 miles in a direction 59° south of east 57. $(15.4, -76.8^\circ)$ 59. $(270.7, 87.5^\circ)$ 61. 63 nm, 40° south of west 63. 99 knots, 47° north of west 65. 26.5 knots, 25.7° north of east 67. 266 volts at 341° 69. 8° west of north, 87 knots 71. 76.3° , 17.7 knots 73. 320 pounds, 50° with the horizontal
 75. Let $A = (a_1, a_2)$, $B = (b_1, b_2)$, and $C = (c_1, c_2)$ be three vectors in rectangular form. Then proceed as follows:
 $(A + B) + C$
 $[(a_1, a_2) + (b_1, b_2)] + (c_1, c_2)$
 The parentheses indicate we add A and B first.
 $(a_1 + b_1, a_2 + b_2) + (c_1, c_2)$
 Two vectors: one is $A + B$, and the other is C .
 $((a_1 + b_1) + c_1, (a_2 + b_2) + c_2)$
 One vector: $(A + B) + C$.
 $(a_1 + (b_1 + c_1), a_2 + (b_2 + c_2))$
 We can rearrange because real numbers are associative, and the components are real numbers.
 $A + (B + C)$

Solutions to skill and review problems

1. $c = 12.0$ and $A = 23^\circ$
 $B = 90^\circ - 23^\circ = 67^\circ$
 $\sin 23^\circ = \frac{a}{12}$, so $a = 12 \sin 23^\circ \approx 4.7$
 $\cos 23^\circ = \frac{b}{12}$, so $b = 12 \cos 23^\circ \approx 11.0$



2. $c = 12.0$, $a = 7.5$, and $A = 23^\circ$. We are not told that this is a right triangle, so we must use either the law of sines or the law of cosines. Since we have an angle (A) and the side opposite that angle (a), we use the law of sines.
 $\frac{\sin 23^\circ}{7.5} = \frac{\sin B}{b} = \frac{\sin C}{12}$
 $\sin C = \frac{\sin 23^\circ}{7.5}(12)$; $C' \approx 38.69^\circ$ or $180^\circ - 38.69^\circ \approx 141.31^\circ$
 Case 1: $C \approx 38.69^\circ$
 $B = 180^\circ - 38.69^\circ - 23^\circ \approx 118.31^\circ$
 $\frac{\sin 23^\circ}{7.5} = \frac{\sin 118.31^\circ}{b}$; $b \approx 16.90$

Case 2: $C \approx 141.31^\circ$
 $B = 180^\circ - 141.31^\circ - 23^\circ \approx 15.69^\circ$
 $\frac{\sin 23^\circ}{7.5} = \frac{\sin 15.69^\circ}{b}$; $b \approx 5.19$

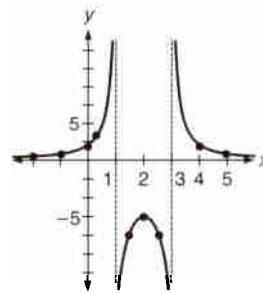
Thus, $b \approx 16.9$, $C \approx 38.7^\circ$, $B \approx 118.3^\circ$, or $b \approx 5.2$, $C \approx 141.3^\circ$, $B \approx 15.7^\circ$.

3. $c = 12.0$, $a = 7.5$, and $B = 23^\circ$. We are not told this is a right triangle. We do not know any angle and the length of the side opposite, so we cannot use the law of sines. Thus, we use the law of cosines.
 $b^2 = a^2 + c^2 - 2ac \cos B$
 $b^2 = 7.5^2 + 12^2 - 2(7.5)(12) \cos 23^\circ$
 $b \approx 5.879$
 $\frac{\sin A}{7.5} = \frac{\sin 23^\circ}{5.879} = \frac{\sin C}{12}$
 Angle C may be obtuse, since it is the largest angle in the triangle (because it is opposite the largest side). Thus, it is better to find angle A next, since it must be acute. $\sin A = \frac{\sin 23^\circ}{5.879}(7.5)$, so $A \approx 29.90^\circ$. Thus, $C \approx 180^\circ - 23^\circ - 29.9^\circ \approx 127.1^\circ$. Therefore, $b \approx 5.9$, $A \approx 29.9^\circ$, and $C \approx 127.1^\circ$.

4. $f(x) = \frac{5}{x^2 - 4x + 3} = \frac{5}{(x-3)(x-1)}$
 Vertical asymptotes at 1 and 3.
 No x -intercepts, since $0 = \frac{5}{x^2 - 4x + 3}$ has no solution.
 y -intercept is $f(0) = \frac{5}{3} = 1\frac{2}{3}$.

Additional points:

x	-2	-1	0.5	1.5	2	2.5	4	5
y	0.3	0.6	4	-6.7	-5	-6.7	1.7	0.6



Solutions to trial exercise problems

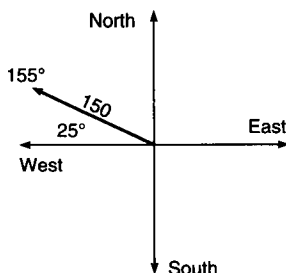
3. $(100.0, 122.3^\circ) = (100 \cos 122.3^\circ, 100 \sin 122.3^\circ) \approx (-53.4, 84.5)$
 23. $(-\sqrt{2}, -\sqrt{8})$; $|A| = \sqrt{(-\sqrt{2})^2 + (-\sqrt{8})^2} \approx \sqrt{10} \approx 3.16$, $\theta' \approx 63.43^\circ$; $\theta \approx 63.43^\circ - 180^\circ \approx -116.57^\circ$; $(3.16, -116.6^\circ)$

39. $(15.3, 311^\circ)$
 $= (15.3 \cos 311^\circ, 15.3 \sin 311^\circ)$
 $\approx (10.038, -11.547)$ [1]
 $(20.9, 117^\circ)$
 $= (20.9 \cos 117^\circ, 20.9 \sin 117^\circ)$
 $\approx (-9.488, 18.622)$ [2]

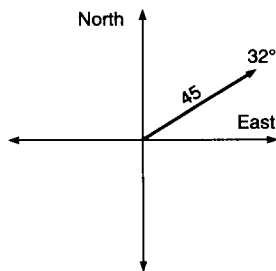
$(13.2, 83^\circ)$
 $= (13.2 \cos 83^\circ, 13.2 \sin 83^\circ)$
 $\approx (1.609, 13.102)$ [3]

Adding [1] + [2] + [3] gives
 $(2.158, 20.177) \approx (20.3, 83.9^\circ)$

43. $V = (150, 155^\circ)$;
 $V_x = 150 \cos 155^\circ \approx -136$ (136 knots due west)
 $V_y = 150 \sin 155^\circ \approx 63$ (63 knots due north)
 The aircraft is moving west at 136 knots and north at 63 knots.

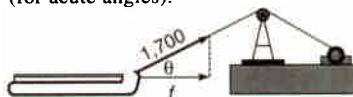


49. 18 knots (18 nautical miles per hour)
 $\times 2.5$ hours = 45 nm (nautical miles).
 $V = (45, 32^\circ)$;
 $V_x = 45 \cos 32^\circ \approx 38$ nm; distance east of the harbor (part b)
 $V_y = 45 \sin 32^\circ \approx 24$ nm; distance north of the harbor (part a).

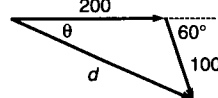


52. $f = 1,700 \cos \theta$. We require $f \geq 1,200$, so $1,200 \geq 1,700 \cos \theta$, or $\frac{12}{17} \geq \cos \theta$. $\cos^{-1} \frac{12}{17} \approx 45.1^\circ$. Thus, $\theta \leq 45.1^\circ$ will move the sled. Note that $\theta \leq 45.1^\circ$ is correct, and not $\theta \geq 45.1^\circ$. This can be seen in the figure. If θ increases, f clearly decreases. Mathematically, the

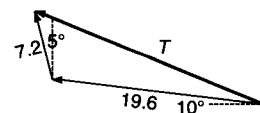
value of $\cos \theta$ increases as θ decreases (for acute angles).



54. At 200 mph, the first leg of the trip is 200 miles. The second is 200 mph $\times 0.5$ hour = 100 miles. To find d and θ , we add the vectors $(200, 0^\circ)$ and $(100, -60^\circ)$.
 $(200, 0^\circ) = (200 \cos 0^\circ, 200 \sin 0^\circ) = (200, 0)$
 $(100, -60^\circ) = (100 \cos(-60^\circ), 100 \sin(-60^\circ)) \approx (50, -86.60)$
 $(250, -86.60) \approx (265, 341^\circ)$
 Thus, $d \approx 265$ miles, and $\theta \approx 341^\circ$. The aircraft is 265 miles from Minneapolis. $360^\circ - 341^\circ = 19^\circ$, so the aircraft is in a direction 19° south of east, relative to the city.



59. $(199, 19.0^\circ) = (199 \cos 19^\circ, 199 \sin 19^\circ) \approx (188.16, 64.79)$
 $(175, 131^\circ) = (175 \cos 131^\circ, 175 \sin 131^\circ) \approx (-114.81, 132.07)$
 $(96, 130^\circ) = (96 \cos 130^\circ, 96 \sin 130^\circ) \approx (-61.71, 73.54)$
 Adding the rectangular form gives
 $(11.64, 270.40) \approx (270.7, 87.5^\circ)$
 64. $(19.6, 170^\circ) = (19.6 \cos 170^\circ, 19.6 \sin 170^\circ) \approx (-19.3, 3.4)$
 $(7.2, 95^\circ) = (7.2 \cos 95^\circ, 7.2 \sin 95^\circ) \approx (0.63, 7.17)$
 $(-19.93, 10.58) \approx (22.6, 152.0^\circ)$
 Thus, its true course is $180^\circ - 152^\circ = 28^\circ$ north of west, at a speed of 22.6 knots.

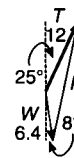


66. $(122, 30^\circ) = (122 \cos 30^\circ, 122 \sin 30^\circ) \approx (105.66, 61.00)$
 $(86, 21^\circ) = (86 \cos 21^\circ, 86 \sin 21^\circ) \approx (80.29, 30.82)$
 Adding the rectangular forms gives
 $(185.94, 91.82) \approx (207, 26^\circ)$
 Magnitude is 207 volts, phase angle is 26° .

71. We let W represent the water current vector.

$H + W = T$
 $H = T - W$
 $= (12, 65^\circ) - (6.4, -82^\circ)$
 $= (12, 65^\circ) + (6.4, -82^\circ + 180^\circ)$
 $= (12, 65^\circ) + (6.4, 98^\circ)$
 $= (5.07, 10.88) + (-0.89, 6.34)$
 $= (4.18, 17.21) \approx (17.7, 76.3^\circ)$

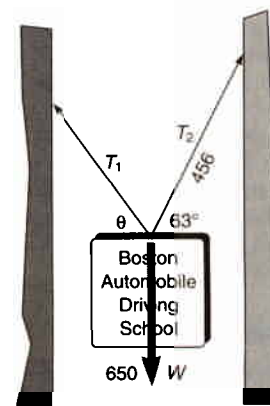
Thus the ship's heading is 76.3° , and its speed is 17.7 knots.



73. The sign is stationary, so the forces acting on it are balanced (they add to zero).

$T_1 + T_2 + W = 0$
 $T_1 = -T_2 - W$
 $= -(456, 63^\circ) - (650, 270^\circ)$
 $= (456, 63^\circ + 180^\circ) + (650, 270^\circ - 180^\circ)$
 $= (-207.02, -406.3) + (0, 650)$
 Convert to rectangular form
 $\approx (-207.02, 243.7)$
 $\approx (320, 130^\circ)$

Convert back to polar form
 Thus, the tension in the second cable is 320 pounds, and it makes an angle θ of 50° ($180^\circ - 130^\circ$) with the horizontal.



74. Let $A = (a_1, a_2)$ and $B = (b_1, b_2)$ be two vectors in rectangular form. Then proceed as follows:

$A + B$

$(a_1, a_2) + (b_1, b_2)$

$(a_1 + b_1, a_2 + b_2)$

Definition of vector addition

$(b_1 + a_1, b_2 + a_2)$

a_1, a_2, b_1, b_2 are real numbers, so their indicated sum commutes

$B + A$

Definition of vector addition

76. The following is for the TI-81:

PRGM EDIT 2

Choose a free location to enter the program.

Say 2 by way of example.

ADDVCTRS Enter these characters.

as the name of the program

:O→A

:O→B

:Lbl 1

:Input R

:If R=0

:Goto 2

:Input θ

:P→R(R,θ)

:A+X→A

:B+Y→B

:Goto 1

:Lbl 2

:A→X

:B→Y

:R→P(X,Y)

:Disp "X,Y"

:Disp X

:Disp Y

:Disp "R,θ"

:Disp R

:Disp θ

To run the program input R , then θ , for each vector in polar form. When all vectors have been entered, enter zero (0) for R . The program converts each vector into rectangular form as it is entered, and accumulates the values (x, y) in variables A and B . When all vectors are entered, the accumulated values in A and B are converted to polar form.

Exercise 8-4

Answers to odd-numbered problems

1. $5.4 \text{ cis } (-21.8^\circ)$ 3. $3.2 \text{ cis } 108.4^\circ$
 5. $5 \text{ cis } 126.9^\circ$ 7. $2 \text{ cis } 30^\circ$
 9. $3\sqrt{2} \text{ cis } 45^\circ$ 11. $\sqrt{2} \text{ cis } (-135^\circ)$
 13. $5 \text{ cis } 90^\circ$ 15. $2.9 + 0.8i$ 17. $3.7 + 2.6i$ 19. $1 - i$ 21. $12.3 - 5.7i$

23. $\frac{3}{2} + \frac{\sqrt{3}}{2}i$ 25. $5 - 5\sqrt{3}i$

27. $-\sqrt{10}$ 29. $2 - 2i$ 31. $15 \text{ cis } 75^\circ$

33. $10.8 \text{ cis } (-120^\circ)$ 35. $4 \text{ cis } 80^\circ$

37. $\frac{20}{7} \text{ cis } (-80^\circ)$ 39. $512 \text{ cis } (-60^\circ)$

41. $27 \text{ cis } (-120^\circ)$ 43. $-8.2 - 0.1i$

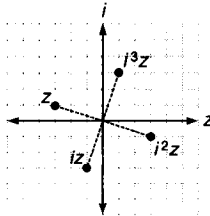
45. $2, -1 + \sqrt{3}i, -1 - \sqrt{3}i$ 47. $3, 3i, -3, -3i$

49. $-1.1 + 4.9i, -3.7 - 3.4i, 4.8 - 1.5i$ 51. $2.5 \text{ cis } (-20^\circ)$

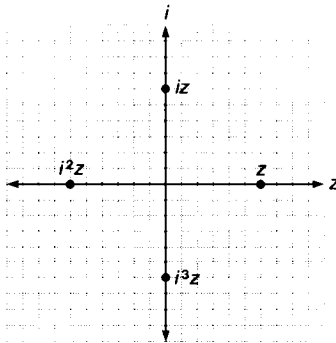
53. $50 \text{ cis } 45^\circ$ 55. $2.04 \text{ cis } 6.19^\circ$

57. $0.75 + 0.86i$ 59. no

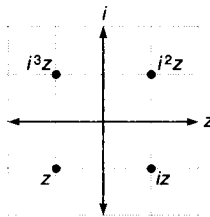
61. $-3 + i, -1 - 3i, 3 - i, 1 + 3i$



63. $6, 6i, -6, -6i$



65. $-1 - i, 1 - i, 1 + i, -1 + i$



67. a. $1 \text{ cis } 30^\circ$ or $\frac{\sqrt{3}}{2} + \frac{1}{2}i$

b. $0.37 + 1.37i$

c. $0.37 - 1.37i$

69. $r^n \text{ cis } \left(\frac{\theta}{n} + \frac{k \cdot 360^\circ}{n} \right)$

Replace k by $an + b$, $b < n$.

$$r^n \text{ cis } \left[\frac{\theta}{n} + \frac{(an + b) \cdot 360^\circ}{n} \right]$$

$$r^n \text{ cis } \left(\frac{\theta}{n} + \frac{an \cdot 360^\circ}{n} + \frac{b \cdot 360^\circ}{n} \right)$$

$$r^n \text{ cis } \left(\frac{\theta}{n} + a \cdot 360^\circ + \frac{b \cdot 360^\circ}{n} \right)$$

$$r^n \text{ cis } \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$r^n \cos \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$+ ir^n \sin \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$\cos \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$= \cos \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) \cos(a \cdot 360^\circ)$$

$$- \sin \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) \sin(a \cdot 360^\circ)$$

$$= \cos \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right), \text{ because}$$

$$\cos(a \cdot 360^\circ) = 1 \text{ and } \sin(a \cdot 360^\circ)$$

$$= 0, \text{ when } a \text{ is an integer.}$$

$$\sin \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$= \sin \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) \cos(a \cdot 360^\circ)$$

$$+ \cos \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) \sin(a \cdot 360^\circ)$$

$$= \sin \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right),$$

$$\text{because } \cos(a \cdot 360^\circ) = 1 \text{ and}$$

$$\sin(a \cdot 360^\circ) = 0, \text{ when } a \text{ is an integer.}$$

Thus,

$$r^n \cos \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$+ ir^n \sin \left[\left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right) + a \cdot 360^\circ \right]$$

$$= r^n \cos \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right)$$

$$+ ir^n \sin \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right)$$

$$= r^n \text{ cis } \left(\frac{\theta}{n} + \frac{b \cdot 360^\circ}{n} \right), \text{ where } b < n.$$

This last expression is one of the previous roots.

Solutions to skill and review problems

1. $(3.8, 28^\circ) = (3.8 \cos 28^\circ, 3.8 \sin 28^\circ)$
 $\approx (3.36, 1.78)$
 $(5.1, 134^\circ) = (5.1 \cos 134^\circ, 5.1 \sin 134^\circ)$
 $\approx (-3.54, 3.67)$; adding the rectangular forms gives $(-0.19, 5.45)$
 $r = \sqrt{0.19^2 + 5.45^2} \approx 5.5$;

$$\theta' = \tan^{-1} \frac{5.45}{-0.19} \approx -88.00^\circ$$

The x-component, -0.19 , is negative, and $\theta' < 0$, so $\theta = \theta' + 180^\circ \approx -88^\circ + 180^\circ = 92^\circ$. Thus the resultant vector is $(5.5, 92^\circ)$.

2. $B = C - A = C + (-A)$; $-A$
 $= (190, 30^\circ + 180^\circ) = (190, 210^\circ)$
 $C: (150, 68^\circ) = (150 \cos 68^\circ, 150 \sin 68^\circ)$
 $\approx (56.19, 139.08)$
 $-A: (190, 210^\circ) = (190 \cos 210^\circ, 190 \sin 210^\circ)$
 $\approx (-164.54, -95)$; adding gives $(-108.35, 44.08) \approx (117, 158^\circ)$

3. $a = 12.6$, $b = 19.1$, and $c = 28.0$; this may not be a right triangle, so we must use the law of sines or the law of cosines. We cannot use the law of sines since we do not have any side and the angle opposite that side. Therefore we use the law of cosines to find one of the angles.

The law of cosines does not have an ambiguous case, so we use it to find the largest angle, which may be acute or obtuse. The largest angle is opposite the longest side, which is c in this case.

$$c^2 = a^2 + b^2 - 2ab \cos C, \text{ so } \cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{12.6^2 + 19.1^2 - 28^2}{2(12.6)(19.1)}$$

$$\approx -0.5411, \text{ so } C \approx 122.757^\circ$$

$$\frac{\sin A}{12.6} = \frac{\sin B}{19.1} = \frac{\sin 122.757^\circ}{28}; \text{ A and B}$$

are both acute, so we can find either next.

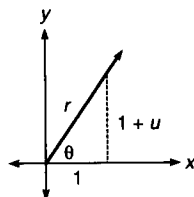
$$\sin A = \frac{\sin 122.757^\circ}{28} (12.6) \approx 0.3784, \text{ so}$$

$$A \approx 22.2^\circ, B \approx 180^\circ - 22.2^\circ - 122.8^\circ \approx 35.0^\circ. \text{ Thus } A \approx 22.2^\circ, B \approx 35.0^\circ, C \approx 122.8^\circ.$$

$$\begin{aligned} 4. \frac{1}{\sqrt[3]{4a^4b}} &= \frac{1}{\sqrt[3]{2^2 a^4 b}} = \frac{1}{a \sqrt[3]{2^3 ab}} \\ &= \frac{1}{a \sqrt[3]{2^3 ab}} \cdot \frac{\sqrt[3]{2a^2b^2}}{\sqrt[3]{2a^2b^2}} = \frac{\sqrt[3]{2a^2b^2}}{a \sqrt[3]{2^3 a^3 b^3}} \\ &= \frac{\sqrt[3]{2a^2b^2}}{a(2ab)} = \frac{\sqrt[3]{2a^2b^2}}{2a^2b} \end{aligned}$$

5. $\tan \theta = 1 + u$ and θ terminates in quadrant I
 $r^2 = (1 + u)^2 + 1^2 = u^2 + 2u + 2$, so
 $r = \sqrt{u^2 + 2u + 2}$

$$\sin \theta = \frac{1 + u}{r} = \frac{1 + u}{\sqrt{u^2 + 2u + 2}}$$



Solutions to trial exercise problems

$$4. \sqrt{3} - 2i$$

$$r = \sqrt{(\sqrt{3})^2 + (-2)^2} = \sqrt{7} \approx 2.6$$

$$\theta' = \tan^{-1} \frac{-2}{\sqrt{3}} \approx -49.1^\circ;$$

$$a > 0 \text{ so } \theta = \theta'$$

The point is $2.6 \text{ cis } (-49.1^\circ)$.

$$7. \sqrt{3} + i$$

$$r = \sqrt{(\sqrt{3})^2 + 1^2} = 2$$

$$\theta' = \tan^{-1} \frac{1}{\sqrt{3}} = \tan^{-1} \frac{\sqrt{3}}{3} = 30^\circ$$

The point is in quadrant I, so it is $2 \text{ cis } 30^\circ$.

$$\begin{aligned} 68. \frac{r_1 \text{cis } \theta_1}{r_2 \text{cis } \theta_2} &= \frac{r_1}{r_2} \cdot \frac{\text{cis } \theta_1}{\text{cis } \theta_2} = \frac{r_1}{r_2} \cdot \frac{\cos \theta_1 + i \sin \theta_1}{\cos \theta_2 + i \sin \theta_2} \\ &= \frac{r_1}{r_2} \cdot \frac{\cos \theta_1 + i \sin \theta_1}{\cos \theta_2 + i \sin \theta_2} \cdot \frac{\cos \theta_2 - i \sin \theta_2}{\cos \theta_2 - i \sin \theta_2} = \frac{r_1}{r_2} \cdot \frac{\cos \theta_1 \cos \theta_2 - i \cos \theta_1 \sin \theta_2 + i \sin \theta_1 \cos \theta_2 - i^2 \sin \theta_1 \sin \theta_2}{\cos^2 \theta_2 - i^2 \sin^2 \theta_2} \\ &= \frac{r_1}{r_2} \cdot \frac{\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 + i(\sin \theta_1 \cos \theta_2 - \cos \theta_1 \sin \theta_2)}{\cos^2 \theta_2 + \sin^2 \theta_2} \\ &= \frac{r_1}{r_2} \cdot \frac{\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)}{1} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2) \end{aligned}$$

$$15. 3 \text{ cis } 15^\circ$$

$$3 \cos 15^\circ + (3 \sin 15^\circ)i = 2.9 + 0.8i$$

$$26. 6 \text{ cis } 135^\circ$$

$$6 \cos 135^\circ + (6 \sin 135^\circ)i$$

$$= 6 \cdot \left(-\frac{\sqrt{2}}{2}\right) + 6 \cdot \frac{\sqrt{2}}{2}i =$$

$$-3\sqrt{2} + 3\sqrt{2}i$$

$$41. (3 \text{ cis } 200^\circ)^3 = 3^3 \text{ cis}(3 \cdot 200^\circ) = 27 \text{ cis } 600^\circ = 27 \text{ cis}(600^\circ - 2 \cdot 360^\circ) = 27 \text{ cis}(-120^\circ)$$

$$44. (0.8 + 0.6i)^{10}$$

Transform into polar form to use De Moivre's theorem.

$$r = \sqrt{0.8^2 + 0.6^2} = 1$$

$$\theta = \theta' = \tan^{-1} \frac{0.6}{0.8} \approx 36.87^\circ$$

$$(0.8 + 0.6i)^{10} \approx (1 \text{ cis } 36.87^\circ)^{10} \approx 1^{10} \text{ cis } 368.7^\circ \approx \text{cis } 8.7^\circ \approx 1.0 + 0.2i$$

$$55. Z_1 = 2 + i \approx \sqrt{5} \text{ cis } 26.565^\circ$$

$$Z_2 = 3 - 5i \approx \sqrt{34} \text{ cis } 300.964^\circ$$

$$Z_1 + Z_2 = 5 - 4i \approx \sqrt{41} \text{ cis } 321.340^\circ$$

$$\frac{Z_1 Z_2}{Z_1 + Z_2} \approx$$

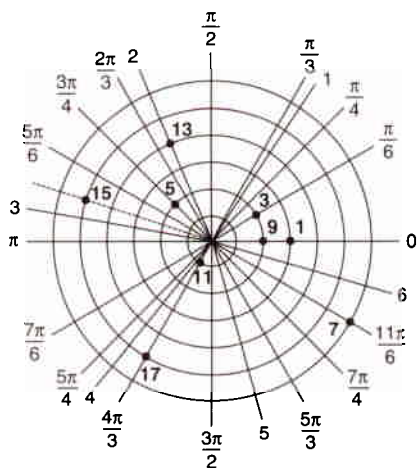
$$\frac{(\sqrt{5} \text{ cis } 26.565^\circ)(\sqrt{34} \text{ cis } 300.964^\circ)}{\sqrt{41} \text{ cis } 321.340^\circ}$$

$$= \frac{\sqrt{170} \text{ cis } 327.529^\circ}{\sqrt{41} \text{ cis } 321.340^\circ} \approx 2.04 \text{ cis } 6.19^\circ$$

Exercise 8-5

Answers to odd-numbered problems

The figure shows the answers to odd-numbered problems 1 through 17.



Many answers are possible in problems 19–23.

19. $\left(-2, \frac{7\pi}{6}\right), \left(2, \frac{13\pi}{6}\right), \left(2, \frac{25\pi}{6}\right)$

21. $\left(-6, \frac{5\pi}{6}\right), \left(6, \frac{23\pi}{6}\right), \left(6, \frac{35\pi}{6}\right)$

23. $(-2, 2 + \pi), (2, 2 + 2\pi), (2, 2 + 4\pi)$

25. $(0, 4)$ 27. $\left(-\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)$

29. $(-2, -2\sqrt{3})$ 31. $(1.08, 1.68)$

33. $(2.05, 2.19)$ 35. $(-2.61, -3.03)$

37. $\left(4, -\frac{5\pi}{6}\right)$ 39. $(2, \pi)$

41. $\left(4\sqrt{2}, -\frac{3\pi}{4}\right)$ 43. $(3.61, 0.98)$

45. $(4.12, -1.33)$ 47. $(6.40, 0.67)$

49. $\tan \theta = 4$ 51. $r = \frac{2}{\sin \theta + 3 \cos \theta}$

53. $r = \frac{b}{\sin \theta - m \cos \theta}$

55. $r^2 = \frac{5}{\sin^2 \theta - 2 \cos^2 \theta}$

57. $r^2 = \frac{1}{3 \cos^2 \theta + 2 \sin^2 \theta}$

59. $x^2 + y^2 - y = 0$ 61. $x = 2$

63. $x^6 + 3x^4y^2 - 36x^2y^4 + y^6 = 0$

65. $x^4 + 2x^2y^2 - 2xy + y^4 = 0$

67. $x^3 + xy^2 - y = 0$

69. $3y^2 + 12y - x^2 + 9 = 0$

71. $2xy = 5$

$2(r \cos \theta)(r \sin \theta) = 5$

$2r^2 \cos \theta \sin \theta = 5$

$r^2(2 \sin \theta \cos \theta) = 5$

$\sin 2\alpha = 2 \sin \alpha \cos \alpha$

$r^2 \sin 2\theta = 5$

$r^2 = \frac{5}{\sin 2\theta}$

$r^2 = 5 \csc 2\theta$

73. Consider a point $P = (r, \theta)$, where $r < 0$. Then $P = (-r, \theta + \pi)$, where $-r > 0$. Therefore, since $-r > 0$, $y = -r \sin(\theta + \pi)$ is true.

$y = -r \sin(\theta + \pi)$

$= -r(\sin \theta \cos \pi + \cos \theta \sin \pi)$

$= -r(\sin \theta(-1) + \cos \theta(0))$

$= -r(-\sin \theta)$

$= r \sin \theta$

Thus, $y = r \sin \theta$, even if $r < 0$.

75. $(x^2 + y^2 + 2y)^2 = x^2 + y^2$

77. $(x^2 + y^2)^3 = (x^2 + y^2 + 4xy)^2$

79. $(x^2 + y^2 + 3x)^2 = x^2 + y^2$

Solutions to skill and review problems

1. $2 \operatorname{cis} 30^\circ \cdot 5 \operatorname{cis} 45^\circ = 2(5)$

$\operatorname{cis}(30^\circ + 45^\circ) = 10 \operatorname{cis} 75^\circ$

2. $1,000 = 1,000 \operatorname{cis} 0^\circ$

Evaluate: $1,000^{1/3} \operatorname{cis}\left(\frac{0^\circ}{3} + \frac{k \cdot 360^\circ}{3}\right)$

for $k = 0, 1, 2$. $1,000^{1/3} = 10$.

$10 \operatorname{cis}(k \cdot 120^\circ)$ for $k = 0, 1, 2$.

$k = 0$: $10(\cos 0^\circ + i \sin 0^\circ) = 10$

$k = 1$: $10(\cos 120^\circ + i \sin 120^\circ) =$

$10\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) = -5 + 5\sqrt{3}i$

$k = 2$: $10(\cos 240^\circ + i \sin 240^\circ) =$

$10\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -5 - 5\sqrt{3}i$

3. $a = 125$, $b = 85$, and $C = 50^\circ$

$c^2 = a^2 + b^2 - 2ab \cos C$

$c^2 = 125^2 + 85^2 - 2(125)(85) \cos 50^\circ$

$c^2 \approx 9190.76$; $c \approx 95.868$

$\frac{\sin A}{125} = \frac{\sin B}{85} \approx \frac{\sin 50^\circ}{95.868}$

Find angle B next; it must be acute.

$\sin B \approx \frac{\sin 50^\circ}{95.868}(85)$; $B \approx 42.8^\circ$

$A \approx 180^\circ - 42.8^\circ - 50^\circ \approx 87.2^\circ$

Thus, $c \approx 96$, $A \approx 87^\circ$, $B \approx 43^\circ$.

4. $f(x) = 2x^2 + 4x - 5$

$= 2(x^2 + 2x) - 5$

$= 2(x^2 + 2x + 1) - 5 - 2(1)$

$= 2(x + 1)^2 - 7$

Vertex: $(-1, -7)$

Intercepts: $f(0) = -5$; $(0, -5)$ is the

y -intercept

$0 = 2(x + 1)^2 - 7$

$2(x + 1)^2 = 7$

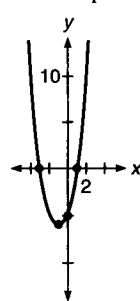
$(x + 1)^2 = \frac{7}{2}$

$x + 1 = \pm \sqrt{\frac{7}{2}} = \pm \frac{\sqrt{14}}{2}$

$x = -1 \pm \frac{\sqrt{14}}{2} \approx -2.87, 0.87$

$(-2.9, 0)$ and $(0.9, 0)$ are the

x -intercepts.



5. $\frac{2x - 3}{x^2 - 16} - \frac{5}{x - 4}$

$\frac{2x - 3}{(x - 4)(x + 4)} - \frac{5(x + 4)}{(x - 4)(x + 4)}$

$\frac{(2x - 3) - 5(x + 4)}{(x - 4)(x + 4)}$

$\frac{-3x - 23}{x^2 - 16}$

6. $\frac{2 + 3i}{5 - 2i} \cdot \frac{5 + 2i}{5 + 2i} = \frac{10 + 4i + 15i + 6i^2}{25 - 4i^2}$

$= \frac{10 + 19i - 6}{25 + 4} = \frac{4 + 19i}{29} = \frac{4}{29} + \frac{19}{29}i$

7. $\left(\frac{x^2x^{-5}}{x^3}\right)^{-2} = \left(\frac{x^{-3}}{x^3}\right)^{-2} = \frac{x^6}{x^{-6}} = x^{12}$

8. $3x^{-2}\left(\frac{1}{3}x^2 - 2x + 1\right)$

$3\left(\frac{1}{3}\right)x^{-2}x^2 - 3(2)x^{-2}x + 3x^{-2}$

$x^0 - 6x^{-1} + 3x^{-2}$

$1 - \frac{6}{x} + \frac{3}{x^2}$

$\frac{x^2}{x^2} - \frac{6x}{x^2} + \frac{3}{x^2}$

$\frac{x^2 - 6x + 3}{x^2}$

Solutions to trial exercise problems

15. $(-5, 6) = (5, 6 - \pi) \approx (5, 2.86)$; $6 - \pi$
 $= (6 - \pi) \cdot \frac{180^\circ}{\pi} \approx 164^\circ$. To plot
 $(-5, 6)$, plot a point 5 units from the
 center, at an angle about 164° . The
 graph is shown in the answer to the
 odd problems for this section.

21. Many answers are possible. To change
 the sign of r add an odd multiple of π
 to θ ; for the rest add an even multiple
 of π .

$$\left(-6, \frac{11\pi}{6} - \pi\right) = \left(-6, \frac{5\pi}{6}\right),$$

$$\left(6, \frac{11\pi}{6} + 2\pi\right) = \left(6, \frac{23\pi}{6}\right),$$

$$\left(6, \frac{11\pi}{6} + 4\pi\right) = \left(6, \frac{35\pi}{6}\right)$$

31. $(2, 1) = (2 \cos 1, 2 \sin 1) \approx (1.08, 1.68)$
 39. $(-2, 0)$; this point is on the x -axis. It is
 easiest to solve by examination; $r = 2$,
 and $\theta = \pi$. Thus, the point is $(2, \pi)$.

45. $(1, -4)$
 $r = \sqrt{1^2 + 4^2} = \sqrt{17} \approx 4.12$
 $\theta' = \tan^{-1}(-4) \approx -1.326$
 $x > 0$ so $\theta = \theta'$; $(4.12, -1.33)$

53. $y = mx + b$, $b \neq 0$
 $r \sin \theta = mr \cos \theta + b$
 $r \sin \theta - mr \cos \theta = b$
 $r(\sin \theta - m \cos \theta) = b$

$$r = \frac{b}{\sin \theta - m \cos \theta}$$

55. $y^2 - 2x^2 = 5$
 $(r \sin \theta)^2 - 2(r \cos \theta)^2 = 5$
 $r^2 \sin^2 \theta - 2r^2 \cos^2 \theta = 5$
 $r^2(\sin^2 \theta - 2 \cos^2 \theta) = 5$
 $r^2 = \frac{5}{\sin^2 \theta - 2 \cos^2 \theta}$

63. $r = 3 \sin 2\theta$
 $r = 3(2 \sin \theta \cos \theta)$
 $r = 6 \frac{y}{r} \cdot \frac{x}{r}$
 $r^3 = 6xy$

Square both members so that we can
 express the left side in terms of r^2 .

$$\begin{aligned} r^6 &= 36x^2y^2 \\ (r^2)^3 &= 36x^2y^2 \\ (x^2 + y^2)^3 &= 36x^2y^2 \\ x^6 + 3x^4y^2 + 3x^2y^4 + y^6 &= 36x^2y^2 \\ x^6 + 3x^4y^2 - 36x^2y^2 + 3x^2y^4 + y^6 &= 0 \end{aligned}$$

$$\begin{aligned} 69. \quad r &= \frac{3}{1 - 2 \sin \theta} \\ r(1 - 2 \sin \theta) &= 3 \\ r\left(1 - \frac{2y}{r}\right) &= 3 \end{aligned}$$

$$\begin{aligned} r - 2y &= 3 \\ r &= 2y + 3 \\ r^2 &= (2y + 3)^2 \\ x^2 + y^2 &= 4y^2 + 12y + 9 \\ 0 &= 3y^2 + 12y - x^2 + 9 \end{aligned}$$

78. Assume the path taken by the
 Scrambler is described by the polar
 equation $r = 2 \cos 3\theta$. Convert this
 equation into rectangular form. It will
 be necessary to rewrite $\cos 3\theta$ in terms
 of $\cos \theta$. Problem 82 in section 7-3
 shows that $\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$.

$$\begin{aligned} r &= 2 \cos 3\theta \\ r &= 2(4 \cos^3 \theta - 3 \cos \theta) \\ r &= 8 \cos^3 \theta - 6 \cos \theta \\ r &= 8\left(\frac{x}{r}\right)^3 - 6\frac{x}{r} \end{aligned}$$

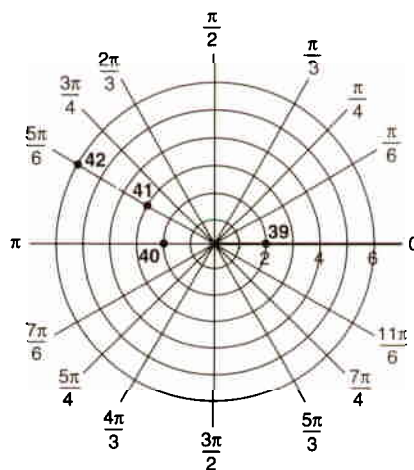
$$\begin{aligned} r &= \frac{8x^3}{r^3} - \frac{6x}{r} \\ \text{Multiply each member by } r^3. \end{aligned}$$

$$\begin{aligned} r^4 &= 8x^3 - 6xr^2 \\ (x^2 + y^2)^2 &= 8x^3 - 6x(x^2 + y^2) \\ x^4 + 2x^2y^2 + y^4 &= 8x^3 - 6x^3 - 6xy^2 \\ x^4 - 2x^3 + 2x^2y^2 + 6xy^2 + y^4 &= 0 \end{aligned}$$

Chapter 8 review

- $C = 121.8^\circ$, $b \approx 2.6$, $c \approx 12.1$
- $A = 151.0^\circ$, $a \approx 4.3$, $c \approx 0.8$
- $A \approx 49.0^\circ$, $C \approx 52.0^\circ$, $c \approx 10.4$
- Case 1: $b \approx 12.2$, $A \approx 76.5^\circ$,
 $B \approx 70.8^\circ$;
 Case 2: $b \approx 9.0$, $A \approx 103.5^\circ$,
 $B \approx 43.8^\circ$
- 48 miles
- $c \approx 3.8$, $A \approx 31.9^\circ$, $B \approx 118.7^\circ$
- $a \approx 63.9$, $B \approx 69.7^\circ$, $C \approx 18.2^\circ$
- $b \approx 40.2$, $A \approx 29.5^\circ$, $C \approx 38.5^\circ$
- $A \approx 106.3^\circ$, $B \approx 23.1^\circ$, $C \approx 50.5^\circ$
- $C \approx 135.5^\circ$, $A \approx 12.2^\circ$, $B \approx 32.3^\circ$
- $a = \sqrt{26}$, $b = \sqrt{58}$, $c = \sqrt{40}$,
 $B \approx 82.9^\circ$, $A \approx 41.6^\circ$, $C \approx 55.5^\circ$
- 29 km
- $V_x \approx 23.8$, $V_y \approx 13.2$
- $|V| \approx 36.3$, $\theta_v \approx 57.3^\circ$

- horizontal: 370 knots; vertical: 256
 knots
- horizontal: 249 pounds; vertical: 60
 pounds
- $(47.6, 20.6^\circ)$ 18. $(10.8, 89.5^\circ)$
- $(7.2, 69.9^\circ)$ 20. $(8.1, -172.4^\circ)$
- magnitude ≈ 205 pounds; direction
 $\approx -87^\circ$
- $\sqrt{13} \operatorname{cis}(-33.7^\circ)$ or $3.6 \operatorname{cis}(-33.7^\circ)$
- $2\sqrt{3} \operatorname{cis} 60^\circ$ 24. $2.2 \operatorname{cis}(-116.6^\circ)$
- $2.5 + 1.7i$ 26. $-2.3 - 4.5i$
- $-1.5 - 1.5\sqrt{3}i$ 28. $5\sqrt{3} - 5i$
- $6 \operatorname{cis} 70^\circ$ 30. $13 \operatorname{cis} 140^\circ$
- $8 \operatorname{cis} 100^\circ$ 32. $0.5 \operatorname{cis} 36^\circ$
- $8 \operatorname{cis} 30^\circ$ 34. $16 \operatorname{cis} 240^\circ$
- $0.42 - 0.91i$ 36. $2.2i, -2, -2i$
- $1.93 - 0.46i, -0.57 + 1.90i$,
 $-1.36 - 1.44i$
- $4\frac{1}{3} \operatorname{cis}(-50^\circ)$ 39. see figure
- see figure 41. see figure
- $\left(-6, \frac{11\pi}{6}\right)$
 $= \left(6, \frac{11\pi}{6} - \pi\right) = \left(6, \frac{5\pi}{6}\right)$; see figure



- $(\frac{3}{2}\sqrt{3}, -\frac{3}{2})$ 44. $(-2.2\sqrt{3})$
- $(-1, \sqrt{3})$ 46. $(1.6, 1.2)$
- $(-2.1, 4.5)$ 48. $(0.7, -0.8)$
- $(2.24, 0.46)$ 50. $(5.83, 2.60)$
- $(4.12, -1.82)$ 52. $\tan \theta = -3$
- $r = \frac{2}{\sin \theta - 4 \cos \theta}$

$$54. r^2 = \frac{5}{2 \sin^2 \theta - \cos^2 \theta}$$

$$55. r = \frac{3 \cos \theta}{\sin^2 \theta}; r = 3 \cot \theta \csc \theta$$

(alternate form of answer)

$$56. r = 3 \quad 57. r = \frac{2}{\cos \theta}; r = 2 \sec \theta$$

(alternate form of answer)

$$58. x^2 + y^2 - y = 0 \quad 59. x = 2$$

$$60. (x^2 + y^2)^2 - 2xy = 0$$

$$61. x^3 + xy^2 - y = 0 \quad 62. y = 2$$

$$63. 4x^2 + 3y^2 - 6y - 9 = 0$$

Chapter 8 test

$$1. B = 84.4^\circ, a \approx 5.3, c \approx 22.5$$

$$2. B \approx 56.3^\circ, A \approx 61.6^\circ, a \approx 23.9$$

$$3. b \approx 32.8, A \approx 50.9^\circ, C \approx 29.1^\circ$$

$$4. C \approx 89.1^\circ, A \approx 39.6^\circ, B \approx 51.3^\circ$$

$$5. 59 \text{ yards} \quad 6. B \approx 53.1^\circ$$

$$7. (\sqrt{3}, 1)$$

$$8. \text{magnitude: } 6.4; \text{direction: } 51.3^\circ$$

$$9. (8.5, 84.9^\circ)$$

$$10. \text{tension: } 584 \text{ pounds; angle above the horizontal: } 65^\circ$$

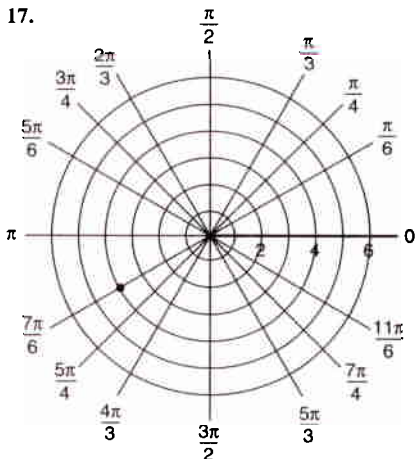
$$11. 6.40 \operatorname{cis}(-51.3) \quad 12. -1 + \sqrt{3}i$$

$$13. 14 \operatorname{cis} 110^\circ \quad 14. 3 \operatorname{cis} 120^\circ$$

$$15. 27 \operatorname{cis} 90^\circ$$

$$16. \sqrt{2} + \sqrt{2}i - \sqrt{2} + \sqrt{2}i, \\ -\sqrt{2} - \sqrt{2}i, \sqrt{2} - \sqrt{2}i$$

17.



$$18. (2.8, 1.1)$$

$$19. \left(2, -\frac{5\pi}{6}\right)$$

$$20. r = \frac{5}{\sin \theta + 3 \cos \theta}$$

$$21. 2r^2 \sin^2 \theta - r \cos \theta - 5 = 0$$

$$22. y = 2$$

$$23. (x^2 + y^2)^2 - x^2 + y^2 = 0$$

Chapter 9

Exercise 9–1

Answers to odd-numbered problems

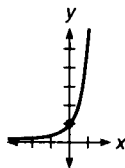
$$1. f(x) = b^x, b > 0 \text{ and } b \neq 1 \quad 3. 2^{x+1}$$

$$5. 2,401^x \quad 7. 4\sqrt{3} \quad 9. 9 \quad 11. 3$$

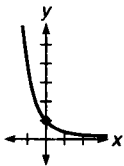
$$13. \frac{3}{2} \quad 15. 6 \quad 17. -3 \quad 19. -\frac{3}{2} \quad 21. \frac{5}{3}$$

$$23. 8$$

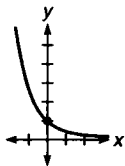
$$25. \text{increasing}$$



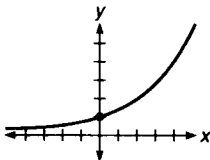
$$27. \text{decreasing}$$



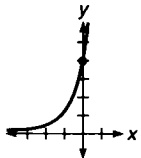
$$29. \text{decreasing}$$



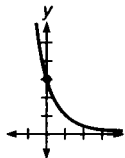
$$31. \text{increasing}$$



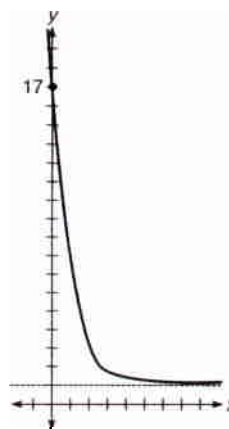
$$33. \text{increasing}$$



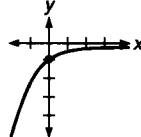
$$35. \text{decreasing}$$



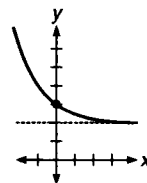
$$37. \text{decreasing}$$



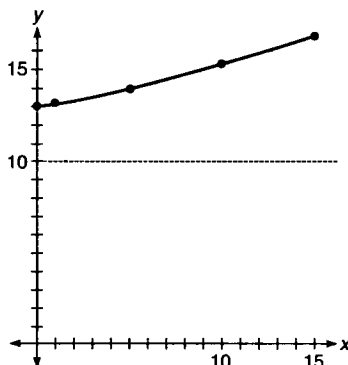
$$39. \text{increasing}$$



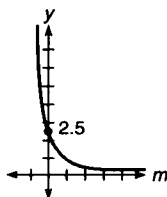
$$41. \text{decreasing}$$



$$43.$$

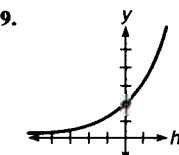


$$45.$$



$$47. \$6,264.08$$

$$49.$$



$$54. r^2 = \frac{5}{2 \sin^2 \theta - \cos^2 \theta}$$

$$55. r = \frac{3 \cos \theta}{\sin^2 \theta}; r = 3 \cot \theta \csc \theta$$

(alternate form of answer)

$$56. r = 3 \quad 57. r = \frac{2}{\cos \theta}; r = 2 \sec \theta$$

(alternate form of answer)

$$58. x^2 + y^2 - y = 0 \quad 59. x = 2$$

$$60. (x^2 + y^2)^2 - 2xy = 0$$

$$61. x^3 + xy^2 - y = 0 \quad 62. y = 2$$

$$63. 4x^2 + 3y^2 - 6y - 9 = 0$$

Chapter 8 test

$$1. B = 84.4^\circ, a \approx 5.3, c \approx 22.5$$

$$2. B \approx 56.3^\circ, A \approx 61.6^\circ, a \approx 23.9$$

$$3. b \approx 32.8, A \approx 50.9^\circ, C \approx 29.1^\circ$$

$$4. C \approx 89.1^\circ, A \approx 39.6^\circ, B \approx 51.3^\circ$$

$$5. 59 \text{ yards} \quad 6. B \approx 53.1^\circ$$

$$7. (\sqrt{3}, 1)$$

$$8. \text{magnitude: } 6.4; \text{direction: } 51.3^\circ$$

$$9. (8.5, 84.9^\circ)$$

$$10. \text{tension: } 584 \text{ pounds; angle above the horizontal: } 65^\circ$$

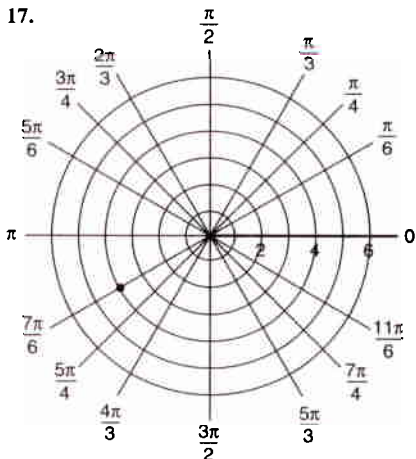
$$11. 6.40 \operatorname{cis}(-51.3) \quad 12. -1 + \sqrt{3}i$$

$$13. 14 \operatorname{cis} 110^\circ \quad 14. 3 \operatorname{cis} 120^\circ$$

$$15. 27 \operatorname{cis} 90^\circ$$

$$16. \sqrt{2} + \sqrt{2}i, -\sqrt{2} + \sqrt{2}i, \sqrt{2} - \sqrt{2}i, -\sqrt{2} - \sqrt{2}i$$

17.



$$18. (2.8, 1.1)$$

$$19. \left(2, -\frac{5\pi}{6}\right)$$

$$20. r = \frac{5}{\sin \theta + 3 \cos \theta}$$

$$21. 2r^2 \sin^2 \theta - r \cos \theta - 5 = 0$$

$$22. y = 2$$

$$23. (x^2 + y^2)^2 - x^2 + y^2 = 0$$

Chapter 9

Exercise 9–1

Answers to odd-numbered problems

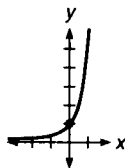
$$1. f(x) = b^x, b > 0 \text{ and } b \neq 1 \quad 3. 2^{\pi+1}$$

$$5. 2,401^\pi \quad 7. 4\sqrt{3} \quad 9. 9 \quad 11. 3$$

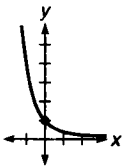
$$13. \frac{3}{2} \quad 15. 6 \quad 17. -3 \quad 19. -\frac{3}{2} \quad 21. \frac{5}{3}$$

$$23. 8$$

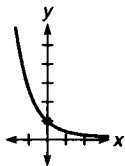
25. increasing



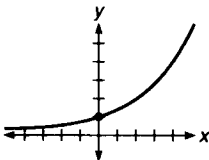
27. decreasing



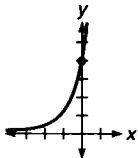
29. decreasing



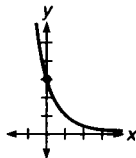
31. increasing



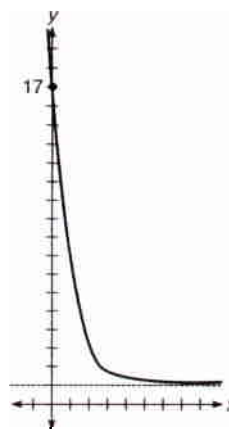
33. increasing



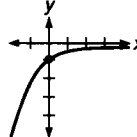
35. decreasing



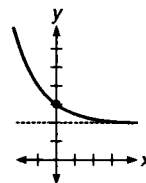
37. decreasing



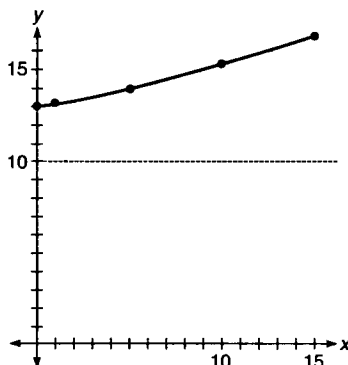
39. increasing



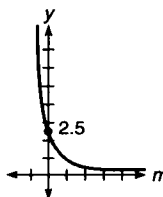
41. decreasing



43.

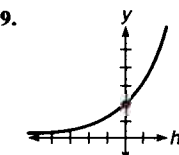


45.



47. \$6,264.08

49.



51. a. 8,000 b. 2,000,000
c. 8,000,000,000,000 (8 trillion)
53. 10^{30} ; b

Solutions to skill and review problems

1. $y = \frac{x-1}{(x-2)(x+2)}$

Vertical asymptotes at ± 2 ; horizontal asymptote is $y = 0$ (the x -axis).

Intercepts:

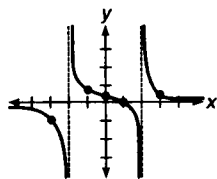
$$x = 0: y = \frac{-1}{-4} = \frac{1}{4}; (0, 0.25)$$

$$y = 0: 0 = \frac{x-1}{x^2-4}$$

$$0 = x - 1$$

$$1 = x; (1, 0)$$

Additional points: $(-3, -0.8)$, $(-1, 0.67)$, $(3, 0.4)$



2. $y = (x-1)(x-2)(x+2)$

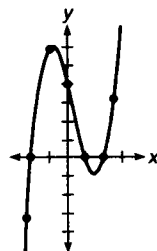
Intercepts:

$$x = 0: y = (-1)(-4) = 4; (0, 4)$$

$$y = 0: 0 = (x-1)(x-2)(x+2)$$

$$x = -2, 1, 2; (-2, 0), (1, 0), (2, 0)$$

Additional points: $(-2.25, -3.45)$, $(-1, 6)$, $(2.5, 3.38)$



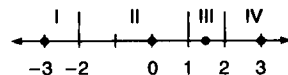
3. $(x-1)(x^2-4) > 0$

To find critical points solve the corresponding equality and find zeros of denominators.

$$(x-1)(x-2)(x+2) = 0$$

$$x = -2, 1, 2$$

Select test points in the intervals determined by the critical points.



Try these values in the original inequality.

$$x = -3: (-4)(5) > 0; \text{false}$$

$$x = 0: (-1)(-4) > 0; \text{true}$$

$$x = 1.5: (0.5)(-1.75) > 0; \text{false}$$

$$x = 3: (2)(5) > 0; \text{true}$$

Solution: $\{x \mid -2 < x < 1 \text{ or } x > 2\}$

4. $6x^3 + 5x^2 - 2x - 1$

Possible zeros are $\pm \frac{1}{6}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm 1$.

Synthetic division will show that -1 is a zero.

	6	5	-2	-1
		-6	1	1
-1	6	-1	-1	0

$$\text{Thus, } 6x^3 + 5x^2 - 2x - 1$$

$$= (x+1)(6x^2 - x - 1)$$

$$= (x+1)(3x+1)(2x-1)$$

5. $y = -x^3 + 1$

This is the graph of $y = x^3$ but "flipped over" and shifted up one unit.

Intercepts:

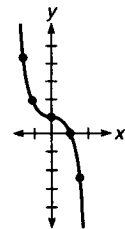
$$x = 0: y = 0^3 + 1 = 1; (0, 1)$$

$$y = 0: 0 = -x^3 + 1$$

$$x^3 = 1$$

$$x = 1; (1, 0)$$

Additional points: $(-1.5, 4.4)$, $(-1, 2)$, $(1.5, -2.4)$



6. $x^{2/3} - x^{1/3} - 6 = 0$

Let $u = x^{1/3}$; then $u^2 = (x^{1/3})^2 = x^{2/3}$.

$$u^2 - u - 6 = 0$$

$$(u-3)(u+2) = 0$$

$$u = 3 \text{ or } u = -2$$

$$x^{1/3} = 3 \text{ or } x^{1/3} = -2$$

Replace u by $x^{1/3}$.

$$x = 27 \text{ or } x = -8$$

Cube each member.

Solution set: $\{-8, 27\}$

Solutions to trial exercise problems

7. $\frac{4\sqrt{12}}{4\sqrt{3}}$

$$4^2\sqrt{3} - \sqrt{3}$$

$$4\sqrt{3}$$

23. $(\sqrt{2})^x = 16$

$$(2^{1/2})^x = 2^4$$

$$2^{x/2} = 2^4$$

$$\frac{x}{2} = 4$$

$$x = 8$$

37. $f(x) = 4^{-x+2} + 1$

$$= 4^2 \cdot 4^{-x} + 1$$

$$= 16 \cdot (4^{-1})^x + 1$$

$$= 16(\frac{1}{4})^x + 1; b = \frac{1}{4}$$

Decreasing since $b < 1$.

This is the graph of $y = (\frac{1}{4})^x$ with a vertical scaling factor of 16 and shifted up one unit.

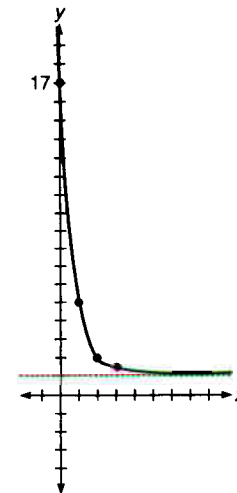
$$y\text{-intercept: } f(0) = 2^2 + 1 = 17; (0, 17)$$

$$x\text{-intercept: } 0 = 16(\frac{1}{4})^x + 1$$

$$-1 = 16(\frac{1}{4})^x; \text{no solution}$$

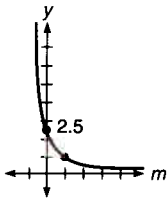
as the left member is negative and the right is nonnegative.

Additional points: $(1, 5)$, $(2, 2)$, $(3, 1.25)$



$$\begin{aligned}
 45. R(m) &= 2.5^{1-m} \\
 &= 2.5^1(2.5^{-m}) \\
 &= 2.5\left(\frac{1}{2.5}\right)^m \\
 &= 2.5(0.4)^m; b = 0.4
 \end{aligned}$$

Additional points: $(-1, 6.25)$, $(0, 2.5)$, $(1, 1)$



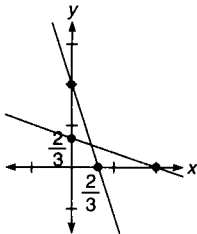
Exercise 9-2

Answers to odd-numbered problems

1. 3 3. 3 5. -3 7. -2 9. -1
 11. 3 13. 4 15. -6 17. 15
 19. -9 21. -55 23. 3 25. $2^3 = 8$
 27. $10^{-1} = 0.1$ 29. $12^2 = x + 3$
 31. $3^{x+2} = 5$ 33. $4 = \log_2 16$ 35. $2 = \log_x(m+3)$ 37. $y = \log_m(x+1)$
 39. $x + y = \log_{2x-3}(y+2)$ 41. 16
 43. 2 45. 2 47. 2 49. 10 51. k^2
 53. $6 < \log_2 100 < 7$ 55. $3 < \log_4 100 < 4$ 57. $-2 < \log_2 0.3 < -1$ 59. 4
 61. 1 63. 5 65. 18 67. 1 69. 0
 71. -5 73. 81 75. 625 77. a. 10 bits b. 14 bits c. 14 bits d. 16 bits
 79. $10^{d/10} = I$ 81. $\log_b x = y$ if and only if $b^y = x$, $b > 0$ and $b \neq 1$.

Solutions to skill and review problems

1. 9^{2x}
 $(3^2)^{2x}$
 3^{4x}
 2. $f(x) = 2 - 3x$
 $y = 2 - 3x$
 $x = 2 - 3y$
 $3y = -x + 2$
 $y = -\frac{1}{3}x + \frac{2}{3}$
 $f^{-1}(x) = -\frac{1}{3}x + \frac{2}{3}$



$$\begin{aligned}
 3. 2x^6 + 15x^3 - 8 &= 0 \\
 \text{Let } u &= x^3, \text{ then } u^2 = x^6. \\
 2u^2 + 15u - 8 &= 0 \\
 (2u - 1)(u + 8) &= 0 \\
 2u - 1 = 0 \text{ or } u + 8 = 0 \\
 2u = 1 \text{ or } u = -8 \\
 u = \frac{1}{2} \text{ or } u = -8
 \end{aligned}$$

$$x^3 = \frac{1}{2} \text{ or } x^3 = -8$$

Replace u by x^3 .

$$x = \sqrt[3]{\frac{1}{2}} \text{ or } x = \sqrt[3]{-8}$$

$$x = \frac{\sqrt[3]{4}}{2} \text{ or } x = -2$$

$$\text{Note: } \sqrt[3]{\frac{1}{2}} = \sqrt[3]{\frac{1}{2} \cdot \frac{4}{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{\sqrt[3]{4}}{2}$$

$$\text{Solution set: } \left\{ -2, \frac{\sqrt[3]{4}}{2} \right\}$$

$$\begin{aligned}
 4. y &= x^4 - x \\
 &= x(x^3 - 1) \\
 &= x(x - 1)(x^2 + x + 1) \\
 x^2 + x + 1 &\text{ is prime on } R.
 \end{aligned}$$

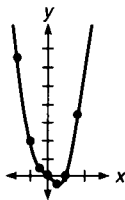
Intercepts:

$$x = 0: y = 0^4 - 0; (0, 0)$$

$$y = 0: 0 = x(x - 1)(x^2 + x + 1)$$

$$x = 0 \text{ or } 1; (0, 0), (1, 0)$$

Additional points: $(-1.5, 6.6)$, $(-1, 2)$, $(-0.5, 0.6)$, $(0.5, -0.4)$, $(1.5, 3.6)$



$$\begin{aligned}
 5. \frac{2x-5}{3} - \frac{3x+12}{2} &= 4 \\
 6 \cdot \frac{2x-5}{3} - 6 \cdot \frac{3x+12}{2} &= 6(4) \\
 2(2x-5) - 3(3x+12) &= 24 \\
 4x - 10 - 9x - 36 &= 24 \\
 -70 &= 5x \\
 -14 &= x \\
 \text{Solution set: } \{-14\}
 \end{aligned}$$

$$\begin{aligned}
 6. 2xy &= \frac{x+y}{3} \\
 3(2xy) &= 3 \cdot \frac{x+y}{3} \\
 6xy &= x + y \\
 6xy - y &= x \\
 y(6x - 1) &= x \\
 y &= \frac{x}{6x - 1}
 \end{aligned}$$

Solutions to trial exercise problems

$$21. 5(3 \log_2 \frac{1}{8} + 2 \log_{10} 0.1) = 5[3(-3) + 2(-1)] = -55 \quad 31. 3^{x+2} = 5$$

$$39. \text{ move the base } 2x - 3 \text{ to the other side.} \\ x + y = \log_{2x-3}(y + 2)$$

$$51. \log_k x = 2$$

$$k^2 = x$$

$$57. \log_2 0.3$$

$$0.25 < 0.3 < 0.5$$

$$\frac{1}{4} < 0.3 < \frac{1}{2}$$

$$2^{-2} < 0.3 < 2^{-1}$$

$$\text{so, } -2 < \log_2 0.3 < -1$$

$$73. 4^{\log_2 9}$$

$$(2^2)^{\log_2 9}$$

$$(2^{\log_2 9})^2, \text{ since } (a^m)^n = (a^n)^m$$

$$9^2$$

$$81$$

$$80. \frac{P}{5} = 1.25^t; \text{ the base is } 1.25, \text{ and } t \text{ is}$$

$$\text{the exponent: } \log_{1.25} \frac{P}{5} = t$$

Exercise 9-3

Answers to odd-numbered problems

$$1. \frac{8}{3} \quad 3. 1 \quad 5. \frac{1}{3} \quad 7. 2 \quad 9. \frac{17}{80}$$

$$11. \frac{1}{40} \quad 13. 24 \quad 15. \frac{-1 + \sqrt{17}}{2}$$

$$17. \frac{9}{10} \quad 19. 48 \quad 21. 6 \quad 23. x > 0$$

$$25. 8 \quad 27. \text{ no solution; solution set is the null set} \quad 29. \log_2 + \log_6 x + \log_6 y$$

$$31. 1 + \log_4 x + \log_4 y$$

$$33. 1 + \log_3 x + \log_3 y - \log_3 2 - \log_3 z$$

$$35. -\log_{10} 3 - \log_{10} x - \log_{10} y - \log_{10} z$$

$$37. 2 + 3 \log_2 x + 2 \log_2 y + 5 \log_2 z$$

$$39. \frac{3}{2} + 4 \log_4 y + 3 \log_4 z - 3 \log_4 x$$

$$41. 0.9208 \quad 43. 1.8416 \quad 45. 2.2584$$

$$47. -0.8271 \quad 49. 7$$

$$51. \alpha = 10 \log_{10} I - 20$$

$$53. \log_a \sqrt[n]{x} = \log_a x^{1/n} = \frac{1}{n} \log_a x$$

$$55. \text{ Let } a = 2, x = y = \frac{1}{2}.$$

$$\log_a(x + y) = \log_a x + \log_a y$$

Assume this is true.

$$\log_2(\frac{1}{2} + \frac{1}{2}) = \log_2 \frac{1}{2} + \log_2 \frac{1}{2}$$

$$\text{Replace } a \text{ by } 2, x \text{ and } y \text{ by } \frac{1}{2}.$$

$$\log_2(1) = \log_2 \frac{1}{2} + \log_2 \frac{1}{2}$$

$$0 = (-1) + (-1)$$

$$\log_2 1 = 0, \log_2 \frac{1}{2} = -1.$$

$$0 = -2$$

A false statement.

The original "identity" does not work

for the selected values of a , x , and y ,

so it is not an identity.

Solutions to skill and review problems

1. x must be between 3 and 4.
2. $3^{2x} = 3^3$; $2x = 3$; $x = 1\frac{1}{2}$
3. Since $5^3 = 125$, the base x must be 5.
4. $x^3 + 2x^{3/2} - 3 = 0$

Let $u = x^{3/2}$; then $u^2 = (x^{3/2})^2 = x^3$.

$$u^2 + 2u - 3 = 0$$

$$(u - 1)(u + 3) = 0$$

$$u = 1 \text{ or } u = -3$$

$$x^{3/2} = 1 \text{ or } x^{3/2} = -3$$

$$(x^{3/2})^2 = 1^2 \text{ or } (x^{3/2})^2 = (-3)^2$$

$$x^3 = 1 \text{ or } x^3 = 9$$

$$x = 1 \text{ or } x = \sqrt[3]{9}$$

However, $\sqrt[3]{9}$ cannot check in $x^{3/2} = -3$ since $\sqrt[3]{9} > 0$ for any exponent.

Thus the solution is $x = 1$.

5. $|2x - 5| = 10$

$$2x - 5 = 10 \text{ or } 2x - 5 = -10$$

$$2x = 15 \text{ or } 2x = -5$$

$$x = 7\frac{1}{2} \text{ or } x = -2\frac{1}{2}$$

6. $f(x) = x^2 + 3x - 5$

This is a parabola. We complete the square.

$$y = x^2 + 3x + \frac{9}{4} - 5 - \frac{9}{4}$$

$$\frac{1}{2} \cdot 3 = \frac{3}{2}; \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{29}{4}$$

$$\text{Vertex: } \left(-1\frac{1}{2}, -7\frac{1}{4}\right).$$

Intercepts:

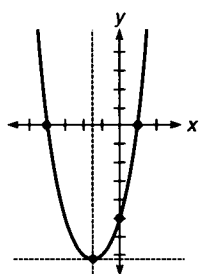
$$x = 0: y = 0^2 + 3(0) - 5 = -5;$$

$$(0, -5)$$

$$y = 0: 0 = x^2 + 3x - 5$$

$$x = \frac{-3}{2} \pm \frac{\sqrt{29}}{2} = -4.2, 1.2;$$

$$(-4.2, 0), (1.2, 0)$$



Solutions to trial exercise problems

13. $\log_5(x + 1) = \log_{10}100$

$$\log_5(x + 1) = 2$$

$$x + 1 = 5^2$$

$$x = 24$$

23. $\log_3 2x = \log_3 2 + \log_3 x$

$$\log_3 2x = \log_3 2x$$

This last equation is an identity and is true for any value for which $\log_3 2x$ is defined. Thus, the solution is all x for which $\log_3 2x$ is defined, which is $\{x | x > 0\}$.

27. $\log_2(x - 2) + \log_2(x + 3)$

$$= \log_2(x^2 - 3x + 2)$$

$$\log_2[(x - 2)(x + 3)]$$

$$= \log_2(x^2 - 3x + 2)$$

$$x^2 + x - 6 = x^2 - 3x + 2$$

$$4x = 8$$

$$x = 2$$

However, the solution 2 is not in the domain of the term $\log_2(x - 2)$ so *there is no solution* (the solution set is the null set).

39. $\log_4 \frac{8y^4z^3}{x^3}$

$$\log_4(8y^4z^3) - \log_4 x^3$$

$$\log_4 8 + \log_4 y^4 + \log_4 z^3 - 3 \log_4 x$$

$$\frac{3}{2} + 4 \log_4 y + 3 \log_4 z - 3 \log_4 x$$

47. $\log_a 0.2$

$$\log_a \frac{1}{5}$$

$$\log_a 1 - \log_a 5$$

$$0 - 0.8271$$

$$-0.8271$$

49. $\log_a 14 = 1.3562$

$$-\log_a 2 = 0.3562$$

$$\log_a 14 - \log_a 2 = 1$$

$$\log_a \frac{14}{2} = 1$$

$$\log_a 7 = 1$$

$$a^1 = 7$$

$$a = 7$$

Exercise 9-4

Answers to odd-numbered problems

1. 1.7160
3. 0.4065
5. 1.0253
7. -0.0706
9. 3.9405
11. 2.8332
13. 5.2470
15. 7.8240
17. -5.8091

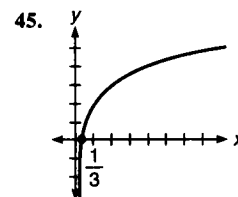
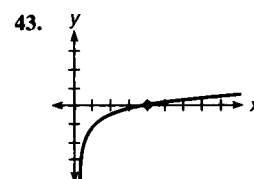
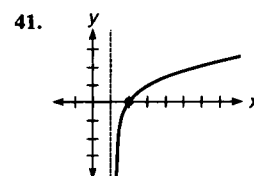
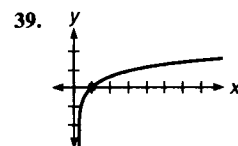
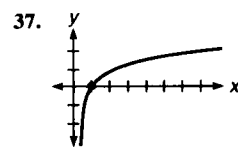
19. 15.8987
21. 19.9542

23. -15.6990
25. -9.9208

27. -26.1785
29. 10.2553

31. 2.5372
33. -0.1195

35. -0.0467



47. 794.33
49. 0.47
51. 6.36
53. 121.51
55. 4.64
57. $3x + \ln 2$
59. $4 - 3x$
61. 100
63. $(x - 1)^2$
65. $x + \ln 5$
67. \$2,872.80
69. \$1,172.89

71. $\text{Nap log } x = 10^7 \log_{10} \left(\frac{x}{10^7} \right)$

Let $y = \text{Nap log } x$.

$$y = 10^7 \log_{10} \left(\frac{x}{10^7} \right)$$

$$\frac{y}{10^7} = \log_{10} \left(\frac{x}{10^7} \right)$$

Divide both members by 10^7 .

$$\left(\frac{1}{e} \right)^{y/10^7} = \frac{x}{10^7}$$

Rewrite as an exponential equation

$$(e^{-1})^{y/10^7} = \frac{x}{10^7}$$

$$e^{-y/10^7} = \frac{x}{10^7}$$

$$\log_e \frac{x}{10^7} = -\frac{y}{10^7}$$

Rewrite as a logarithmic equation

with base e and exponent $-\frac{y}{10^7}$

$$\ln \frac{x}{10^7} = -\frac{y}{10^7}$$

$$\log_e z = \ln z$$

$$-10^7 \ln \frac{x}{10^7} = y$$

Multiply each member by -10^7 .

$$y = -10^7 (\ln x - \ln 10^7)$$

$$\ln \frac{x}{10^7} = \ln x - \ln 10^7$$

$$y = 10^7 (-\ln x + 7 \ln 10)$$

$$y = 10^7 (7 \ln 10 - \ln x)$$

Then, $\text{Nap log } x = 10^7 (7 \ln 10 - \ln x)$.

73. 9.3 75. 215 ohms 77. -223

BTU/hour 79. 0.32 centiliters per second

Solutions to skill and review problems

1. $2x^2 - 9x + 4 = 0$

$$(2x - 1)(x - 4) = 0$$

$$2x - 1 = 0 \text{ or } x - 4 = 0$$

$$x = \frac{1}{2} \text{ or } x = 4$$

2. $2x^4 - 9x^2 + 4 = 0$

$$\text{Let } u = x^2; \text{ then } u^2 = x^4.$$

$$2u^2 - 9u + 4 = 0$$

$$u = \frac{1}{2} \text{ or } u = 4$$

Solve as in the previous problem.

$$x^2 = \frac{1}{2} \text{ or } x^2 = 4$$

$$u = x^2$$

$$x = \pm \frac{\sqrt{2}}{2} \text{ or } x = \pm 2$$

Extract square root of both sides.

3. $2x - 9\sqrt{x} + 4 = 0$

$$\text{Let } u = \sqrt{x}; \text{ then } u^2 = x.$$

$$2u^2 - 9u + 4 = 0$$

$$u = \frac{1}{2} \text{ or } u = 4$$

See previous two problems.

$$\sqrt{x} = \frac{1}{2} \text{ or } \sqrt{x} = 4$$

$$u = \sqrt{x}$$

$$x = \frac{1}{4} \text{ or } x = 16$$

Square both sides.

4. $2(x - 3)^2 - 9(x - 3) + 4 = 0$

$$\text{Let } u = x - 3.$$

$$2u^2 - 9u + 4 = 0$$

$$u = \frac{1}{2} \text{ or } u = 4$$

See previous three problems.

$$x - 3 = \frac{1}{2} \text{ or } x - 3 = 4$$

$$u = x - 3.$$

$$x = 3\frac{1}{2} \text{ or } x = 7$$

5. $\log_a \frac{2x^4}{3y^3z}$

$$\log_a 2x^4 - \log_a 3y^3z$$

$$\log_a 2 + \log_a x^4 - (\log_a 3 + \log_a y^3 +$$

$$\log_a z)$$

$$\log_a 2 + 4 \log_a x - \log_a 3 - 3 \log_a y -$$

$$\log_a z$$

6. $\log_2 x = -3$

$$2^{-3} = x$$

$$\frac{1}{8} = x$$

7. $f(x) = \log_3(x - 1)$

We compute points for the inverse function and reverse them.

Find f^{-1} .

$$y = \log_3(x - 1)$$

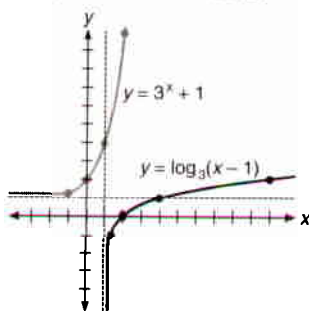
$$x = \log_3(y - 1)$$

$$y - 1 = 3^x$$

$$y = 3^x + 1$$

Computed points:

$y = 3^x + 1$	$y = \log_3(x - 1)$
$(-1, 1\frac{1}{3})$	$(1\frac{1}{3}, -1)$
$(0, 2)$	$(2, 0)$
$(1, 4)$	$(4, 1)$
$(2, 10)$	$(10, 2)$



8. $\frac{x^2 - 4}{x^2 - 1} > 2$

Find critical points by (a) solving the corresponding equality and (b) finding zeros of denominators.

Solve the corresponding equality.

$$\frac{x^2 - 4}{x^2 - 1} = 2$$

$$x^2 - 4 = 2x^2 - 2$$

$$-2 = x^2$$

No real solutions.

Find zeros of denominators.

$$x^2 - 1 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Critical points are ± 1 .

I	II	III
-3	-2	-1
0	1	2
3		

Select trial points from each interval I, II, and III. We will test 0, ± 2 .

$$\frac{x^2 - 4}{x^2 - 1} > 2$$

$$x = -2: \frac{(-2)^2 - 4}{(-2)^2 - 1} > 2; \frac{0}{3} > 2; \text{false}$$

$$x = 0: \frac{0 - 4}{0 - 1} > 2; 4 > 2; \text{true}$$

$$x = 2: \frac{2^2 - 4}{2^2 - 1} > 2; \frac{0}{3} > 2; \text{false}$$

Thus, interval II is the solution set:

$$\{x \mid -1 < x < 1\}$$

Solutions to trial exercise problems

25. $\log 0.000\,000\,000\,120\,004$

$$\log (1.20004 \times 10^{-10})$$

$$\log 1.20004 + \log 10^{-10}$$

$$0.0792 + (-10)$$

$$-9.9208$$

31. $\log_{20} 2,000 = \frac{\log 2,000}{\log 20} \approx 2.5372$

$$2,000 \quad \boxed{\log} \quad \boxed{\div} \quad 20 \quad \boxed{\log} \quad \boxed{=}$$

$$\text{TI-81: } \boxed{\log} \quad 2000 \quad \boxed{\div} \quad \boxed{\log} \quad 20$$

$$\boxed{\text{ENTER}}$$

45. $f(x) = \log_2 3x$

Calculate inverse function.

$$y = \log_2 3x$$

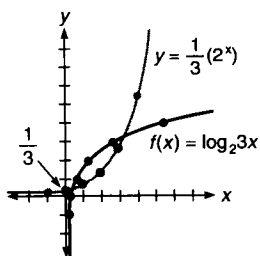
$$x = \log_2 3y$$

$$3y = 2^x$$

$$y = \frac{1}{3}(2^x)$$

Calculated points:

$y = \frac{1}{3}(2^x)$	$y = \log_2 3x$
$(-1, \frac{1}{6})$	$(\frac{1}{6}, -1)$
$(0, \frac{1}{3})$	$(\frac{1}{3}, 0)$
$(1, \frac{2}{3})$	$(\frac{2}{3}, 1)$
$(2, 1\frac{1}{3})$	$(1\frac{1}{3}, 2)$
$(3, 2\frac{2}{3})$	$(2\frac{2}{3}, 3)$
$(4, 5\frac{1}{3})$	$(5\frac{1}{3}, 4)$



49. $10^{-0.33} \approx 0.47$

63. $10^{\log(x-1)^2} = (x-1)^2$, since $10^{\log x} = x$

73. $k = 12$, and $I = 6I_0$.

$$S = 12 \log \left(\frac{6I_0}{I_0} \right) \\ = 12 \log 6 \approx 9.3$$

77. $L = 80$, $T_{in} = 30$, $T_{out} = 42$, $T_{earth} = 54$

$$Q = 0.07(80) \frac{30 - 42}{\log \frac{54 - 30}{54 - 42}} \\ = 5.6 \frac{-12}{\frac{24}{\log 12}} = \frac{-67.2}{\log 12} \\ \approx -223 \text{ BTU/hour}$$

Exercise 9-5

Answers to odd-numbered problems

1. $\frac{15}{13}$ 3. $\frac{2}{5}$ 5. -1 7. $2\sqrt{2} - 1$
 9. $\frac{13}{3}$ 11. $2\sqrt{26} - 1$ 13. $\frac{299}{99}$
 15. $\frac{\log 14.2}{\log 2} \approx 3.8$ 17. $\pm \sqrt[4]{25} \approx \pm 2.2$
 19. $\frac{\log 34}{\log 17} \approx 1.2$ 21. $-2 \pm \sqrt[4]{200}$
 ≈ -5.8 or 1.8 23. $\frac{\log 8}{\log 25} \approx 0.6$
 25. $\frac{\log 41}{2 \log 41 - \log 2} \approx 0.6$
 27. $\frac{2 \log 5}{\log 57 - 2 \log 5} \approx 3.9$
 29. $\frac{\log 3 + \log 5}{\log 3 - \log 5} \approx -5.3$
 31. $2^{0.33} \approx 1.26$ 33. $\sqrt[5]{10} = 10^{1/5} \approx 1.58$
 35. $\frac{\log 30}{2 \log 3} \approx 1.55$ 37. $\frac{\sqrt[3]{14}}{2} \approx 1.21$
 39. 1 or 100 41. $10^{1,000}$ 43. $\frac{100}{3}$
 $\frac{5(\log 2)(\log 3)}{\log 3 + \log 2}$ 47. $\ln 2$ 49. $\ln 4$
 51. $10^{10^{34}-4}$ 53. \$1,000.61

55. \$3,512.37

57. \$2,744.06

59. 5.78% 61. 21.97 years

63. 37.08 mg 65. 9,709; the charcoal is about 10,000 years old.

67. 5,589.9 or about 5,600 years 69. $10 \log 20 \approx 13$

71. $I = 10^{0.3} I_0$. Thus, the power of a sound must change by a factor of $10^{0.3} \approx 2$ for a 3-decibel change in intensity.

73. 95% 75. 0.60 time constants

77. $t = -\ln(1 - q)$

79. Let $y = b^x$.

$$\ln y = \ln b^x$$

$$\ln y = x \ln b$$

$$e^{\ln y} = e^{x \ln b}$$

$$y = e^{x \ln b}$$

$$b^x = e^{x \ln b}$$

81. $\pm \sqrt{-2 \ln(y \sqrt{2\pi})}$

83. $\frac{\log M}{\log \left(1 - \frac{1}{h} \right)} = N$

85.	x	x^2	2^x
	5	25	32
	10	100	1,024
	20	400	1,048,576
	40	1,600	1.09951×10^{12}

87. $5 \ln 2 \approx 3.4657359$ Thus, it takes about $3\frac{1}{2}$ years. The Mesopotamian value

is $3 + \frac{47}{60} + \frac{13}{60^2} + \frac{20}{60^3} \approx 3.787$

89. 74.4 hours 91. a. 80.0 μg

b. after 3.3 hours of growth (or 1.3 hours after it reaches 40 μg)

93. 60.1 talents

Solutions to skill and review problems

1. $y = 2^{1-x}$

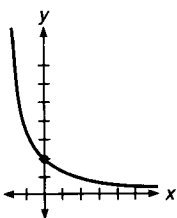
$$= 2(2^{-x})$$

$$= 2\left(\frac{1}{2}\right)^x; b = \frac{1}{2}$$

$$y\text{-intercept: } f(0) = 2; (0, 2)$$

$$\text{Additional points: } (-2, 8), (-1, 4),$$

$$(1, 1), (2, \frac{1}{2})$$



2. $f(x) = \frac{2}{(x-1)(x-5)}$

Vertical asymptotes: 1 and 5.

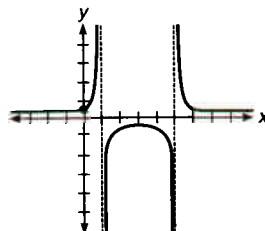
 Horizontal asymptote: x -axis.

$$y\text{-intercept: } f(0) = \frac{2}{5}; (0, \frac{2}{5})$$

$$\text{Additional points: } (-1, 0.2), (0.5, 0.9),$$

$$(2, -0.67), (3, -0.5), (4, -0.67),$$

$$(5.5, 0.9), (6, 0.4)$$



3. $|3 - 2x| < 13$

$$-13 < 3 - 2x < 13$$

$$-16 < -2x < 10$$

$$8 > x > -5$$

$$\{x | -5 < x < 8\}$$

4. $\left| \frac{3-2x}{x} \right| < 13$

This is nonlinear so it is solved by the critical point/test point method.

Critical points:

Solve the corresponding equality.

$$\left| \frac{3-2x}{x} \right| = 13$$

$$\frac{3-2x}{x} = 13 \text{ or } \frac{3-2x}{x} = -13$$

$$3-2x = 13x \text{ or } 3-2x = -13x$$

$$3 = 15x \text{ or } 11x = -3$$

$$x = \frac{1}{5} \text{ or } x = -\frac{3}{11}$$

Find zeros of denominators.

$$x = 0$$

Critical points are $-\frac{3}{11}$, 0 , $\frac{1}{5}$.



Test points are -1 , -0.1 , 0.1 , 1 .

$$\left| \frac{3-2x}{x} \right| < 13$$

$$x = -1: |-5| < 13; \text{ true}$$

$$x = -0.1: |-32| < 13; \text{ false}$$

$$x = 0.1: |28| < 13; \text{ false}$$

$$x = 1: |1| < 13; \text{ true}$$

The solution set is intervals I and IV:

$$\{x \mid x < -\frac{3}{11} \text{ or } x > \frac{1}{5}\}.$$

5. $y = x^5 - 4x^4 + 2x^3 + 4x^2 - 3x$
 $= x(x^4 - 4x^3 + 2x^2 + 4x - 3)$

Possible zeros of $x^4 - 4x^3 + 2x^2 + 4x - 3$ are $\pm 1, \pm 3$. Using synthetic division produces the following factorization.

$$y = x(x-1)^2(x+1)(x-3)$$

Since 1 is a root of even multiplicity the function does not cross the x -axis there.

Intercepts:

$$x = 0: y = 0; (0,0)$$

$$y = 0: 0 = x(x-1)^2(x+1)(x-3);$$

$$(0,0), (1,0), (-1,0), (3,0)$$

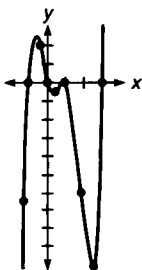
Additional points:

$$(-1.25, -6.7),$$

$$(-0.5, 1.97),$$

$$(0.5, -0.47),$$

$$(2, -6), (2.5, -9.8)$$



6. $\frac{2}{x-3} + \frac{2}{x+3} - \frac{5}{x+1}$
 $\frac{2(x+3) + 2(x-3) - 5(x+1)}{(x+3)(x-3)} - \frac{5}{x+1}$
 $\frac{4x - 5}{x^2 - 9} - \frac{5}{x+1}$
 $\frac{4x(x+1) - 5(x^2-9)}{(x^2-9)(x+1)}$
 $\frac{-x^2 + 4x + 45}{x^3 + x^2 - 9x - 9}$

7. $(\frac{2}{5} - \frac{3}{4}) \div 2$
 $(\frac{2(4) - 3(5)}{5(4)}) \cdot \frac{1}{2}$
 $-\frac{7}{20} \cdot \frac{1}{2} = -\frac{7}{40}$

Solutions to trial exercise problems

5. $(\sqrt{8})^{2x-2} = 4^{3x}$
 $[(2^3)^{1/2}]^{2x-2} = (2^2)^{3x}$
 $2^{3x-3} = 2^{6x}$
 $3x - 3 = 6x$
 $x = -1$

7. $\log(x-1) + \log(x+3) = \log 4$
 $\log[(x-1)(x+3)] = \log 4$
 $(x-1)(x+3) = 4$
 $x^2 + 2x - 3 = 4$
 $x^2 + 2x - 7 = 0$
 $x = -1 \pm 2\sqrt{2}$

We require $x-1 > 0$ or $x > 1$ so we choose $x = 2\sqrt{2} - 1$.

11. $\log(x-1) + \log(x+3) = 2$
 $\log[(x-1)(x+3)] = 2$
 $10^2 = x^2 + 2x - 3$
 $x^2 + 2x - 103 = 0$
 $x = -1 \pm 2\sqrt{26}$. Because we require $x-1 > 0$, or $x > 1$, we choose the solution $x = 2\sqrt{26} - 1$.

23. $25 = 8^{1/x}$
 $\log 25 = \log 8^{1/x}$
 $\log 25 = \frac{1}{x} \log 8$
 $\frac{\log 25}{\log 8} = \frac{1}{x}$
 $x = \frac{\log 8}{\log 25} \approx 0.6$

27. $57^{x/2} = 5^{x+1}$
 $\log 57^{x/2} = \log 5^{x+1}$
 $\frac{x}{2} \log 57 = (x+1) \log 5$
 $x \log 57 = 2(x+1) \log 5$
 $x \log 57 = 2x \log 5 + 2 \log 5$
 $x \log 57 - 2x \log 5 = 2 \log 5$
 $x(\log 57 - 2 \log 5) = 2 \log 5$
 $x = \frac{2 \log 5}{\log 57 - 2 \log 5} \approx 3.9$

37. $\log_2 14 = 3$
 $(2x)^3 = 14$
 $2x = \sqrt[3]{14}$
 $x = \frac{\sqrt[3]{14}}{2} \approx 1.21$

45. $\log_2 x + \log_3 x = 5$
 Use the change-to-common log formula.
 $\frac{\log x}{\log 2} + \frac{\log x}{\log 3} = 5$
 $\frac{\log x}{\log 2} (\log 2)(\log 3) + \frac{\log x}{\log 3} (\log 2)(\log 3)$
 $= 5(\log 2)(\log 3)$
 $(\log 3)(\log x) + (\log 2)(\log x) = 5(\log 2)(\log 3)$
 $\log x(\log 3 + \log 2) = 5(\log 2)(\log 3)$
 $\log x = \frac{5(\log 2)(\log 3)}{\log 3 + \log 2}$

$x = 10^{\frac{5(\log 2)(\log 3)}{\log 3 + \log 2}}$
 49. $e^{2x} - 3e^x = 4$
 Let $u = e^x$ so that $u^2 = e^{2x}$.
 $u^2 - 3u - 4 = 0$
 $(u-4)(u+1) = 0$
 $u = -1$ or 4
 $e^x = -1$ or 4
 -1 is not in the range of e^x so we proceed with $e^x = 4$.
 $x = \ln 4$ (by the property that if $b^x = y$ then $\log_b y = x$, with base e).

52. $\ln x = \frac{8}{\ln x - 2}$
 $(\ln x)^2 - 2 \ln x = 8$
 $(\ln x)^2 - 2 \ln x - 8 = 0$
 Let $u = \ln x$.
 $u^2 - 2u - 8 = 0$
 $(u-4)(u+2) = 0$
 $\ln x = 4$ or $\ln x = -2$
 so $x = e^4$ or e^{-2}

57. $A = Pe^{it}$, $A = 5,000$, $i = 0.1$, $t = 6$
 $5,000 = Pe^{(0.1)(6)}$
 $P = \frac{5,000}{e^{0.6}} \approx \$2,744.06$

61. $A = Pe^{it}$, $A = 3P$, $i = 0.05$; find t .
 $3P = Pe^{0.05t}$
 $3 = e^{0.05t}$
 $\ln 3 = 0.05t$; $\ln e^x = x$
 $t = \frac{\ln 3}{0.05} = 20 \ln 3 \approx 21.97$ years

63. Use $q = q_0 e^{-0.000124t}$, $q_0 = 100$,
 $t = 8,000$; $q = 100e^{-0.000124(8,000)} \approx$
 37.08 mg

68. Use $q = q_0 e^{-0.000124t}$, $t = 1,200$,

$$q = 18; \text{ find } q_0.$$

$$18 = q_0 e^{-0.000124(1,200)}$$

$$q_0 = \frac{18}{e^{-0.000124(1,200)}} \approx 20.89 \mu\text{g}$$

75. $q = 1 - e^{-t}$, $q = 0.45$

$$0.45 = 1 - e^{-t}$$

$$e^{-t} = 1 - 0.45$$

$$e^{-t} = 0.55$$

$$\ln e^{-t} = \ln 0.55$$

$$-t = \ln 0.55$$

$$t = -\ln 0.55 \approx 0.60 \text{ time constants}$$

89. $q = q_0 e^{rt}$, $q = 1.15q_0$ (increase of 15% puts the population at 115%), $t = 15$.

$$1.15q_0 = q_0 e^{15r}$$

$$1.15 = e^{15r}$$

$$\ln 1.15 = \ln e^{15r}$$

$$\ln 1.15 = 15r$$

$$r = \frac{\ln 1.15}{15} \approx 0.009317. \text{ Thus, for this}$$

bacteria $q = q_0 e^{0.009317t}$. Now find t for which $q = 2q_0$.

$$2q_0 = q_0 e^{0.009317t}$$

$$2 = e^{0.009317t}$$

$$\ln 2 = \ln e^{0.009317t}$$

$$\ln 2 = 0.009317t$$

$$t = \frac{\ln 2}{0.009317} \approx 74.4 \text{ hours}$$

91. $q_0 = 10$ and $q = 40$ when $t = 2$.

Basic growth/decay formula:

$$q = q_0 e^{rt}$$

$$40 = 10e^{2r}$$

$$4 = e^{2r}$$

$$\ln 4 = \ln e^{2r}$$

$$\ln 4 = 2r$$

$$r = \frac{1}{2} \ln 4 \approx 0.6931$$

Thus, the equation is $q = 10e^{0.6931t}$.

a. $q(3) = 10e^{0.6931(3)} \approx 80.0 \mu\text{g}$

b. Find t for $q = 100$.

$$100 = 10e^{0.6931t}$$

$$10 = e^{0.6931t}$$

$$\ln 10 = 0.6931t$$

$$t = \frac{\ln 10}{0.6931} \approx 3.32$$

Thus, the population will be 100 μg after 3.3 hours of growth (or 1.3 hours after it reaches 40 μg).

92. $10^{6.2-4.5} = 10^{1.7} \approx 50.1$, so the second is about 50 times stronger than the first.

93. Method 1: $\ln x = \log_x x = \frac{\log x}{\log e}$. The

value of $\log e$ (the common logarithm of the value e) could be stored in the calculator. Then to compute $\ln x$,

compute $\log x$ as described in the problem and divide by $\log e$.

Method 2: Store roots of e instead of 10:

$$e^{1/2} = 1.6487213$$

$$e^{1/4} = 1.2840254$$

$$e^{1/8} = 1.1331484$$

$$e^{1/16} = 1.0644945$$

$$e^{1/32} = 1.0317434$$

$$e^{1/64} = 1.0157477$$

etc.

Then to compute say $\ln 6$ we find by successive divisions that

$$6 = e \cdot e^{1/2} \cdot e^{1/4} \cdot e^{1/32} \cdot e^{1/128} \cdot e^{1/512} \cdot$$

$$e^{1/2,048} \cdot e^{1/4,096} \cdot \dots$$

so

$$\ln 6 \approx \ln(e \cdot e^{1/2} \cdot e^{1/4} \cdot e^{1/32} \cdot e^{1/128} \cdot$$

$$e^{1/512} \cdot e^{1/2,048} \cdot e^{1/4,096})$$

$$= \ln e + \ln e^{1/2} + \ln e^{1/4} +$$

$$+ \ln e^{1/32} + \ln e^{1/128} + \ln e^{1/512} +$$

$$+ \ln e^{1/2,048} + \ln e^{1/4,096}$$

$$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{32} + \frac{1}{128}$$

$$+ \frac{1}{512} + \frac{1}{2,048} + \frac{1}{4,096}$$

$$\approx 1.7917$$

which is correct to four decimal places.

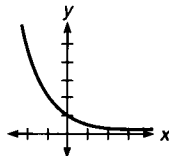
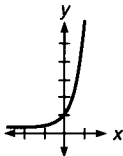
Chapter 9 review

1. If $b > 1$ the exponential function is increasing; if $0 < b < 1$ the function is decreasing. 2. $57\sqrt{2}$ 3. $43\sqrt{2}$

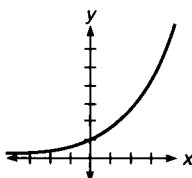
4. $3^{4\pi}$

5. increasing

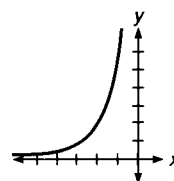
6. decreasing



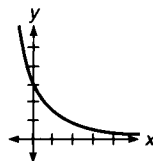
7. increasing



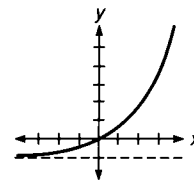
8. increasing; y-intercept: (0, 16)



9. decreasing

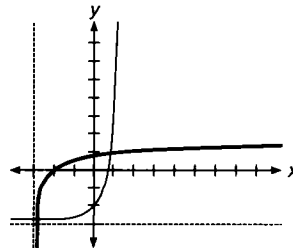


10. increasing



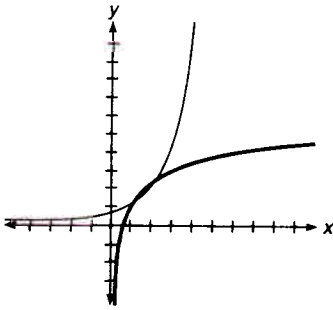
11. $\frac{3}{2}$ 12. $\frac{3}{4}$ 13. $-\frac{3}{2}$ 14. $-\frac{3}{4}$
 15. 4 16. 6 17. 3 18. $\frac{7}{2}$ 19. -3
 20. $-\frac{9}{2}$ 21. $-\frac{5}{2}$ 22. Definition: $\log_x b = y$ if and only if $x^y = b$, $b > 0$, $b \neq 1$.
 23. $4^{-1} = 0.25$ 24. $5^2 = x - 3$
 25. $2^8 = y$ 26. $3^{x+2} = 9$ 27. $m^{y+1} = x$
 28. $\log_x(m-3) = 3$ 29. $\log_5 5 = 2x$
 30. $\log_4 4 = 2x - 1$ 31. $\log_{x-1} 5 = y$
 32. $\log_{x+3}(y-2) = x + y$
 33. $\log_{5x} 3y = 2$ 34. $\frac{3}{2}$
 35. 2 36. $-\frac{99}{100}$ 37. $\frac{1}{2}$ 38. $\sqrt[3]{k}$
 39. $3 < \log_4 100 < 4$ 40. $4 < \log_{10} 15,600 < 5$ 41. m 42. 5 43. $\frac{1}{24}$
 44. -2 or 3 45. $\frac{5}{2}$ 46. $\frac{17}{5}$ 47. $-\frac{1}{24}$
 48. $25\frac{1}{2}$ 49. $2\frac{1}{3}$ 50. $\frac{7}{6}$ 51. $\frac{15}{11}$
 52. 6 53. 7 54. 64 55. 4,096
 56. $3 + 4 \log_2 x + 2 \log_2 y + \log_2 z$
 57. $\frac{3}{2} + \log_4 y + 3 \log_4 z - 2 \log_4 x$
 58. $5 \log_{10} x + 3 \log_{10} y + \log_{10} z - 2$
 59. 1.7479 60. 1.8416 61. -0.8271
 62. -0.0292 63. 3.7959 64. 21.7008
 65. -17.3840 66. 2.2920 67. 2.3059
 68. 0.1950 69. -2.1610 70. 2.5104
 71. -5.6550 72. 1.3854557
 73. 0.0032 74. 121.5104

75.

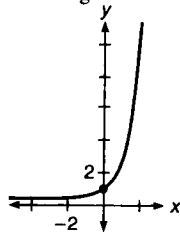


Chapter 9 test

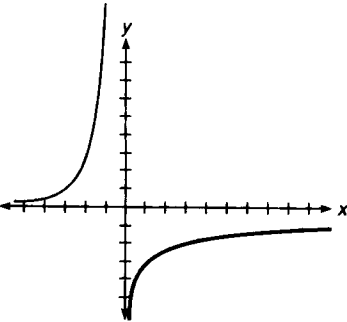
76.



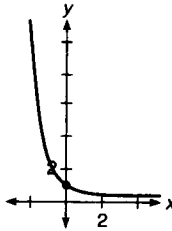
1. $\frac{1}{8\sqrt{2}}$ 2. 16
3. increasing



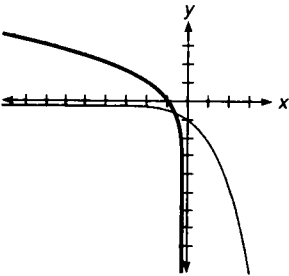
77.



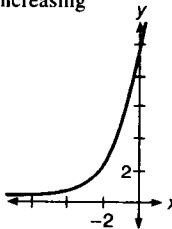
4. decreasing



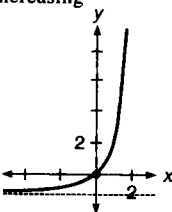
78.



5. increasing



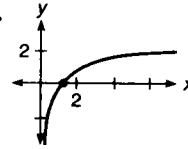
6. increasing



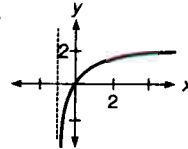
79. 4 80. $6x$ 81. $\sqrt{3}$ 82. $1 - 3x$
83. 30 84. $3x^2$ 85. $4x$
86. $\frac{1}{2} \ln 5 + x$ 87. \$3,111.30
88. $-\frac{16}{17}$ 89. $-\frac{2}{5}$ 90. 10.01
91. 3.32 92. -0.37 93. 3.34
94. 1.52 95. 2.11 96. 2.51
97. 3.03 98. 3.40 99. 1 or 100
100. $\frac{\log 3 - \log 2}{2 \log 3 - \log 2}$ 101. 10^{100}
102. $\pm 10^5$ 103. $\frac{100}{3}$ 104. 10^{-3} or 10^5
105. $10^{\frac{8(\log 3)(\log 2)}{\log 2 + \log 3}}$ 106. $\frac{10 - \log 3}{\log 6}$
107. $\ln 4$ 108. $\frac{1}{2} \ln \frac{3}{2}$ 109. $\frac{\ln 5 + 1}{\ln 5 - 1}$
110. 0.0001 or 1 111. e^{-2} or e^6
112. \$2,744.06 113. 7.7%
114. 79.6 hours

7. $\frac{1}{2}$ 8. $\frac{1}{2}$ 9. $-\frac{3}{2}$ 10. 4
11. $\frac{3}{2}$ 12. $\frac{7}{3}$ 13. -12 14. -5
15. $\log_a x = y$ if and only if $a^y = x$, $a > 0$, $a \neq 1$ 16. $5^{-x} = 0.25$ 17. $3^2 = x - 3$
18. $m^{y/2} = x$ 19. $\log_2 32 = 5$
20. $\log_{x-1} m = 3$ 21. $\log_y z = 2x - 1$
22. $-\frac{3}{2}$ 23. $502\frac{1}{2}$ 24. $-\frac{1}{2}$

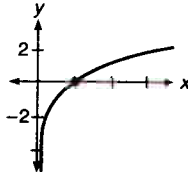
25.



26.



27.



28. $\frac{1}{4}$ 29. -2 or 16 30. 5 31. $\frac{2}{9}$
32. $\frac{19}{27}$ 33. $\frac{38}{31}$ 34. 5 35. 2 36. 32
37. $2 + 3 \log_3 x + \log_3 y - 4 \log_3 z$
38. $10 \log_{10} x + 3 \log_{10} y + \log_{10} z - 3$
39. 1.2770 40. 1.8957 41. -0.4709
42. 13.4916 43. -8.9872
44. 2.4307 45. 6.9078 46. 13.8546
47. 4.9462 48. $2x$ 49. x^2 50. 18
51. 27 52. $10x$ 53. \$3,190.06
54. -1.14 55. 15.29 56. 5.99
57. 125 58. 3.59 59. 4.10
60. 0.01, 1, or 100
61. $\frac{2 \log 4 - \log 3}{\log 4 - 6 \log 3} \approx -0.3216$
62. 512 63. $\pm 10^5$
64. 10,000,000 or 0.00001
65. $10^{3\left(\frac{1}{\log 2} + \frac{1}{\log 3}\right)^{-1}} \approx 3.5787$
66. $\frac{1}{2} \ln 1.25 \approx 0.1116$
67. $\frac{\ln 3 + 1}{\ln 3 - 1} \approx 21.2814$
68. \$1,573.26 69. 5.8%
70. 17.7 grams

Chapter 10

Exercise 10–1

Answers to odd-numbered problems

1. $(-3, -\frac{2}{5})$ 3. $(-5, 2)$ 5. $(-2, 10)$
7. $(8, \frac{1}{3})$ 9. $(\frac{3}{10}, -\frac{1}{2})$ 11. dependent
13. $(11, -4)$ 15. $(\frac{155}{47}, \frac{261}{47})$
17. inconsistent 19. dependent
21. $(8, \frac{1}{8})$ 23. $(\frac{3}{2}, -\frac{2}{3})$ 25. $(-2, 3, 2)$

27. $(6, -5, 2)$ 29. $(1, 5, -5)$ 31. $(-4, 6, -1)$
 33. $(0, -5, 5)$ 35. $(-10, 3, -1)$
 37. $L = 13\frac{7}{13}$ cm, $W = 8\frac{6}{13}$ cm
 39. $L = 11\frac{1}{2}$ in, $W = 6\frac{1}{2}$ in
 41. $L = 56\frac{2}{3}$ mm, $W = 18\frac{1}{3}$ mm
 43. \$8,000 at 5%, \$4,000 at 10%
 45. \$5,000 for each investment
 47. $y = \frac{1}{6}x^2 + \frac{7}{6}x + 3$ 49. $y = \frac{7}{3}x - \frac{38}{3}$
 51. $(-\frac{28}{11}, \frac{29}{11})$ 53. $y = \frac{5}{2}x + 2$

Solutions to skill and review problems

1. $\sqrt{\frac{4x^2}{27y^3z}} = \frac{\sqrt{4x^2}}{\sqrt{27y^3z}} = \frac{2x}{3y\sqrt{3yz}} \cdot \frac{\sqrt{3yz}}{\sqrt{3yz}} = \frac{2x\sqrt{3yz}}{3y(3yz)} = \frac{2x\sqrt{3yz}}{9y^2z}$
2. $\frac{2x-1}{3} = \frac{5-3x}{4}$
 $4(2x-1) = 3(5-3x)$
 Cross multiply.
 $8x-4 = 15-9x$
 $17x = 19$
 $x = \frac{19}{17}$
3. $\frac{2x-1}{3} = \frac{5-3x}{x}$
 $x(2x-1) = 3(5-3x)$
 Cross multiply.
 $2x^2 - x = 15 - 9x$
 $2x^2 + 8x - 15 = 0$
 $x = \frac{-4 \pm \sqrt{46}}{2}$
 Quadratic formula.
4. $\left| \frac{2x-1}{3} \right| > 5$
 $\frac{2x-1}{3} > 5$ or $\frac{2x-1}{3} < -5$
 If $|x| > a$ then $x > a$ or $x < -a$.
 $2x-1 > 15$ or $2x-1 < -15$
 $2x > 16$ or $2x < -14$
 $x > 8$ or $x < -7$
5. $\left| \frac{2x-1}{x} \right| < 5$
 This inequality is nonlinear. It must be solved using the critical point/test point method. Critical points:
 Solve the corresponding equality.
 $\left| \frac{2x-1}{x} \right| = 5$
 $\frac{2x-1}{x} = 5$ or $\frac{2x-1}{x} = -5$
 $2x-1 = 5x$ or $2x-1 = -5x$
 $-1 = 3x$ or $7x = 1$
 $-\frac{1}{3} = x$ or $x = \frac{1}{7}$

Find zeros of denominators.

$x = 0.$

Critical points: $-\frac{1}{3}, 0, \frac{1}{7}$

We choose test points from each interval: $-1, -0.1, 0.1, 1$.

$\left| \frac{2x-1}{x} \right| < 5$

$x = -1: |3| < 5$; true

$x = -0.1: |12| < 5$; false

$x = 0.1: |-8| < 5$; false

$x = 1: |1| < 5$; true

The solution set is intervals I and IV:

$\{x | x < -\frac{1}{3} \text{ or } x > \frac{1}{7}\}.$

Solutions to trial exercise problems

7. [1] $-1 = -\frac{1}{2}x + 9y$
 [2] $\frac{57}{14} = \frac{1}{2}x + \frac{3}{14}y$
 Multiply [1] by 2 and multiply [2] by 14.
 [1] $-x + 18y = -2$
 [2] $7x + 3y = 57$
 Add 7 times [1] to [2].
 [3] $129y = 43$
 $y = \frac{43}{129}$
 Add -6 times [2] to [1].
 [4] $-43x = -344$
 $x = 8$
 $(8, \frac{1}{3})$
25. [1] $x + y - 5z = -9$
 [2] $-x + y + 2z = 9$
 [3] $5x + 2y = -4$
 Add [1] to [2].
 [4] $2y - 3z = 0$
 Add 5 times [2] to [3].
 [5] $7y + 10z = 41$
 Add 7 times [4] to -2 times [5].
 [6] $-41z = -82$
 $z = 2$
 [7] $2y - 6 = 0$
 Insert value of z into [4].
 $y = 3$
 Insert value of y and z into [1].
 [8] $x + 2 = 0$
 $x = -2$
 $(-2, 3, 2)$
41. $W = \frac{1}{2}L - 10$
 $P = 150 = 2L + 2W$
 $75 = L + W$
 Solve $\begin{cases} L - 2W = 20 \\ L + W = 75 \end{cases}$ to find $L = 56\frac{2}{3}$ mm, $W = 18\frac{1}{3}$ mm.
45. If the two investments are x and y then $x + y = 10,000$ and $0.06x + 0.12y = 900$, or $x + 2y = 15,000$, so we solve the system $\begin{cases} x + y = 10,000 \\ x + 2y = 15,000 \end{cases}$ to find $x = y = \$5,000$.
49. Since the points satisfy $y = mx + b$, we know $\begin{cases} -1 = 5m + b \\ 6 = 8m + b \end{cases}$, which we solve to find m and b : $m = \frac{7}{3}$, $b = -\frac{38}{3}$, so the equation is $y = \frac{7}{3}x - \frac{38}{3}$.

Exercise 10–2

Answers to odd-numbered problems

1. $(-3, 6)$ 3. $(\frac{1}{2}, 2)$ 5. $(3, 2)$
 7. $(5, -3)$ 9. $(8, -3)$ 11. $(6, 6)$
 13. $(-8, 3)$ 15. $(-6, 6)$ 17. $(5, 5)$
 19. $(\frac{7}{3}, 3)$ 21. $(6, -2)$
 23. $(-2, 3, 2)$ 25. $(6, -5, 2)$
 27. $(1, 5, -\frac{1}{3})$ 29. $(-\frac{4}{3}, 6, -1)$
 31. $(0, -5, 5)$ 33. $(-10, 3, -1)$
 35. $(3, 2, -2, 1)$ 37. $(1, -2, 3, 4)$
 39. $(1, -2, \frac{1}{3}, 2)$ 41. inconsistent
 43. $(5, -1, 2, -4)$ 45. $(3, -2, \frac{3}{4}, 1)$
 47. $i_1 = 2, i_2 = \frac{2}{3}$
 49. $I_1 = 180, I_2 = 200, I_3 = 210$
 51. $333\frac{1}{3}$ liters of the 20% solution and $166\frac{2}{3}$ liters of the 50% solution
 53. $49\frac{3}{13}$ gallons of 25% solution, and $30\frac{10}{13}$ gallons of 90% solution.
 55. $(\frac{27}{4}, \frac{17}{4}, \frac{11}{4})$

Solutions to skill and review problems

1. Solve the system
 [1] $2x - y = -3$
 [2] $2x - 3y = 6$
 $2y = -9 \leftarrow [1] - [2]$
 $y = -\frac{9}{2}$
 [1] $2x - (-\frac{9}{2}) = -3$
 $2x = -\frac{15}{2}$
 $x = -\frac{15}{4}$
 Multiply each member by $\frac{1}{2}$.
 $(-\frac{3}{4}, -\frac{9}{2})$

2. $3^4 = 81$, so $\log_3 81 = 4$

3. $9^{3x+1} = 27^x$

$(3^2)^{3x+1} = (3^3)^x$

$3^{6x+2} = 3^{3x}$

$6x + 2 = 3x$

$x = -\frac{2}{3}$

4. $\log(x-1) - \log(x+1) = 2$

$\log \frac{x-1}{x+1} = 2$ since $\log \frac{m}{n}$

$= \log m - \log n$

$\frac{x-1}{x+1} = 10^2$ because if $\log x = y$, then

$x = 10^y$

$x-1 = 100x+100$

$-\frac{101}{99} = x$

This solution makes both expressions $\log(x-1)$ or $\log(x+1)$ undefined, since $\log m$ is only defined if $m > 0$. Thus, there is no solution.

5. $\log(x-1) + \log(x+1) = \log 2$

$\log(x-1)(x+1) = \log 2$

$\log mn = \log m + \log n$

$(x-1)(x+1) = 2$

If $\log m = \log n$ then $m = n$.

$x^2 - 1 = 2$

$x^2 = 3$

$x = \pm\sqrt{3}$

The value $-\sqrt{3}$ makes $\log(x-1)$ and $\log(x+1)$ undefined, so the solution is $\sqrt{3}$.

6. $x^3 - x^2 - x + 1 < 0$

This is a nonlinear inequality. It must be solved by the critical point/test point method. Critical points:

Solve the corresponding equality.

$x^3 - x^2 - x + 1 = 0$

$x^2(x-1) - 1(x-1) = 0$

$(x-1)(x^2-1) = 0$

Factor by grouping.

$(x-1)(x-1)(x+1) = 0$

$x = \pm 1$.

Find zeros of denominators; in this case there are none.

Critical points are ± 1 . I | II | III
-2 -1 0 1 2

Choose test points in each interval and try in the original inequality.

$x^3 - x^2 - x + 1 < 0$

$x = -2: -9 < 0$; true

$x = 0: 1 < 0$; false

$x = 2: 3 < 0$; false

Thus the solution is interval I:

$x < -1$.

Solutions to trial exercise problems

7.
$$\begin{bmatrix} 2 & 1 & 1 \\ 5 & 3 & 1 \\ -2 & 2 & -16 \end{bmatrix}$$

Multiply [1] by 15. $\begin{bmatrix} 6 & 5 & 15 \\ -2 & 2 & -16 \end{bmatrix}$

Divide [2] by 2.

$\begin{bmatrix} 6 & 5 & 15 \\ -1 & 1 & -8 \end{bmatrix}$ [1] \leftarrow 6[2] + [1]

Note: This notation means to add 6 times row 2 to row 1 and replace 1 with this result.

$\begin{bmatrix} 0 & 11 & -33 \\ -1 & 1 & -8 \end{bmatrix}$ Divide [1] by 11.

$\begin{bmatrix} 0 & 1 & -3 \\ -1 & 1 & -8 \end{bmatrix}$ [2] \leftarrow [1] - [2]

$\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 5 \end{bmatrix}$

Rearrange rows and set coefficients to 1.

$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \end{bmatrix}$ Solution: (5, -3)

29.
$$\begin{bmatrix} -3 & 0 & 1 & 3 \\ 9 & 2 & 3 & -3 \\ 3 & 1 & -2 & 4 \end{bmatrix}$$

[2] \leftarrow 2[3] - [2]

$\begin{bmatrix} -3 & 0 & 1 & 3 \\ -3 & 0 & -7 & 11 \\ 3 & 1 & -2 & 4 \end{bmatrix}$

[2] \leftarrow 7[1] + [2]

[3] \leftarrow 2[1] + [3]

$\begin{bmatrix} -3 & 0 & 1 & 3 \\ -24 & 0 & 0 & 32 \\ -3 & 1 & 0 & 10 \end{bmatrix}$

Divide [2] by 8.

$\begin{bmatrix} -3 & 0 & 1 & 3 \\ -3 & 0 & 0 & 4 \\ -3 & 1 & 0 & 10 \end{bmatrix}$

[1] \leftarrow -[2] + [1]

[3] \leftarrow -[2] + [3]

$\begin{bmatrix} 0 & 0 & 1 & -1 \\ -3 & 0 & 0 & 4 \\ 0 & 1 & 0 & 6 \end{bmatrix}$

Rearrange rows and set coefficients to 1.

$\begin{bmatrix} 1 & 0 & 0 & -\frac{4}{3} \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 \end{bmatrix}$

Solution: $(-\frac{4}{3}, 6, -1)$

39.
$$\begin{bmatrix} 1 & \frac{1}{2} & 3 & -3 & -5 \\ 2 & -\frac{3}{2} & 3 & 5 & 16 \\ -6 & 0 & 0 & -1 & -8 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$$

Multiply [1] by 2, and [2] by 2.

$\begin{bmatrix} 2 & 1 & 6 & -6 & -10 \\ 4 & -3 & 6 & 10 & 32 \\ -6 & 0 & 0 & -1 & -8 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

[2] \leftarrow 3[1] + [2]

$\begin{bmatrix} 2 & 1 & 6 & -6 & -10 \\ 10 & 0 & 24 & -8 & 2 \\ -6 & 0 & 0 & -1 & -8 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

Divide [2] by 2.

$\begin{bmatrix} 2 & 1 & 6 & -6 & -10 \\ 5 & 0 & 12 & -4 & 1 \\ -6 & 0 & 0 & -1 & -8 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

[1] \leftarrow 2[4] - [1]

[2] \leftarrow 5[4] - [2]

[3] \leftarrow 6[4] + [3]

$\begin{bmatrix} 0 & -1 & -18 & 6 & 8 \\ 0 & 0 & -42 & 4 & -6 \\ 0 & 0 & -36 & -1 & -14 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

Divide [2] by 2.

$\begin{bmatrix} 0 & -1 & -18 & 6 & 8 \\ 0 & 0 & -21 & 2 & -3 \\ 0 & 0 & -36 & -1 & -14 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

[1] \leftarrow 6[3] + [1]

[2] \leftarrow 2[3] + [2]

$\begin{bmatrix} 0 & -1 & -234 & 0 & -76 \\ 0 & 0 & -93 & 0 & -31 \\ 0 & 0 & -36 & -1 & -14 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

Divide [2] by 31.

$\begin{bmatrix} 0 & -1 & -234 & 0 & -76 \\ 0 & 0 & -3 & 0 & -1 \\ 0 & 0 & -36 & -1 & -14 \\ 1 & 0 & -6 & 0 & -1 \end{bmatrix}$

[1] \leftarrow -78[2] + [1]

[3] \leftarrow -12[2] + [3]

[4] \leftarrow -2[2] + [4]

$\begin{bmatrix} 0 & -1 & 0 & 0 & 2 \\ 0 & 0 & -3 & 0 & -1 \\ 0 & 0 & 0 & -1 & -2 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$

Rearrange rows and set coefficients to 1.

$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$

Solution: $(1, -2, \frac{1}{3}, 2)$

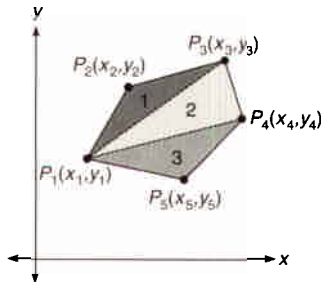
51. Let t = required amount of 20% solution and f = required amount of 50% solution. Then $t + f = 500$. Now, 30% of the 500 liters is to be alcohol, or 150 liters. This alcohol comes from 20% of t and 50% of f , so that we also have the equation $0.20t + 0.50f = 150$, or $2t + 5f = 1,500$. Thus, we solve
- $$\begin{aligned} t + f &= 500 \\ 2t + 5f &= 1,500 \end{aligned}$$
- for t and f . The solution is $\left(\frac{1,000}{3}, \frac{500}{3}\right)$, so we need $333\frac{1}{3}$ liters of the 20% solution and $166\frac{2}{3}$ liters of the 50% solution.

Exercise 10–3

Answers to odd-numbered problems

1. -9 3. $-42\frac{1}{2}$ 5. $-\pi$ 7. 149
 9. $\frac{34}{3}$ 11. -6 13. 7 15. 105
 17. $-54\sqrt{2}$ 19. -2 21. 74

81. The area of the five-sided polygon is the sum of the areas marked 1, 2, and 3 in the figure. Each of these is a triangle.



23. 406 25. -220 27. -112
 29. $x = -\frac{16}{31}, y = \frac{12}{31}$ 31. $x = -\frac{17}{18}, y = -\frac{5}{9}$ 33. $x = \frac{92}{19}, y = -\frac{71}{19}$
 35. $x = -\frac{59}{110}, y = \frac{17}{110}, z = -\frac{37}{55}$
 37. $x = -\frac{11}{10}, y = \frac{3}{20}, z = -\frac{29}{20}$
 39. $x = \frac{8}{5}, y = -\frac{1}{5}, z = -\frac{6}{5}$
 41. $x = -\frac{3}{4}, y = -\frac{19}{4}, z = -\frac{49}{4}$
 43. inconsistent 45. $x = -\frac{5}{4}, y = -\frac{27}{2}, z = 2, w = -\frac{51}{4}$ 47. $x = \frac{417}{2}, y = \frac{19}{2}, z = -\frac{1,329}{4}, w = -\frac{567}{2}$ 49. $x = \frac{17}{13}, y = -\frac{29}{13}, z = -\frac{14}{13}, w = -\frac{17}{26}$ 51. $\frac{87}{28}$
 53. $y = 1.16x + 0.57$ 55. $y = 2.13x - 8.18$ 57. The line is $y = 1.9x + 1$. For the fifth year the line predicts $y = 1.9(5) + 1 = 10.5\%$ failures, and for the sixth year it predicts $y = 1.9(6) + 1 = 12.4\%$ failures. 59. a. $y = -0.3426 + 0.8851x$. This assumes x is the year less 1,875, and y

- is the time in seconds less 4:24.5 (in seconds, or 264.5). b. 2,022 61. 47
 63. 31 65. $-7x + 3y + 23 = 0$
 67. $y = 0.8x^2 + 2.6x - 0.4$
 69. $y = \frac{7}{3}x - \frac{38}{3}$ 71. $(-\frac{57}{26}, -\frac{29}{26})$
 73. $i_1 = \frac{4}{3}, i_2 = -\frac{55}{3}, i_3 = \frac{134}{3}$
 75. $x = 9,000, y = 12,000, z = 15,000$
 77. (1,1) is the correct solution, which can be verified by substitution into the system itself.
 79. Define the determinant of an order 1 matrix as the value of the single element. This permits order 2 determinants to be evaluated by expansion about rows and columns, just like the determinants of order greater than 2.

$$\text{Area}_{\text{total}} = \text{Area}_1 + \text{Area}_2 + \text{Area}_3$$

$$= \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[(x_2y_3 - x_3y_2) - (x_1y_3 - x_3y_1) + (x_1y_2 - x_2y_1)] +$$

$$\frac{1}{2}[(x_3y_4 - x_4y_3) - (x_1y_4 - x_4y_1) + (x_1y_3 - x_3y_1)] +$$

$$\frac{1}{2}[(x_4y_5 - x_5y_4) - (x_1y_5 - x_5y_1) + (x_1y_4 - x_4y_1)]$$

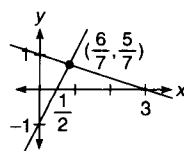
$$= \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_4 - x_4y_3) + (x_4y_5 - x_5y_4) + (x_5y_1 - x_1y_5)].$$

The solution for the four-sided figure is similar.

Solutions to skill and review problems

1. $2x - 3 < 8$
 $2x < 11$
 $x < 5\frac{1}{2}$
 2. $2x + y > 2, x = 2, y = -1$
 $2(2) + (-1) > 2$
 $3 > 2$; true
 Thus, $(2, -1)$ is a solution to the statement $2x + y > 2$.

3. $y = 2x - 1$ and
 $y = -\frac{1}{3}x + 1$ so
 $2x - 1 = -\frac{1}{3}x + 1$
 $6x - 3 = -x + 3$
 $7x = 6$
 $x = \frac{6}{7}$
 $y = 2x - 1 = 2(\frac{6}{7}) - 1 = \frac{5}{7}$.



4. $0.075(1,200) + 0.05(1,800) = \180
 5. $3(2x + 3) - 2(5x + 3) = x$
 $6x + 9 - 10x - 6 = x$
 $3 = 5x$
 $\frac{3}{5} = x$

6. $\frac{4x-1}{x} < -5x$

This is nonlinear. Use the critical point/test point method.

Critical points:

Solve corresponding equality:

$$\frac{4x-1}{x} = -5x$$

$$4x-1 = -5x^2$$

$$5x^2 + 4x - 1 = 0$$

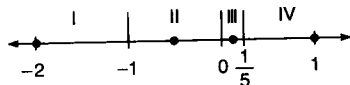
$$(5x-1)(x+1) = 0$$

$$5x-1 = 0 \text{ or } x+1 = 0$$

$$x = \frac{1}{5} \text{ or } x = -1$$

Find zeros of denominators: $x = 0$

Critical points: $-1, 0, \frac{1}{5}$.



Test points (one from each interval):

$-2, -0.5, 0.1, 1$

$$\frac{4x-1}{x} < -5x$$

$$x = -2: 4.5 < 10; \text{ true}$$

$$x = -0.5: 6 < 2.5; \text{ false}$$

$$x = 0.1: -6 < -0.5; \text{ true}$$

$$x = 1: 3 < -5; \text{ false}$$

The solution is intervals I and III:

$$x < -1 \text{ or } 0 < x < \frac{1}{5}.$$

7. $\log^2 x - \log x = 6$

Let $u = \log x$:

$$u^2 - u = 6$$

$$u^2 - u - 6 = 0$$

$$(u-3)(u+2) = 0$$

$$u = 3 \text{ or } u = -2$$

$$\log x = 3 \text{ or } \log x = -2$$

$$x = 10^3 \text{ or } x = 10^{-2}$$

8. $y = \log_2(x+1)$

Find inverse function.

$$x = \log_2(y+1)$$

$$y+1 = 2^x$$

Inverse function:

$$y = 2^x - 1$$

Computed values:

$$y = 2^x - 1 \quad y = \log_2(x+1)$$

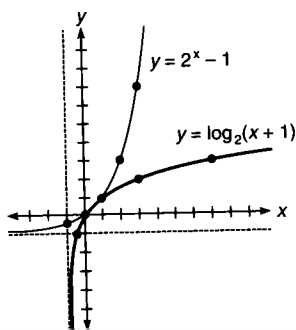
$$(-1, -\frac{1}{2}) \quad (-\frac{1}{2}, -1)$$

$$(0, 0) \quad (0, 0)$$

$$(1, 1) \quad (1, 1)$$

$$(2, 3) \quad (3, 2)$$

$$(3, 7) \quad (7, 3)$$



9. $y = x^2 + 3x - 4$

Parabola

$$y = x^2 + 3x - 4$$

Complete the square.

$$y = x^2 + 3x + \frac{9}{4} - 4 - \frac{9}{4}$$

$$\frac{1}{2} \cdot 3 = \frac{3}{2}; \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$y = \left(x + \frac{3}{2}\right)^2 - \frac{25}{4}$$

$$\text{Vertex: } \left(-1\frac{1}{2}, -6\frac{1}{4}\right)$$

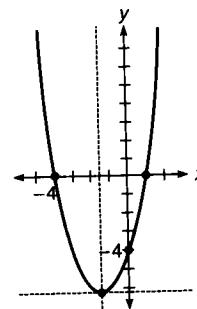
Intercepts:

$$x = 0: y = 0 + 0 - 4 = -4; (0, -4)$$

$$y = 0: 0 = x^2 + 3x - 4$$

$$0 = (x+4)(x-1)$$

$$x = -4 \text{ or } 1; (-4, 0), (1, 0)$$



Solutions to trial exercise problems

$$9. \begin{vmatrix} 2 & \frac{2}{3} & -1 \\ 4 & -1 & \frac{1}{2} \\ -3 & 0 & -2 \end{vmatrix} = -3 \begin{vmatrix} \frac{2}{3} & -1 \\ -1 & \frac{1}{2} \end{vmatrix} + (-2) \begin{vmatrix} 2 & \frac{2}{3} \\ 4 & -1 \end{vmatrix}$$

$$= -3\left(-\frac{2}{3}\right) - 2\left(-\frac{14}{3}\right) = \frac{34}{3}$$

$$23. \begin{vmatrix} 4 & 5 & 1 & 0 \\ -2 & 1 & 3 & 7 \\ 0 & 1 & 2 & 0 \\ 4 & -2 & 0 & 3 \end{vmatrix} = -1 \begin{vmatrix} 4 & 1 & 0 \\ -2 & 3 & 7 \\ 4 & 0 & 3 \end{vmatrix} + 2 \begin{vmatrix} 4 & 5 & 0 \\ -2 & 1 & 7 \\ 4 & -2 & 3 \end{vmatrix}$$

$$= -\left\{4 \begin{vmatrix} 3 & 7 \\ 0 & 3 \end{vmatrix} - \begin{vmatrix} -2 & 7 \\ 4 & 3 \end{vmatrix}\right\} + 2 \left\{4 \begin{vmatrix} 1 & 7 \\ -2 & 3 \end{vmatrix} - 5 \begin{vmatrix} -2 & 7 \\ 4 & 3 \end{vmatrix}\right\}$$

$$= -[4(9) - (-34)] + 2[4(17) - 5(-34)]$$

$$= -[70] + 2[238] = 406$$

$$37. D_x = \begin{vmatrix} -6 & -3 & -3 \\ 7 & -4 & -6 \\ -3 & 2 & 0 \end{vmatrix} = -132; D_y = \begin{vmatrix} 9 & -6 & -3 \\ 1 & 7 & -6 \\ 3 & -3 & 0 \end{vmatrix} = 18$$

$$D_z = \begin{vmatrix} 9 & -3 & -6 \\ 1 & -4 & 7 \\ 3 & 2 & -3 \end{vmatrix} = -174; D = \begin{vmatrix} 9 & -3 & -3 \\ 1 & -4 & -6 \\ 3 & 2 & 0 \end{vmatrix} = 120$$

$$x = \frac{D_x}{D} = -\frac{11}{10}, y = \frac{D_y}{D} = \frac{3}{20}, z = \frac{D_z}{D} = -\frac{29}{20}$$

$$45. D_x = \begin{vmatrix} 2 & 2 & 3 & -2 \\ -3 & 2 & 5 & -1 \\ -2 & 4 & -2 & -4 \\ 2 & 4 & 0 & -4 \end{vmatrix} = -40, D_y = \begin{vmatrix} 2 & 2 & 3 & -2 \\ -1 & -3 & 5 & -1 \\ -4 & -2 & -2 & -4 \\ -4 & 2 & 0 & -4 \end{vmatrix} = -432$$

$$D_z = \begin{vmatrix} 2 & 2 & 2 & -2 \\ -1 & 2 & -3 & -1 \\ -4 & 4 & -2 & -4 \\ -4 & 4 & 2 & -4 \end{vmatrix} = 64, D_w = \begin{vmatrix} 2 & 2 & 3 & 2 \\ -1 & 2 & 5 & -3 \\ -4 & 4 & -2 & -2 \\ -4 & 4 & 0 & 2 \end{vmatrix} = -408$$

$$D = \begin{vmatrix} 2 & 2 & 3 & -2 \\ -1 & 2 & 5 & -1 \\ -4 & 4 & -2 & -4 \\ -4 & 4 & 0 & -4 \end{vmatrix} = 32, x = -\frac{5}{4}, y = -\frac{27}{2}, z = 2, w = -\frac{51}{4}$$

$$52. D = \begin{vmatrix} 2 & -1 & 3 & -1 & 0 \\ 1 & 1 & 0 & -2 & 1 \\ 0 & -1 & -1 & 0 & 0 \\ 3 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 0 & -1 \end{vmatrix} = 1 \begin{vmatrix} 2 & -1 & -1 & 0 \\ 1 & 1 & -2 & 1 \\ 0 & -1 & 0 & 0 \\ 3 & 0 & 1 & 1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -1 & 3 & -1 \\ 1 & 1 & 0 & -2 \\ 0 & -1 & -1 & 0 \\ 3 & 0 & -1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} 2 & -1 & -1 & 0 \\ 1 & 1 & -2 & 1 \\ 0 & -1 & 0 & 0 \\ 3 & 0 & 1 & 1 \end{vmatrix} = -(-1) \begin{vmatrix} 2 & -1 & 0 \\ 1 & -2 & 1 \\ 3 & 1 & 1 \end{vmatrix} = (1) \left[2 \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} - (-1) \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} \right] = 2(-3) + (-2) = -8,$$

$$\begin{vmatrix} 2 & -1 & 3 & -1 \\ 1 & 1 & 0 & -2 \\ 0 & -1 & -1 & 0 \\ 3 & 0 & -1 & 1 \end{vmatrix} = -(3) \begin{vmatrix} -1 & 3 & -1 \\ 1 & 0 & -2 \\ -1 & -1 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 2 & -1 & -1 \\ 1 & 1 & -2 \\ 0 & -1 & 0 \end{vmatrix} + \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 0 \\ 0 & -1 & -1 \end{vmatrix}$$

$$= -3 \left[(-1) \begin{vmatrix} 3 & -1 \\ 0 & -2 \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix} \right] + \left[-(-1) \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} \right] + \left[-(-1) \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \right]$$

$$= -3[-1(-6) - (-1)(3)] + [(-1)(-3)] + [(-3) + (-1)(3)] = -3[9] + [-3] + [-6] = -36,$$

$$\text{so } D = 1(-8) - 1(-36) = 28$$

$$D_E = \begin{vmatrix} 2 & -1 & 3 & -1 & 5 \\ 1 & 1 & 0 & -2 & 0 \\ 0 & -1 & -1 & 0 & 10 \\ 3 & 0 & -1 & 1 & -4 \\ 0 & 0 & 1 & 0 & -20 \end{vmatrix} = 1 \begin{vmatrix} 2 & -1 & -1 & 5 \\ 1 & 1 & -2 & 0 \\ 0 & -1 & 0 & 10 \\ 3 & 0 & 1 & -4 \end{vmatrix} + (-20) \begin{vmatrix} 2 & -1 & 3 & -1 \\ 1 & 1 & 0 & -2 \\ 0 & -1 & -1 & 0 \\ 3 & 0 & -1 & 1 \end{vmatrix}$$

$$= (-73) - 20(-36) = 647$$

$$\text{Therefore, } E = \frac{D_E}{D} = \frac{647}{28}$$

$$55. (1.5, -4.8), (2, -4.0), (3, -2.0), (4.5, 1.4), (6, 4.7), (6.5, 5.6)$$

$$X = 1.5 + 2 + 3 + 4.5 + 6 + 6.5 = 23.5$$

$$Y = -4.8 - 4 - 2 + 1.4 + 4.7 + 5.6 = 0.9$$

$$P = (1.5)(-4.8) + (2)(-4.0) + (3)(-2.0) + (4.5)(1.4) + (6)(4.7) + (6.5)(5.6) = 49.7$$

$$S = 1.5^2 + 2^2 + 3^2 + 4.5^2 + 6^2 + 6.5^2 = 113.75$$

$$N = 6$$

$$\text{Solve } \begin{matrix} 23.5m + 6b = 0.9 \\ 113.75m + 23.5b = 49.7 \end{matrix}$$

$$D_m = \begin{vmatrix} 0.9 & 6 \\ 49.7 & 23.5 \end{vmatrix} = -277.05$$

$$D_b = \begin{vmatrix} 23.5 & 0.9 \\ 113.75 & 49.7 \end{vmatrix} = 1065.575$$

$$D = \begin{vmatrix} 23.5 & 6 \\ 113.75 & 23.5 \end{vmatrix} = -130.25$$

$$m = \frac{D_m}{D} \approx 2.13, b = \frac{D_b}{D} \approx -8.18, \text{ so the line is } y = 2.13x - 8.18.$$

64. We know that one equation for a straight line is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ if $x_2 \neq x_1$.

We can transform this to $(y - y_1)(x_2 - x_1)$

$= (y_2 - y_1)(x - x_1)$, which is true even when $x_2 = x_1$.

$$x_2y - x_1y - x_2y_1 + x_1y_1 = xy_2 - x_1y_2 - xy_1 + x_1y_1$$

$$x_2y - x_1y - x_2y_1 = xy_2 - x_1y_2 - xy_1$$

We put the terms on the same side of the equation, in descending order of variable names and subscripts, to make comparison with other equations easier:

$$-x_2y_1 + x_2y + x_1y_2 - x_1y - xy_2 + xy_1 = 0$$

Expanding $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}$ will give the left member of this last equation.

71. First, find the equation of the line passing through $(-2, -1)$ and $(3, 2)$:

$$\text{Solve } \begin{cases} -1 = -2m + b \\ 2 = 3m + b \end{cases}; m = \frac{3}{5}, b = \frac{1}{5}, \text{ so the first line is } y = \frac{3}{5}x + \frac{1}{5}.$$

$$\text{Now, the second line, through } (-6, 2) \text{ and } (5, -7): \text{ Solve } \begin{cases} 2 = -6m + b \\ -7 = 5m + b \end{cases};$$

$$m = -\frac{9}{11}, b = -\frac{32}{11}, \text{ so the second line is } y = -\frac{9}{11}x - \frac{32}{11}.$$

To find the point of intersection we solve the system $\begin{cases} y = \frac{3}{5}x + \frac{1}{5} \\ y = -\frac{9}{11}x - \frac{32}{11} \end{cases}$.

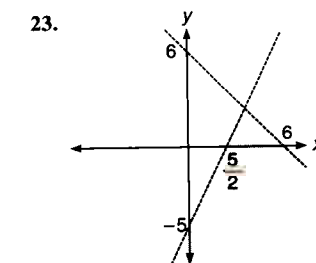
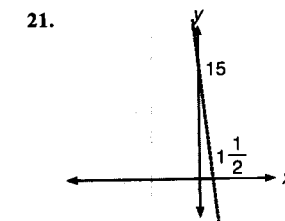
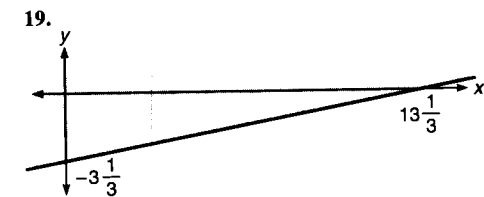
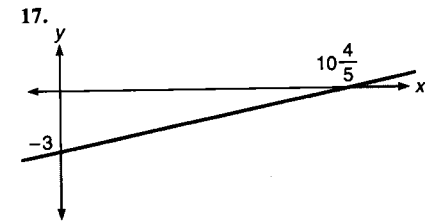
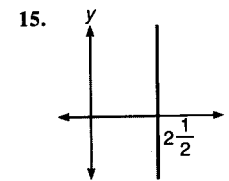
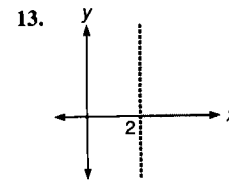
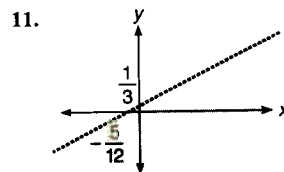
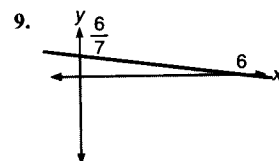
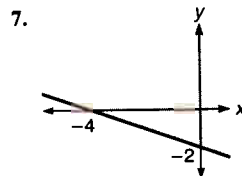
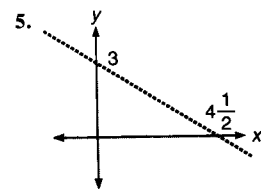
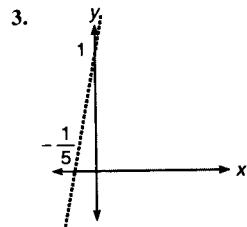
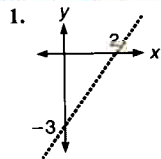
To use Cramer's rule it is easier to rewrite this system as $\begin{cases} 3x - 5y = -1 \\ 9x + 11y = -32 \end{cases}$.

$$D = \begin{vmatrix} 3 & -5 \\ 9 & 11 \end{vmatrix} = 78, D_x = \begin{vmatrix} -1 & -5 \\ -32 & 11 \end{vmatrix} = -171,$$

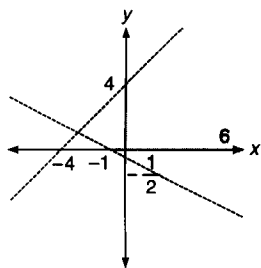
$$D_y = \begin{vmatrix} 3 & -1 \\ 9 & -32 \end{vmatrix} = -87, x = -\frac{57}{26}, y = -\frac{29}{26}, \text{ so the point is } (-\frac{57}{26}, -\frac{29}{26}).$$

Exercise 10-4

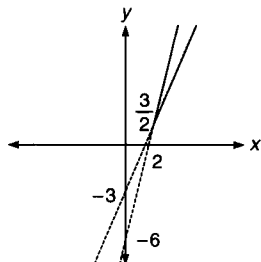
Answers to odd-numbered problems



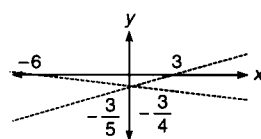
25.



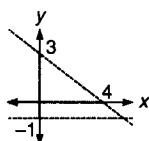
27.



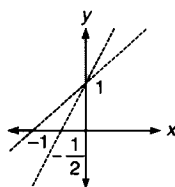
29.



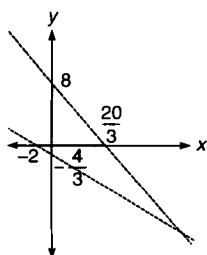
31.



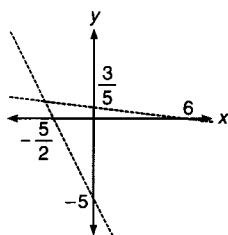
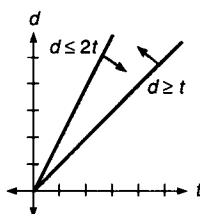
33.



35.



37.

39. a. $r \leq 1.5 + 0.5 = 2$, so $d \leq 2t$.b. Graph the system $d \geq t$ and $d \leq 2t$.41. $P = 12$ at $(6, 0)$ 43. $P = 16$ at $(2, 4)$ 45. $P = 12$ at $(6, 0)$ 47. $P = 24\frac{1}{2}$ at $(6, \frac{25}{8})$ 49. $P = 4$ at $(12, 0)$ 51. $P = 7$ at $(7, 0)$ 53. $P = 14$ at $(4, 8)$ 55. $P = 9$ at $(5, 2)$ 57. $P = 18$ at $(9, 0)$ 59. $P = 22$ at $(7\frac{1}{3}, 0)$ 61. $P = 1\frac{1}{2}$ at $(7\frac{1}{2}, 0)$ 63. $P = 36$ at $(6, 2)$ or $(7\frac{1}{5}, 0)$ 65. $P = 24$ at $(8, 0)$ 67. $C = 6$ at $(0, 6)$ 69. $C = 16$ at $(2, 4)$ 71. $C = 8\frac{2}{3}$ at $(3\frac{1}{2}, 5)$ 73. $C = 7$ at $(5, 1)$ 75. $C = 5\frac{3}{5}$ at $(1\frac{3}{5}, 2\frac{2}{5})$ 77. $C = 12\frac{2}{5}$ at $(5\frac{1}{5}, 3\frac{1}{5})$

79. The maximum income is \$2,980 and

comes from producing 20 tables and

240 chairs.

81. Production is maximized at 315 tons

with 5 type-A crews and 10 type-B

crews.

83. We minimize cost at $52\frac{16}{17}$ cents byusing $\frac{36}{17}$ lb of A and $\frac{20}{17}$ lb of B.

Solutions to skill and review problems

$$1. \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$2. \left| 3 - \frac{\sqrt{2}}{2} \right| = 3 - \frac{\sqrt{2}}{2},$$

since $|x| = x$ if $x \geq 0$

$$3. |4 - x| < 10$$

$$-10 < 4 - x < 10$$

$$-14 < -x < 6$$

$$14 > x > -6$$

$$-6 < x < 14$$

$$4. 2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

$$1\frac{1}{2}$$

$$5. 2x^2 - 3x = 5$$

$$2x^2 - 3x - 5 = 0$$

$$(2x - 5)(x + 1) = 0$$

$$2x - 5 = 0 \text{ or } x + 1 = 0$$

$$2x = 5 \text{ or } x = -1$$

$$x = \frac{5}{2} \text{ or } x = -1$$

$$6. |2x^2 - 3x| = 5$$

$$2x^2 - 3x = 5$$

$$x = -1 \text{ or } 2\frac{1}{2} \text{ (problem 5)}$$

$$2x^2 - 3x = -5$$

$$2x^2 - 3x + 5 = 0$$

$$x = \frac{3}{4} \pm \frac{\sqrt{31}}{4}i, \text{ quadratic formula}$$

$$x = -1, 2\frac{1}{2}, \frac{3}{4} \pm \frac{\sqrt{31}}{4}i$$

$$7. \sqrt[3]{16x^5y^2z}$$

$$\sqrt[3]{2^4x^5y^2z}$$

$$\sqrt[3]{2^3x^3\sqrt[3]{2x^2y^2z}}$$

$$2x\sqrt[3]{2x^2y^2z}$$

$$8. f(x) = |x - 2|$$

This is the graph of $y = |x|$ shifted

two units to the right.

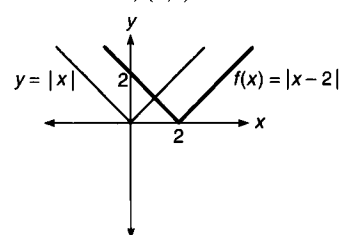
Intercepts:

$$x = 0: y = |-2| = 2; (0, 2)$$

$$y = 0: 0 = |x - 2|$$

$$0 = x - 2$$

$$2 = x; (2, 0)$$

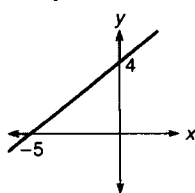


Solutions to trial exercise problems

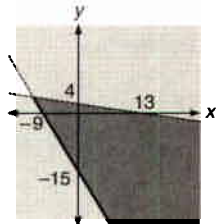
22. $(x + 5)(y - 4) \geq xy$

$xy - 4x + 5y - 20 \geq xy$

$-4x + 5y - 20 \geq 0$

Thus the solution is also described by the statement $-4x + 5y - 20 \geq 0$.Graph $-4x + 5y = 20$.Test point $(0,0)$: $-20 \geq 0$; false

30.

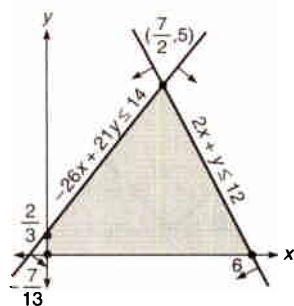


45. $-26x + 21y \leq 14$

$2x + y \leq 12$

x	y	P
0	0	0
0	$\frac{2}{3}$	$\frac{2}{3}$
$\frac{7}{2}$	5	$8\frac{2}{3}$
6	0	12

Solution:

 $P = 12$ at $(6,0)$ 

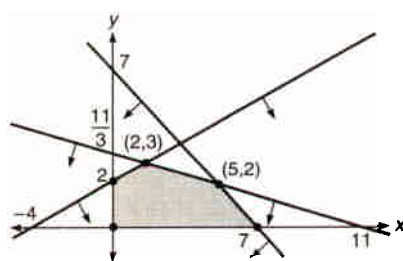
55. $-x + 2y \leq 4$

$x + 3y \leq 11$

$x + y \leq 7$

$P = x + 2y$

Solution:

 $P = 9$ at $(5,2)$ 

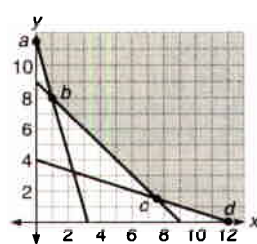
74. $x + y \geq 9$

$\frac{7}{2}x + y \geq \frac{23}{2}$

$x + 3y \geq 12$

$C = 2x + 3y$

	x	y	C
a	0	11.5	34.5
b	1	8	26
c	7.5	1.5	19.5
d	12	0	24

 C is a minimum of 19.5 at $(7.5, 1.5)$.

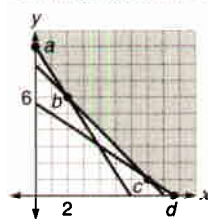
76. $x + y \geq 8$

$2x + 3y \geq 17$

$\frac{3}{2}x + y \geq 9$

$C = 5x + 4y$

	x	y	C
a	0	9	36
b	2	6	34
c	7	1	39
d	8.5	0	42.5

 C is a minimum of 34 at $(2,6)$.

$x + 2y = P$

x y P

0 0 0

0 2 4

2 3 8

5 2 9

7 0 7

79. Let x = number of tables to produce per production run and y = number of chairs, and C = income per production run. Then $C = 29x + 10y$. The number of hours required to produce x tables is $3x$, and for chairs it is y hours. Therefore, $3x + y \leq 300$. The restrictions on finishing are $2x + \frac{5}{6}y \leq 200$.

Thus, we have the system

$3x + y \leq 300$

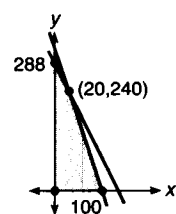
$2x + \frac{5}{6}y \leq 240$

$C = 29x + 10y$

$C(0,0) = 0$, $C(0,288) = 2,880$,

$C(20,240) = 2,980$, $C(100,0) = 2,900$.

The maximum value for C is 2,980 at $(20,240)$. Thus, the maximum income is \$2,980 and comes from producing 20 tables and 240 chairs.



83. x = amount of A, y = amount of B.

Based on protein we need $5x + 8y \geq 20$, and based on carbohydrates we need $4x + 3y \geq 12$. Total cost C is $C = 15x + 18y$. Minimize C for the system

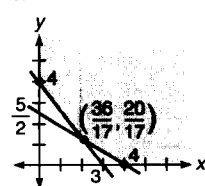
$5x + 8y \geq 20$

$4x + 3y \geq 12$

$C = 15x + 18y$

$C(0,4) = 72$, $C(\frac{36}{17}, \frac{20}{17}) = 52\frac{16}{17}$,

$C(4,0) = 60$. Thus, we minimize cost at $52\frac{16}{17}$ cents by using $\frac{36}{17}$ lb of A and $\frac{20}{17}$ lb of B.



Exercise 10-5

Answers to odd-numbered problems

1. $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

5. $\begin{bmatrix} -1 & 1 & -3 \\ 4 & -4 & 2 \end{bmatrix}$

7. $\begin{bmatrix} -3 & -1 & 4 \\ -4 & -18 & 7 \end{bmatrix}$

$$9. \begin{bmatrix} -1 & 1 \\ -3 & 4 \\ -4 & 2 \end{bmatrix} \quad 11. \begin{bmatrix} -3 & -1 \\ 4 & -4 \\ -18 & 7 \end{bmatrix}$$

$$13. \begin{bmatrix} -4 & 8 \\ 16 & 20 \end{bmatrix} \quad 15. \begin{bmatrix} 0 & -15 \\ 10 & -25 \end{bmatrix}$$

$$17. \begin{bmatrix} 2 & \frac{1}{2} & -1 \\ 1 & 3 & -1 \end{bmatrix} \quad 19. \begin{bmatrix} -2 & 13 \\ -5 & 0 \\ -3 & -2 \end{bmatrix}$$

$$21. -26 \quad 23. -2 \quad 25. 6 - \pi$$

27. There are an unlimited number of solutions; an obvious one is $[0, 1, 0, 0]$.

$$29. \begin{bmatrix} 1 & -3 \\ 5 & -21 \end{bmatrix} \quad 31. \begin{bmatrix} -7 & 15 \\ -10 & 18 \end{bmatrix}$$

$$33. \begin{bmatrix} 9 & 9 & 2 \\ 13 & -2 & -9 \\ 27 & 0 & -4 \end{bmatrix} \quad 35. \begin{bmatrix} 5 & 1 \\ -25 & 16 \\ -2 & -1 \end{bmatrix}$$

$$37. \begin{bmatrix} 6 & -2 \\ -6 & -8 \end{bmatrix} \quad 39. \begin{bmatrix} 47 & 1 \\ 70 & -22 \end{bmatrix}$$

$$41. \begin{bmatrix} -7 & -30 & 22 \\ 8 & 33 & -26 \end{bmatrix}$$

$$43. \begin{bmatrix} -20x^2 + y & 15x + 9 \\ -16xy - 3y & 12y - 27 \end{bmatrix}$$

$$45. \begin{bmatrix} 19 & -2 \\ 13 & -55 \end{bmatrix} \quad 47. \begin{bmatrix} 15 & -13 & 8 \\ -74 & -34 & -36 \end{bmatrix}$$

$$49. AB = \begin{bmatrix} 7 & -13 & 10 & 13 \\ 6 & -2 & -28 & 34 \\ -17 & 35 & -38 & -23 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 93 & 63 & -69 \\ 226 & -42 & -114 \\ -171 & -189 & 147 \end{bmatrix}$$

$$BC = \begin{bmatrix} 49 & 21 & -33 \\ 5 & -21 & 3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 93 & 63 & -69 \\ 226 & -42 & -114 \\ -171 & -189 & 147 \end{bmatrix}, \text{ so}$$

$$(AB)C = A(BC).$$

$$51. \begin{bmatrix} \frac{1}{27} & -\frac{5}{27} \\ -\frac{1}{9} & -\frac{4}{9} \end{bmatrix} \quad 53. \begin{bmatrix} -\frac{3}{10} & \frac{1}{20} \\ \frac{1}{5} & \frac{3}{10} \end{bmatrix}$$

$$55. \begin{bmatrix} \frac{1}{10} & \frac{2}{5} & \frac{1}{10} \\ -\frac{1}{3} & 0 & 0 \\ \frac{1}{5} & -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

$$57. \begin{bmatrix} \frac{1}{7} & \frac{3}{7} & 0 \\ -\frac{2}{21} & -\frac{2}{7} & \frac{1}{3} \\ \frac{2}{7} & -\frac{1}{7} & 0 \end{bmatrix}$$

$$59. \begin{bmatrix} -\frac{5}{37} & \frac{1}{37} & -\frac{7}{37} & 0 \\ -\frac{13}{37} & \frac{10}{37} & \frac{4}{37} & 0 \\ -\frac{11}{222} & \frac{17}{222} & -\frac{4}{111} & -\frac{1}{6} \\ -\frac{1}{37} & \frac{15}{37} & \frac{6}{37} & 0 \end{bmatrix}$$

$$61. \begin{bmatrix} -3 & \frac{6}{7} & -\frac{11}{7} & 2 \\ 3 & -1 & 2 & -2 \\ -\frac{1}{2} & \frac{1}{7} & -\frac{3}{7} & \frac{1}{2} \\ 3 & -\frac{4}{7} & \frac{12}{7} & -2 \end{bmatrix}$$

$$63. x = \frac{1}{3}, y = 2 \quad 65. x = 2, y = -2$$

$$67. x = 2, y = -1, z = 3$$

$$69. x = -3, y = \frac{2}{3}, z = -1$$

$$71. x = 1, y = -2, z = 2, w = 1$$

$$73. x = 1, y = 3, z = -1, w = 2$$

$$75. x = -3, y = 3 \quad 77. x = \frac{1}{2}, y = 2$$

$$79. x = -2, y = 3, z = 2$$

$$81. x = 2, y = -4, z = \frac{1}{2}$$

$$83. \begin{bmatrix} -24 & 18 \\ 32 & -14 \end{bmatrix} \quad 85. \begin{bmatrix} 4 & 3 \\ 10 & 5 \end{bmatrix}$$

$$87. \begin{bmatrix} 34 & -42 \\ -35 & 55 \end{bmatrix} \quad 89. \begin{bmatrix} 4 & 1 & 2 & 4 \\ 1 & 2 & 2 & 2 \\ 2 & 2 & 3 & 3 \\ 4 & 2 & 3 & 5 \end{bmatrix}$$

$$91. \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}; \text{ The row 1 column 2 entry is 1 so there is one path of length 2 from node 1 to node 2.}$$

$$93. \begin{bmatrix} 1 & \boxed{1} & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}; \text{ 1 path of length 3 from node 1 to node 2.}$$

$$95. \begin{bmatrix} \frac{3}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

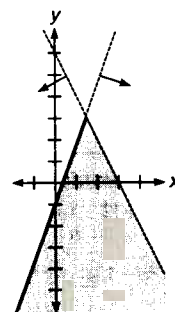
The probability that a mouse which started in room 2 is in room 3 after two moves is $\frac{1}{4}$, the entry in row 2 column 3.

$$97. L^3V = \begin{bmatrix} 6,498 \\ 296 \\ 200 \end{bmatrix}$$

Thus, there are 6,498, 296, and 200 females in each stage after three life cycles.

Solutions to skill and review problems

1. Graph the lines $2x + y = 6$ and $y = 3x - 1$. Use a test point for each half-plane. The origin $(0,0)$ is the easiest. Since $y = 3x - 1$ is part of the solution set where it overlaps the solution to $2x + y < 6$, this part of the line is made darker.



2. Use equation [1] to remove the variable z from equations [2] and [3]:
- [1] $2x - y - 2z = -7$
 [2] $x + y + 4z = 2$
 [3] $3x + 2y - 2z = -3$
- Add twice [1] to [2].
 [4] $5x - y = -12$
 Subtract [1] from [3].
 [5] $x + 3y = 4$
- Remove y from equations [4] and [5]:
 Add 3 times [4] to [5].
 $16x = -32$
 $x = -2$
 Substitute x into equation [4]:
 $5(-2) - y = -12$
 $2 = y$
 Substitute x and y into equation [1]:
 $2(-2) - 2 - 2z = -7$
 $1 = 2z$
 $\frac{1}{2} = z$
 Thus, $x = -2, y = 2, z = \frac{1}{2}$.
3. Solve
- [1] $2x + 3y = -6$
 [2] $x - 4y = 8$
 Subtract twice [2] from [1].
 $11y = -22$
 $y = -2$
 Substitute y into equation [2]:
 $x - 4(-2) = 8$
 $x = 0$
 Thus $x = 0, y = -2$, and the point is $(0, -2)$.

4. Let $P_1 = (x_1, y_1) = (-2, 4)$; $P_2 = (x_2, y_2) = (3, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{3 - (-2)} = \frac{4}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{4}{5}(x - (-2))$$

$$5y - 20 = 4(x + 2)$$

$$5y - 4x - 28 = 0$$

5. $(x - h)^2 + (y - k)^2 = r^2$, where (h, k)

is the center and r = radius.

r = distance from center $(-2, 4)$ to

$(3, 8)$

$$= \sqrt{(3 - (-2))^2 + (8 - 4)^2}$$

$$= \sqrt{41}$$

$$(x - (-2))^2 + (y - 4)^2 = (\sqrt{41})^2$$

$$(x + 2)^2 + (y - 4)^2 = 41$$

$$6. \sqrt{\frac{3}{8x^2y}} = \frac{\sqrt{3}}{\sqrt{8x^2y}} = \frac{\sqrt{3}}{2x^2\sqrt{2xy}}$$

$$\frac{\sqrt{2xy}}{\sqrt{2xy}} = \frac{\sqrt{6xy}}{2x^2(2xy)} = \frac{\sqrt{6xy}}{4x^3y}$$

7. $81x^4 - 1$

$$(9x^2 - 1)(9x^2 + 1)$$

$$(3x - 1)(3x + 1)(9x^2 + 1)$$

$$8. \frac{3a}{2b} - \frac{5a}{3c} + \frac{1}{a}$$

$$\frac{3a(3c) - 5a(2b)}{2b(3c)} + \frac{1}{a}$$

$$\frac{9ac - 10ab}{6bc} + \frac{1}{a}$$

$$\frac{a(9ac - 10ab) + 1(6bc)}{a(6bc)}$$

$$\frac{9a^2c - 10a^2b + 6bc}{6abc}$$

Solutions to trial exercise problems

$$8. \begin{bmatrix} -5 & 2 & 6 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 13 & -12 & 5 \\ 10 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -5 + 13 & 2 - 12 & 6 + 5 \\ 1 + 10 & 2 + 1 & 1 + 0 \end{bmatrix} = \begin{bmatrix} 8 & -10 & 11 \\ 11 & 3 & 1 \end{bmatrix}$$

$$18. \frac{2}{3} \begin{bmatrix} -15 & 9 & 3 \\ 1 & 6 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}(-15) & \frac{2}{3}(9) & \frac{2}{3}(3) \\ \frac{2}{3}(1) & \frac{2}{3}(6) & \frac{2}{3}(1) \end{bmatrix} = \begin{bmatrix} -10 & 6 & 2 \\ \frac{2}{3} & 4 & \frac{2}{3} \end{bmatrix}$$

28. We want a vector $[a, b, c, d]$ such that $[5, 2, -4, 3][a, b, c, d] = \frac{1}{2}$. Of the unlimited number of possibilities, an obvious choice is $[0, \frac{1}{4}, 0, 0]$. Another is $[0, 0, -\frac{1}{8}, 0]$.

$$35. \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & -2 \\ -3 & 5 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 15 & -2 \end{bmatrix} = \begin{bmatrix} (1)(4) + (-1)(-1) + (0)(15) & (1)(3) + (-1)(2) + (0)(-2) \\ (2)(4) + (3)(-1) + (-2)(15) & (2)(3) + (3)(2) + (-2)(-2) \\ (-3)(4) + (5)(-1) + (1)(15) & (-3)(3) + (5)(2) + (1)(-2) \end{bmatrix} = \begin{bmatrix} 5 & 1 \\ -25 & 16 \\ -2 & -1 \end{bmatrix}$$

$$61. \begin{bmatrix} 0 & 0 & 4 & 1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 3 & 2 & 0 & 1 & 0 & 0 & 1 & 0 \\ 2 & 2 & 6 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{array}{l} [4] \leftarrow 3[1] + -2[4] \end{array}$$

$$\begin{bmatrix} 0 & 0 & 4 & 1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 3 & 2 & 0 & 1 & 0 & 0 & 1 & 0 \\ -4 & -4 & 0 & 1 & 3 & 0 & 0 & -2 \end{bmatrix} \begin{array}{l} [3] \leftarrow 2[2] + 1[3] \\ [4] \leftarrow -4[2] + 1[4] \end{array}$$

$$\begin{bmatrix} 0 & 0 & 4 & 1 & 1 & 0 & 0 & 0 \\ 2 & -1 & 0 & 3 & 0 & 1 & 0 & 0 \\ 7 & 0 & 0 & 7 & 0 & 2 & 1 & 0 \\ -12 & 0 & 0 & -11 & 3 & -4 & 0 & -2 \end{bmatrix} \begin{array}{l} [2] \leftarrow 2[3] + -7[2] \\ [4] \leftarrow 12[3] + 7[4] \end{array}$$

$$\begin{bmatrix} 0 & 0 & 4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 7 & 0 & -7 & 0 & -3 & 2 & 0 \\ 7 & 0 & 0 & 7 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 7 & 21 & -4 & 12 & -14 \end{bmatrix} \begin{array}{l} [1] \leftarrow 1[4] + -7[1] \\ [2] \leftarrow 1[4] + 1[2] \\ [3] \leftarrow 1[4] + -1[3] \end{array}$$

$$\begin{bmatrix} 0 & 0 & -28 & 0 & 14 & -4 & 12 & -14 \\ 0 & 7 & 0 & 0 & 21 & -7 & 14 & -14 \\ -7 & 0 & 0 & 0 & 21 & -6 & 11 & -14 \\ 0 & 0 & 0 & 7 & 21 & -4 & 12 & -14 \end{bmatrix} \begin{array}{l} \text{Rearrange rows and} \\ \text{set coefficients to 1} \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 & \frac{6}{7} & -\frac{11}{7} & 2 \\ 0 & 1 & 0 & 0 & 3 & -1 & 2 & -2 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{7} & -\frac{3}{7} & \frac{1}{2} \\ 0 & 0 & 0 & 1 & 3 & -\frac{4}{7} & \frac{12}{7} & -2 \end{bmatrix}$$

$$\text{The inverse is } \begin{bmatrix} -3 & \frac{6}{7} & -\frac{11}{7} & 2 \\ 3 & -1 & 2 & -2 \\ -\frac{1}{2} & \frac{1}{7} & -\frac{3}{7} & \frac{1}{2} \\ 3 & -\frac{4}{7} & \frac{12}{7} & -2 \end{bmatrix}$$

71. The inverse matrix is the answer to problem 59.

$$\begin{bmatrix} -\frac{5}{37} & \frac{1}{37} & -\frac{7}{37} & 0 \\ -\frac{13}{37} & \frac{10}{37} & \frac{4}{37} & 0 \\ -\frac{11}{222} & \frac{17}{222} & -\frac{4}{111} & -\frac{1}{6} \\ -\frac{1}{37} & \frac{15}{37} & \frac{6}{37} & 0 \end{bmatrix} \begin{bmatrix} 8 \\ 7 \\ -10 \\ -9 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 1 \end{bmatrix};$$

81. The inverse of $\begin{bmatrix} -1 & -1 & -2 \\ 1 & 1 & -4 \\ 0 & -\frac{1}{2} & 2 \end{bmatrix}$ is $\begin{bmatrix} 0 & 1 & 2 \\ -\frac{2}{3} & -\frac{2}{3} & -2 \\ -\frac{1}{6} & -\frac{1}{6} & 0 \end{bmatrix}$.

$$\begin{bmatrix} 0 & 1 & 2 \\ -\frac{2}{3} & -\frac{2}{3} & -2 \\ -\frac{1}{6} & -\frac{1}{6} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ \frac{1}{2} \end{bmatrix}; x = 2, y = -4, z = \frac{1}{2}$$

$$\begin{aligned}
 86. & -3 \begin{bmatrix} 2 & -3 \\ -1 & 4 \end{bmatrix}^2 - 2 \begin{bmatrix} 0 & 3 \\ 4 & -1 \end{bmatrix}^2 - 5 \begin{bmatrix} 0 & 3 \\ 4 & -1 \end{bmatrix} \\
 & = -3 \begin{bmatrix} 7 & -18 \\ -6 & 19 \end{bmatrix} - 2 \begin{bmatrix} 12 & -3 \\ -4 & 13 \end{bmatrix} - \begin{bmatrix} 0 & 15 \\ 20 & -5 \end{bmatrix} \\
 & = \begin{bmatrix} -21 & 54 \\ 18 & -57 \end{bmatrix} - \begin{bmatrix} 24 & -6 \\ -8 & 26 \end{bmatrix} - \begin{bmatrix} 0 & 15 \\ 20 & -5 \end{bmatrix} = \begin{bmatrix} -45 & 45 \\ 6 & -78 \end{bmatrix}
 \end{aligned}$$

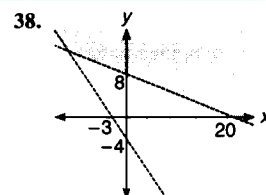
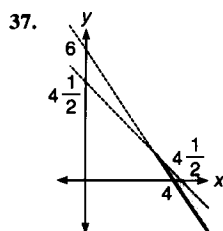
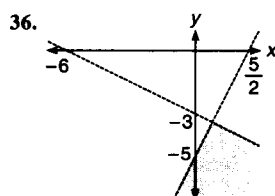
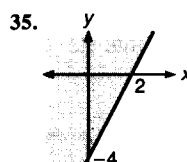
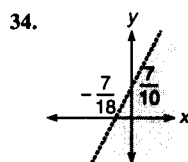
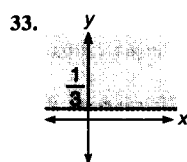
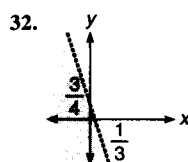
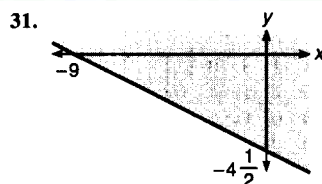
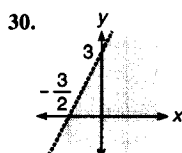
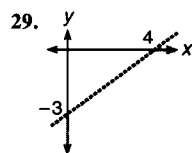
$$91. \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}; \text{ The row 1 column 2 entry is 1 so there is one path of length 2 from node 1 to node 2.}$$

$$95. A^2 = AA = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \end{bmatrix} \text{ The probability that a mouse that started in room 2 is in room 3 after two moves is } \frac{1}{4}, \text{ the entry in row 2 column 3.}$$

Chapter 10 review

1. $(-3, \frac{3}{2})$ 2. $(1, 2)$ 3. $(-12, 2)$
4. $(-8, \frac{3}{5})$ 5. $(-2, 3, 1)$ 6. $(1, -3, -2)$
7. $(3, 2, -2, 1)$ 8. $(-1, -2, 3, 1)$
9. $L = 56$ cm, $W = 35$ cm
10. $L = 52$ in., $W = 27$ in.
11. \$4,500 at 6%, \$10,500 at 12%
12. $y = 2x^2 - \frac{1}{2}x + 3$ 13. $(\frac{3}{2}, 2)$
14. $(-1, -3)$ 15. $(-2, 0, -3)$
16. dependent 17. $(1, -4, 0, -2)$
18. $(1, -4, \frac{1}{3}, 2)$ 19. $i_1 = 2\frac{3}{5}$,
 $i_2 = -4\frac{7}{10}$, $i_3 = 15\frac{4}{5}$ 20. $666\frac{2}{3}$ gallons
 of 8% solution and $333\frac{1}{3}$ gallons of 20%
 solution 21. $x = -\frac{16}{23}$, $y = \frac{12}{23}$

22. $x = \frac{571}{304}$, $y = \frac{2,379}{304}$
23. $x = \frac{115}{59}$, $y = -\frac{60}{59}$, $z = \frac{43}{59}$
24. $x = \frac{1}{4}$, $y = -\frac{1}{2}$, $z = \frac{7}{8}$
25. $x = \frac{23}{18}$, $y = \frac{59}{108}$, $z = \frac{1}{27}$, $w = -\frac{43}{54}$
26. $x = \frac{16}{25}$, $y = -\frac{6}{25}$, $z = -\frac{46}{25}$, $w = -\frac{18}{25}$
27. $D = \frac{6}{7}$; complete solution:
 $(-\frac{58}{7}, -\frac{367}{28}, \frac{87}{28}, \frac{6}{7}, \frac{647}{28})$ 28. $9\frac{1}{4}$



39. P is maximized for $x = 5\frac{1}{3}$, $y = 0$; Its value is $21\frac{1}{3}$.

40. P is maximized at either of the points $(2, 2\frac{2}{3})$ or $(4, 2)$; its value is 10.

41. Income is maximized at \$225 by producing 75 tables and no chairs per run.

42. Production is maximized at 580 trees by using 20 one-supervisor crews and $6\frac{2}{3}$ three-supervisor crews.

43. $4\frac{3}{4}$ 44. -17 45. -13

46. $6 - 3\sqrt{\pi}$

47. There are an unlimited number of solutions; one is $(0, 0, \frac{1}{2}, 0)$.

48. $\begin{bmatrix} -11 & -22 & 32 \\ 0 & 0 & -6 \end{bmatrix}$

49. $\begin{bmatrix} -4x^2 + 2y & -3x + 18 \\ 16xy - 3y & 12y - 27 \end{bmatrix}$

50. $\begin{bmatrix} -15 & -4 & 15 & \frac{3}{4} \\ -21 & -34 & \frac{131}{2} & \frac{83}{8} \\ -7 & 8 & 6 & \frac{3}{2} \\ -44 & -76 & 105 & \frac{45}{4} \end{bmatrix}$

51. $\begin{bmatrix} \frac{1}{17} & -\frac{5}{17} \\ -\frac{3}{17} & -\frac{2}{17} \end{bmatrix}$

$$52. \begin{bmatrix} \frac{5}{38} & \frac{6}{19} & \frac{3}{38} \\ -\frac{11}{38} & \frac{2}{19} & \frac{1}{38} \\ \frac{5}{19} & -\frac{7}{19} & \frac{3}{19} \end{bmatrix}$$

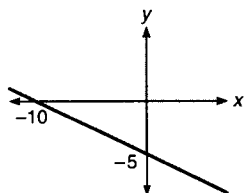
$$53. x = -1, y = -1$$

$$54. x = 5, y = 5, z = -1$$

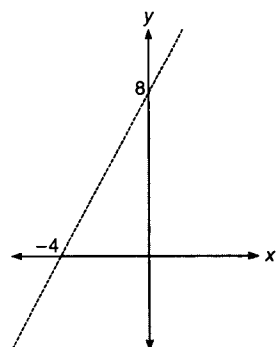
Chapter 10 test

1. $(\frac{1}{2}, \frac{2}{3})$ 2. $(2, \frac{1}{2}, -1)$ 3. L is length, W is width: $(L, W) = (80'', 24'')$ 4. S is amount invested at 6%, T = amount invested at 10%, $(S, T) = (\$4,500, \$7,500)$.
 5. $y = 2x^2 - x + 3$ 6. $(2, -2, 3)$
 7. $(0, -2, 1, -1)$ 8. Let T = amount of 30% solution to use, and S = amount of 70% solution; $(T, S) = (312.5 \text{ gallons}, 187.5 \text{ gallons})$ 9. $x = \frac{13}{4}, y = \frac{9}{4}$
 10. $D = 0, D_x = 0, D_y = 0, D_z = 0$. Since all the determinants are 0, the system is dependent. 11. $8\frac{1}{2}$

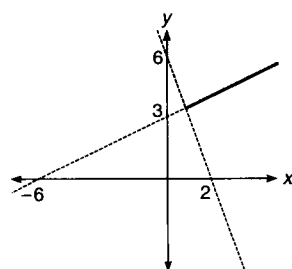
12.



13.



14.



15. C is maximized at $x = 4\frac{1}{5}, y = 1\frac{3}{5}$, with a value of $7\frac{2}{5}$

16. $\frac{12}{19}$ gm of Prime, $2\frac{18}{19}$ gm of Regular

17. 12 of the first type of crews and $5\frac{1}{3}$ of the second type of crews. $429\frac{1}{3}$ trees logged per day.

$$18. -17 \quad 19. 17 \quad 20. \begin{bmatrix} 3 & -10 & 20 \\ 8 & 8 & -16 \end{bmatrix}$$

$$21. \begin{bmatrix} -3 & 1 \\ 14 & -22 \end{bmatrix} \quad 22. \begin{bmatrix} 0 & -\frac{1}{3} \\ \frac{1}{5} & \frac{2}{15} \end{bmatrix}$$

$$23. \begin{bmatrix} \frac{1}{3} & \frac{5}{9} & -\frac{2}{9} \\ \frac{1}{3} & -\frac{1}{9} & -\frac{5}{9} \\ \frac{1}{3} & -\frac{4}{9} & -\frac{2}{9} \end{bmatrix}$$

$$24. x = -\frac{4}{9}, y = -\frac{10}{9}$$

$$25. a = 1, b = -2, c = 2, d = 3$$

$$26. A^2 = \begin{bmatrix} 1 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

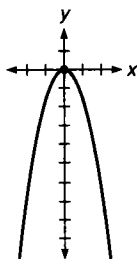
The entry in $A^2_{1,5}$ is 2, so there are 2 paths of length 2 from node 1 to node 5.

Chapter 11

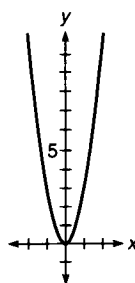
Exercise 11–1

Answers to odd-numbered problems

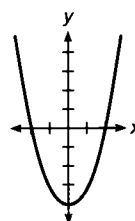
1. focus: $(0, -\frac{1}{8})$; directrix: $y = \frac{1}{8}$



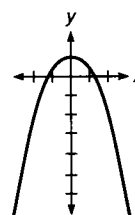
3. focus: $(0, \frac{1}{12})$; directrix: $y = -\frac{1}{12}$



5. focus: $(0, -3\frac{3}{4})$; directrix: $y = -4\frac{1}{4}$



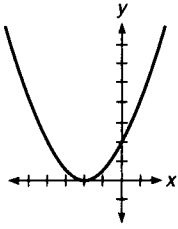
7. focus: $(0, \frac{3}{4})$; directrix: $y = 1\frac{1}{4}$



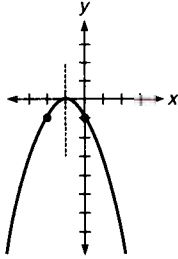
9. focus: $(3, \frac{1}{8})$; directrix: $y = -\frac{1}{8}$



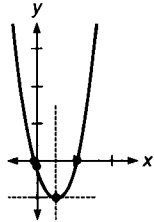
11. focus: $(-2, \frac{1}{2})$; directrix: $y = -\frac{1}{2}$



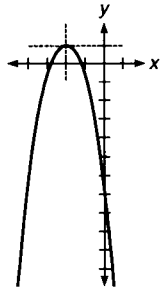
13. focus: $(-1, -\frac{1}{4})$; directrix: $y = \frac{1}{4}$



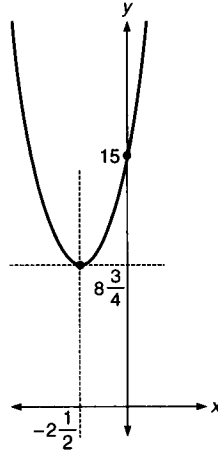
15. focus: $(\frac{1}{2}, -\frac{11}{12})$; directrix: $y = -\frac{13}{12}$
intercepts: $(0, -\frac{1}{4})$, $(\frac{1}{2} \pm \frac{\sqrt{3}}{3}, 0)$
vertex: $(\frac{1}{2}, -1)$



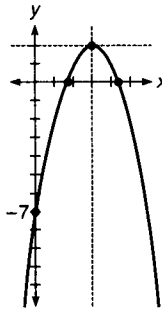
17. focus: $(-2, \frac{7}{8})$; directrix: $y = 1\frac{1}{8}$
intercepts: $(0, -7)$, $(-2 \pm \sqrt{\frac{1}{2}}, 0)$



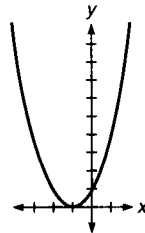
19. focus: $(-2\frac{1}{2}, 9)$; directrix: $y = 8\frac{1}{2}$
vertex: $(-2\frac{1}{2}, 8\frac{3}{4})$



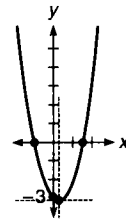
21. focus: $(3, 1\frac{3}{4})$; directrix: $y = 2\frac{1}{4}$
intercepts: $(0, -7)$, $(3 - \sqrt{2}, 0)$, $(3 + \sqrt{2}, 0)$; vertex: $(3, 2)$



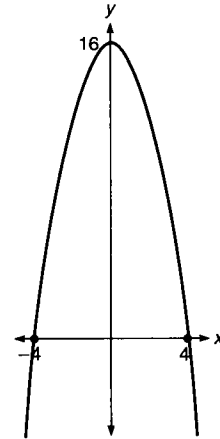
23. focus: $(-1, \frac{1}{4})$; directrix: $y = -\frac{1}{4}$



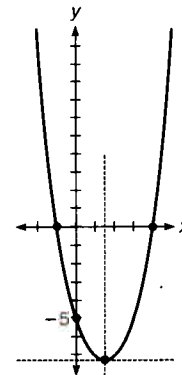
25. focus: $(\frac{1}{4}, -3)$; directrix: $y = -3\frac{1}{4}$
intercepts: $(0, -3)$, $(-1, 0)$, $(1\frac{1}{2}, 0)$
vertex: $(\frac{1}{4}, -3\frac{1}{8})$



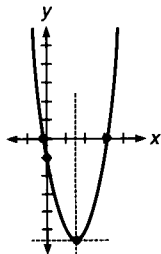
27. focus: $(0, 15\frac{3}{4})$; directrix: $y = 16\frac{1}{4}$



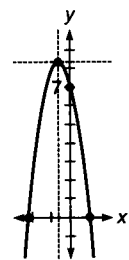
29. vertex: $(1\frac{1}{2}, -7\frac{1}{4})$; focus: $(1\frac{1}{2}, -7)$
directrix: $y = -7\frac{1}{2}$; intercepts: $(0, -5)$, $(\frac{3 - \sqrt{29}}{2}, 0)$, $(\frac{3 + \sqrt{29}}{2}, 0)$



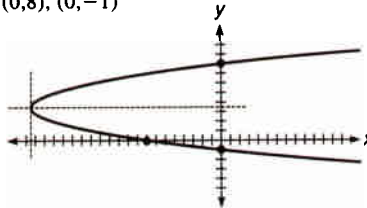
31. vertex: $(1\frac{1}{2}, -5\frac{1}{2})$; focus: $(1\frac{1}{2}, -5\frac{3}{8})$;
directrix: $y = -5\frac{5}{8}$; intercepts: $(0, -1)$,
 $(\frac{3}{2} - \frac{\sqrt{11}}{2}, 0)$, $(\frac{3}{2} + \frac{\sqrt{11}}{2}, 0)$



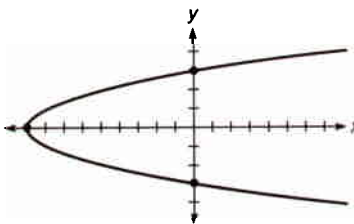
33. vertex: $(-\frac{2}{3}, 8\frac{1}{3})$; focus: $(-\frac{2}{3}, 8\frac{1}{4})$;
directrix: $y = 8\frac{5}{12}$; intercepts: $(0, 7)$,
 $(-2\frac{1}{3}, 0)$, $(1, 0)$



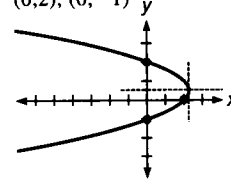
35. vertex: $(-20\frac{1}{4}, 3\frac{1}{2})$; focus: $(-20, 3\frac{1}{2})$;
directrix: $x = -20\frac{1}{2}$; intercepts: $(-8, 0)$,
 $(0, 8)$, $(0, -1)$



37. vertex: $(-9, 0)$; focus: $(-8\frac{3}{4}, 0)$;
directrix: $x = -9\frac{1}{4}$;
intercepts: $(-9, 0)$, $(0, \pm 3)$



39. vertex: $(2\frac{1}{4}, \frac{1}{2})$; focus: $(2, \frac{1}{2})$;
directrix: $x = 2\frac{1}{2}$; intercepts: $(2, 0)$,
 $(0, 2)$, $(0, -1)$



41. $y = \frac{1}{6}x^2 - \frac{2}{3}x - \frac{23}{6}$

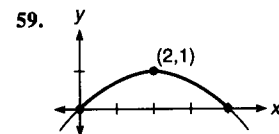
43. $y = \frac{1}{6}x^2 - \frac{2}{3}x - \frac{23}{6}$

45. $y = \frac{1}{8}x^2 - \frac{3}{4}x + \frac{1}{8}$

47. $y = \frac{1}{12}x^2$ 49. $y = x^2 - 6x + 8$

51. $32\sqrt{3}$ 53. $\frac{9}{5}$ 55. $\frac{9}{5}$

57. $y = \frac{3}{3,125}x^2$



61. $4\sqrt{10} \approx 12.6$ feet; the horizontal distance traveled did double also

63. $\frac{4}{5}\sqrt{10} \approx 2.5$ ft/s

Solutions to skill and review problems

1. $\frac{x^2}{4} + 3y^2 = 1$
 $3y^2 = 1 - \frac{x^2}{4}$
 $3y^2 = \frac{4 - x^2}{4}$
 $y^2 = \frac{4 - x^2}{12}$
 $y = \pm \sqrt{\frac{4 - x^2}{12}} = \pm \frac{\sqrt{4 - x^2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm \frac{\sqrt{3}\sqrt{4 - x^2}}{6}$

2. $\begin{bmatrix} -2(-2) + 3(1) + 0(-3) & -2(3) + 3(5) + 0(2) \\ 1(-2) + 5(1) - 3(-3) & 1(3) + 5(5) - 3(2) \\ 4(-2) + 2(1) + 6(-3) & 4(3) + 2(5) + 6(2) \end{bmatrix}$
 $= \begin{bmatrix} 7 & 9 \\ 12 & 22 \\ -24 & 34 \end{bmatrix}$

3. Use equation [1] to remove z from equations [2] and [3]:

[1] $2x + 3y - z = 5$

[2] $4x - 6y + z = -4$

[3] $2x + 6y - 3z = 11$

[4] $6x - 3y = 1$ ([1] + [2])

[5] $-4x - 3y = -4$ (-3 [1] + [3])

$10x = 5$ ([4] - [5])

$x = \frac{1}{2}$

$-30y = -20$ (4 [4] + 6 [5])

$y = \frac{2}{3}$

Use equation [1] to find z :

$2x + 3y - z = 5$

$2x + 3y - 5 = z$

$2(\frac{1}{2}) + 3(\frac{2}{3}) - 5 = z$

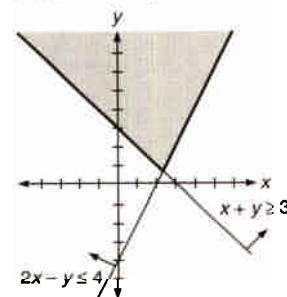
$-2 = z$

Thus, the solution is $(\frac{1}{2}, \frac{2}{3}, -2)$.

4. $2x - y \leq 4$

$x + y \geq 3$

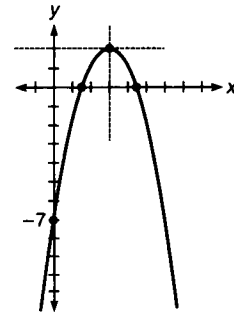
Graph the straight lines $2x - y = 4$ and $x + y = 3$. Use a test point such as $(0, 0)$ to determine which half-plane is applicable to each inequality. Darken in the area in which these two half-planes overlap.



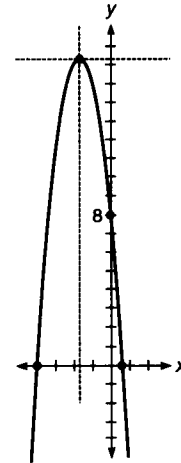
5. $\log_5 10 = \frac{\log 10}{\log 5} = \frac{1}{\log 5} \approx 1.4307$
6. $\log(2x - 1) + \log(3x + 1) = \log 4$
 $\log[(2x - 1)(3x + 1)] = \log 4$
 $(2x - 1)(3x + 1) = 4$
 $6x^2 - x - 5 = 0$
 $(6x + 5)(x - 1) = 0$
 $6x + 5 = 0$ or $x - 1 = 0$
 $x = -\frac{5}{6}$ or $x = 1$
 The negative value is not in the domain of $\log(2x - 1)$ or $\log(3x + 1)$, so the solution is 1.
7. Solve $|2x - 5| < 10$.
 $-10 < 2x - 5 < 10$
 $-5 < 2x < 15$
 $-\frac{5}{2} < x < \frac{15}{2}$
 $-2\frac{1}{2} < x < 7\frac{1}{2}$

Solutions to trial exercise problems

21. $y = -x^2 + 6x - 7$
 $y = -(x^2 - 6x) - 7$
 $y = -(x^2 - 6x + 9) - 7 + 9$
 $y = -(x - 3)^2 + 2; V(3, 2)$
 $\frac{1}{4p} = -1; p = -\frac{1}{4}$
 focus: $(2, 2 - \frac{1}{4}) = (2, 1\frac{3}{4})$;
 directrix: $y = 2 - (-\frac{1}{4}) = 2\frac{1}{4}; y = 2\frac{1}{4}$;
 intercepts:
 $x = 0: y = -7; (0, -7)$
 $y = 0: 0 = -(x - 3)^2 + 2$
 $(x - 3)^2 = 2$
 $x - 3 = \pm \sqrt{2}$
 $x = 3 \pm \sqrt{2} \approx 4.4, 1.6$
 $(1.6, 0), (4.4, 0)$



26. $y = -3x^2 - 10x + 8$
 $y = -3(x^2 + \frac{10}{3}x) + 8$
 $y = -3(x^2 + \frac{10}{3}x + \frac{25}{9}) + 8 + 3(\frac{25}{9})$
 $y = -3(x + 1\frac{2}{3})^2 + 16\frac{1}{3}; V(-1\frac{2}{3}, 16\frac{1}{3})$
 $\frac{1}{4p} = -3; p = -\frac{1}{12}$;
 focus: $(-\frac{5}{3}, \frac{49}{3} - \frac{1}{12}) = (-1\frac{2}{3}, 16\frac{1}{4})$;
 directrix: $y = \frac{49}{3} - (-\frac{1}{12}) = 16\frac{5}{12}$
 $y = 16\frac{5}{12}$
 intercepts:
 $x = 0: y = 8; (0, 8)$
 $y = 0: 0 = -3x^2 - 10x + 8$
 $0 = 3x^2 + 10x - 8$
 $0 = (3x - 2)(x + 4)$
 $x = \frac{2}{3}$ or $-4; (-4, 0), (\frac{2}{3}, 0)$



40. $x = -y^2 - 4y + 8$; since this relation expresses x as a function of y we first graph its inverse relation then reflect the graph about the line $y = x$.

$$\begin{aligned} y &= -x^2 - 4x + 8 \\ y &= -1(x^2 + 4x) + 8 \\ y &= -1(x^2 + 4x + 4) + 8 + 4 \\ y &= -1(x + 2)^2 + 12; V(-2, 12) \\ \frac{1}{4p} &= -1; p = -\frac{1}{4} \\ \text{focus: } &(-2, 12 - \frac{1}{4}); (-2, 11\frac{3}{4}) \\ \text{directrix: } &y = 12 - (-\frac{1}{4}) = 12\frac{1}{4} \\ \text{intercepts:} & \\ x = 0: &y = 8; (0, 8) \\ y = 0: &0 = -1(x + 2)^2 + 12 \\ &(x + 2)^2 = 12 \\ &x + 2 = \pm \sqrt{12} \\ &x = -2 \pm 2\sqrt{3} \\ &x \approx -5.5, 1.5; (-5.5, 0), \\ &(1.5, 0) \end{aligned}$$

$$x = -y^2 - 4y + 8$$

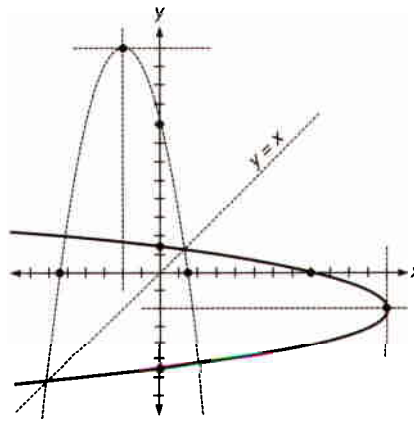
$$V(12, -2)$$

$$(11\frac{3}{4}, -2)$$

$$x = 12\frac{1}{4}$$

$$(8, 0)$$

$$(0, -5.5), (0, 1.5)$$



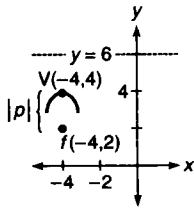
48. focus: $(-4, 2)$; vertex: $(-4, 4) = (h, k)$
 $|p| = 2$, and $p < 0$ since the parabola opens downward (toward the focus).
 Thus $p = -2$, and the equation is

$$y = \frac{1}{4p}(x - h)^2 + k$$

$$y = \frac{1}{4(-2)}(x - (-4))^2 + 4$$

$$y = -\frac{1}{8}(x^2 + 8x + 16) + 4$$

$$y = -\frac{1}{8}x^2 - x + 2$$



49. vertex: $(3, -1)$; x -intercepts: 2, 4

$$y = \frac{1}{4p}(x - h)^2 + k$$

Replace h by 3, k by -1 .

$$[1] \quad y = \frac{1}{4p}(x - 3)^2 - 1$$

To find the value of p we can use the fact that we know the point $(x, y) = (2, 0)$ satisfies the equation (since it is one of the equation's x -intercepts).
 Thus we know that

$$0 = \frac{1}{4p}(2 - 3)^2 - 1$$

Replace x by 2, y by 0 in [1].

$$1 = \frac{1}{4p}(2 - 3)^2$$

$$1 = \frac{1}{4p}$$

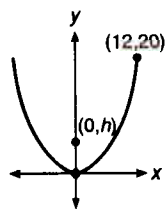
No need to find p itself.

$$y = 1(x - 3)^2 - 1$$

Replace $\frac{1}{4p}$ by 1 in [1].

$$y = x^2 - 6x + 8$$

55. $w = 24$, $d = 20$; find h .



Referring to the figure we see that the vertex is $(0, 0)$, so the equation is of the

form $y = \frac{1}{4p}x^2$; the point $(12, 20)$ satisfies the equation so

$$y = \frac{1}{4p}x^2$$

$$20 = \frac{1}{4p}(144)$$

$$p = \frac{144}{80} = \frac{9}{5} = h$$

58. $y = -\frac{1}{6}x^2 + \frac{1}{\sqrt{3}}x$

$$y = -\frac{1}{6}\left(x^2 - \frac{6}{\sqrt{3}}x\right), \text{ since } \left(-\frac{1}{6}\right)\left(-\frac{6}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}}$$

$$y = -\frac{1}{6}(x^2 - 2\sqrt{3}x), \text{ since } \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

$$y = -\frac{1}{6}(x^2 - 2\sqrt{3}x + 3) + \frac{1}{6}(3),$$

since $\frac{1}{2}(-2\sqrt{3}) = -\sqrt{3}$; $(-\sqrt{3})^2 = 3$

$$y = -\frac{1}{6}(x - \sqrt{3})^2 + \frac{1}{2}$$

vertex: $(\sqrt{3}, \frac{1}{2}) = (1.7, 0.5)$

$$\text{intercepts: } x = 0: y = \frac{1}{\sqrt{3}} \cdot 0 - \frac{1}{6}$$

$$0^2 = 0; (0, 0)$$

$$y = 0: 0 = \frac{1}{\sqrt{3}}x - \frac{1}{6}x^2$$

Multiply each term by $6\sqrt{3}$.

$$(6\sqrt{3})(0) = (6\sqrt{3})\left(\frac{1}{\sqrt{3}}x\right) -$$

$$(6\sqrt{3})\left(\frac{1}{6}x^2\right)$$

$$0 = 6x - \sqrt{3}x^2$$

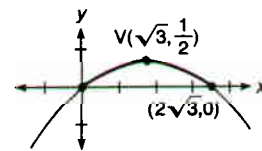
$$0 = x(6 - \sqrt{3}x)$$

$$x = 0 \text{ or } 6 - \sqrt{3}x = 0$$

$$6 = \sqrt{3}x$$

$$x = \frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}; (0, 0)$$

$$\text{and } (2\sqrt{3}, 0) \approx (3.5, 0)$$



60. $y = -\frac{16}{v^2}x^2$

$$v = 4, y = -40.$$

$$-40 = -\frac{16}{4^2}x^2$$

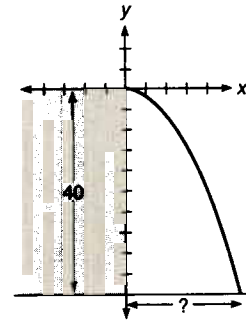
$$40 = x^2$$

$$\pm \sqrt{40} = x$$

Assume $x > 0$.

$$x = 2\sqrt{10} \approx 6.3$$

Thus, the stuntperson will land about 6.3 feet from the base of the building.



64. Let $-h$ be an initial height, and r a fixed velocity.

$$y = -\frac{16}{v^2}x^2$$

$$y = -h, v = r$$

$$-h = -\frac{16}{r^2}x^2$$

$$hr^2 = 16x^2$$

$$x^2 = \frac{hr^2}{16}$$

Assume $x > 0$

$$x = \sqrt{\frac{hr^2}{16}} = \frac{1}{4}r\sqrt{h}$$

Now double the height to $-2h$.

$$y = -\frac{16}{v^2}x^2$$

$$y = -h, v = r$$

$$-2h = -\frac{16}{r^2}x^2$$

$$2hr^2 = 16x^2$$

$$x^2 = \frac{hr^2}{8}$$

Assume $x > 0$

$$x = \sqrt{\frac{hr^2}{8}}$$

$$x = r\sqrt{\frac{h}{8}} \cdot \frac{2}{2} = r\frac{\sqrt{2h}}{4} = \frac{\sqrt{2}}{4}r\sqrt{h}$$

$$\frac{\sqrt{2}}{4}r\sqrt{h}$$

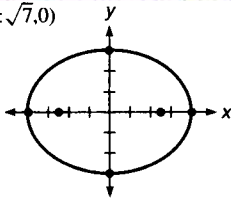
Dividing $\frac{\frac{\sqrt{2}}{4}r\sqrt{h}}{\frac{1}{4}r\sqrt{h}} = \sqrt{2}$ shows that the

horizontal distance did *not* double when the height doubled. It increased by a factor of $\sqrt{2} \approx 1.4$.

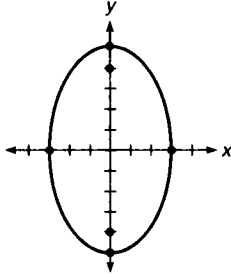
Exercise 11-2

Answers to odd-numbered problems

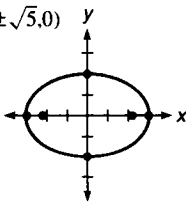
1. foci: $(\pm\sqrt{7}, 0)$



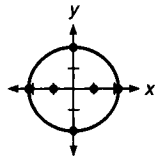
3.



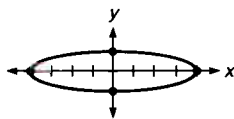
5. foci: $(\pm\sqrt{5}, 0)$



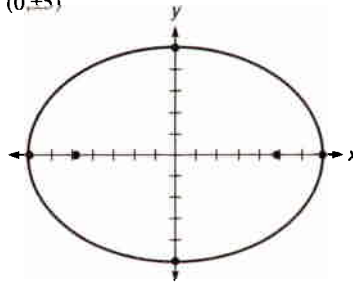
7. foci: $(\pm\frac{1}{2}, 0)$; intercepts: $(\pm\frac{\sqrt{5}}{2}, 0)$, $(0, \pm 1)$



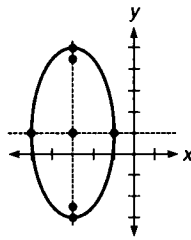
9. foci $(\pm\sqrt{15}, 0)$; intercepts: $(\pm 4, 0)$, $(0, \pm 1)$



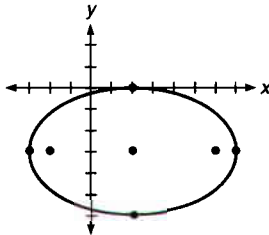
11. foci: $(\pm 2\sqrt{6}, 0)$; intercepts: $(\pm 7, 0)$, $(0, \pm 5)$



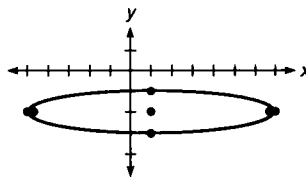
13. foci: $(-3, 1 \pm 2\sqrt{3})$



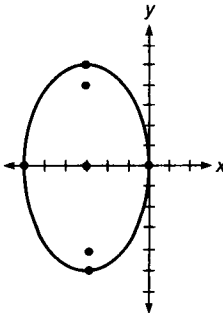
15.



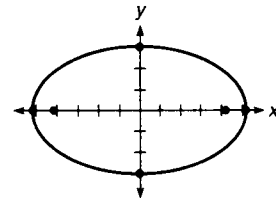
17.



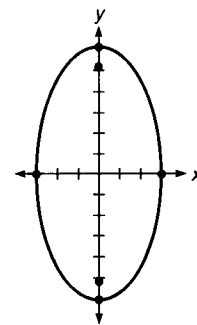
19.



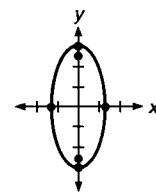
21. $\frac{x^2}{27} + \frac{y^2}{9} = 1$; foci: $(\pm 3\sqrt{2}, 0)$; intercepts: $(\pm 3\sqrt{3}, 0)$, $(0, \pm 3)$



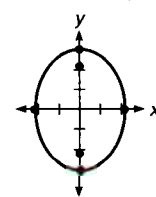
23. $\frac{x^2}{9} + \frac{y^2}{36} = 1$; foci: $(0, \pm 3\sqrt{3})$; intercepts: $(\pm 3, 0)$, $(0, \pm 6)$



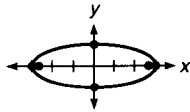
25. $\frac{x^2}{2} + \frac{y^2}{9} = 1$; foci: $(0, \pm\sqrt{7})$; intercepts: $(\pm\sqrt{2}, 0)$, $(0, \pm 3)$



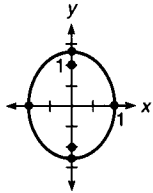
27. $\frac{x^2}{9} + \frac{y^2}{9} = 1$; foci: $(0, \pm\frac{3\sqrt{2}}{2})$; intercepts: $(\pm\frac{3\sqrt{2}}{2}, 0)$, $(0, \pm 3)$



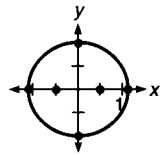
29. $\frac{x^2}{9} + y^2 = 1$; foci: $(\pm 2\sqrt{2}, 0)$;
intercepts: $(\pm 3, 0)$, $(0, \pm 1)$



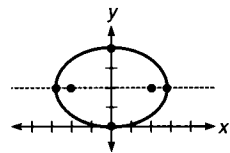
31. $x^2 + \frac{y^2}{2} = 1$; foci: $(0, -1)$ and $(0, 1)$;
intercepts: $(0, \pm\sqrt{2})$, $(\pm 1, 0)$



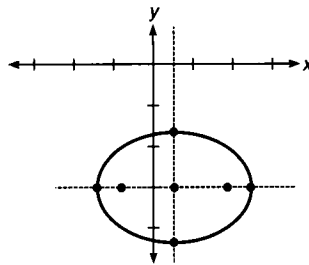
33. $\frac{x^2}{5} + y^2 = 1$; foci: $(\pm \frac{1}{2}, 0)$;
intercepts: $(0, \pm 1)$, $(\pm \frac{\sqrt{5}}{2}, 0)$



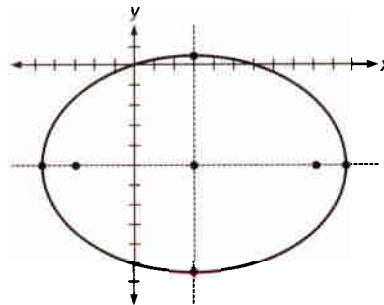
35. $\frac{x^2}{8} + \frac{(y-2)^2}{4} = 1$; center: $(0, 2)$; foci:
 $(\pm 2, 2)$; end points of major/minor axes:
 $(0 \pm 2\sqrt{2}, 2)$, $(0, 0)$, and $(0, 4)$



37. $\frac{(x - \frac{1}{2})^2}{4} + \frac{(y + 3)^2}{2} = 1$;
center: $(\frac{1}{2}, -3)$; foci: $(\frac{1}{2} \pm \sqrt{2}, -3)$;
end points of major/minor axes:
 $(-1\frac{1}{2}, -3)$, $(2\frac{1}{2}, -3)$, $(\frac{1}{2}, -3 \pm \sqrt{2})$

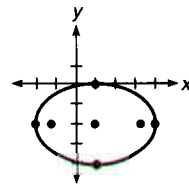


39. $\frac{(x-3)^2}{60} + \frac{(y+5)^2}{30} = 1$; center:
 $(3, -5)$; foci: $(3 \pm \sqrt{30}, -5)$; end points
of major/minor axes: $(3 \pm 2\sqrt{15}, -5)$,
 $(3, -5 \pm \sqrt{30})$

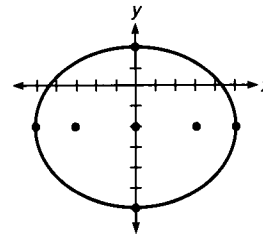


41. $x^2 + 2y^2 + 8 = 0$
 $x^2 + 2y^2 = -8$
There is no real solution to this
equation, so there is no graph for this
relation.

43. $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$; center:
 $(1, -2)$; foci: $(1 \pm \sqrt{5}, -2)$; end points
of major/minor axes: $(-2, -2)$, $(4, -2)$,
 $(1, -4)$, $(1, 0)$



45. $\frac{x^2}{25} + \frac{(y+2)^2}{16} = 1$; center: $(0, -2)$;
foci: $(\pm 3, -2)$; end points of major/
minor axes: $(\pm 5, -2)$, $(0, -6)$, $(0, 2)$



47. $\frac{x^2}{13} + \frac{y^2}{9} = 1$ 49. $\frac{x^2}{48} + \frac{y^2}{64} = 1$

51. $\frac{x^2}{9} + \frac{y^2}{4} = 1$ 53. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

55. $\frac{x^2}{22,500} + \frac{y^2}{20,000} = 1$

57. $\frac{x^2}{4} + (y-1)^2 = 1$ 59. $\frac{1}{2}$ 61. $\frac{\sqrt{5}}{5}$

63. $\frac{3}{5}$ 65. $\frac{\sqrt{3}}{2}$ 67. $\frac{2\sqrt{2}}{3}$

69. $\frac{\sqrt{6}}{3}$ 71. $\frac{\sqrt{2}}{2}$

Solutions to skill and review problems

1. $y = 2(x-1)^2 - 4$

Using $y = a(x-h)^2 + k$ we see that
this is a parabola with vertex at
 $(1, -4)$.

intercepts:

$$y = 0: 0 = 2(x-1)^2 - 4$$

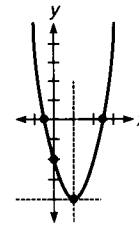
$$4 = 2(x-1)^2$$

$$2 = (x-1)^2$$

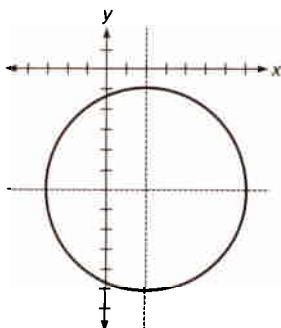
$$\pm\sqrt{2} = x-1$$

$$1 \pm \sqrt{2} = x = (2.4, 0), (-0.4, 0)$$

$$x = 0: y = 2(-1)^2 - 4 = -2; (0, -2)$$



2. $x^2 - 4x + y^2 + 12y + 12 = 0$
 $x^2 - 4x + 4 + y^2 + 12y + 36 = -12$
 $+ 4 + 36$
 $(x - 2)^2 + (y + 6)^2 = 28$
circle; center: $(2, -6)$; radius =
 $\sqrt{28} = 2\sqrt{7} \approx 5.3$

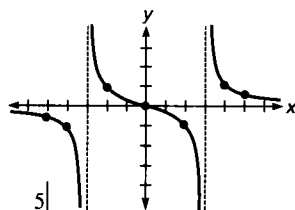


3. $x^{\frac{2}{3}} + 7x^{\frac{1}{3}} = 8$
Let $u = x^{\frac{1}{3}}$; then $u^2 = x^{\frac{2}{3}}$.
 $u^2 + 7u = 8$
 $u^2 + 7u - 8 = 0$
 $(u + 8)(u - 1) = 0$
 $u = -8$ or $u = 1$
 $x^{\frac{1}{3}} = -8$ or $x^{\frac{1}{3}} = 1$
Cube each member.
 $x = -512$ or $x = 1$

4. $\frac{2\sqrt{3}}{\sqrt{6} - \sqrt{2}} \cdot \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} + \sqrt{2}}$
 $= \frac{2\sqrt{3}(\sqrt{6} + \sqrt{2})}{2\sqrt{18} + 2\sqrt{6}}$
 $= \frac{6 + \sqrt{12} + \sqrt{12} + 2}{2(3\sqrt{2}) + 2\sqrt{6}}$
 $= \frac{8 + 2\sqrt{3}}{4(3\sqrt{2} + \sqrt{6})} = \frac{3\sqrt{2} + \sqrt{6}}{2}$

5. $y = \frac{2x}{(x - 3)(x + 3)}$
vertical asymptotes: $x = \pm 3$
horizontal asymptote: $y = 0$ (x -axis)
intercepts:
 $x = 0$: $y = 0$; $(0, 0)$
 $y = 0$: $0 = \frac{2x}{(x - 3)(x + 3)}$
 $0 = 2x$
 $0 = x$; $(0, 0)$
additional points:

x	-5	-4	-2	2	4	5
y	-0.6	-1.1	0.8	-0.8	1.1	0.6



6. $x^3 - 3x^2 + x + 2$
Possible rational zeros are $\pm 1, \pm 2$. Using synthetic division with $x = 2$:

	1	-3	1	2
		2	-2	-2
2	1	-1	-1	0

Thus $x^3 - 3x^2 + x + 2 = (x - 2)(x^2 - x - 1)$. The zeros of $x^2 - x - 1$ are not real. Thus the factorization above is complete over R .

Solutions to trial exercise problems

7. $\frac{4x^2}{5} + y^2 = 1$
 $\frac{x^2}{\frac{5}{4}} + y^2 = 1$

center: $(0, 0)$

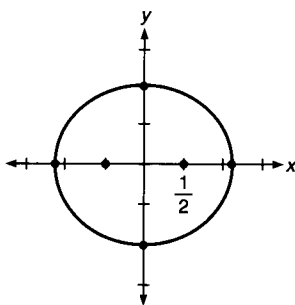
$a = \frac{\sqrt{5}}{2} \approx 1.1$, $b = 1$, $c =$

$\sqrt{\frac{5}{4} - 1} = \frac{1}{2}$

major axis: x -axis

foci: $(\pm \frac{1}{2}, 0)$

intercepts: $(\pm \frac{\sqrt{5}}{2}, 0)$, $(0, \pm 1)$



15. $\frac{(x - 2)^2}{25} + \frac{(y + 3)^2}{9} = 1$

$a = 5$, $b = 3$, $c = 4$; major axis parallel to x -axis.

center: $(h, k) = (2, -3)$

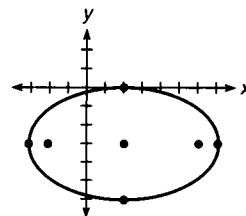
foci: $(h \pm c, k) = (2 \pm 4, -3)$; $(6, -3)$, $(-2, -3)$

end points of major/minor axes:

$(h \pm a, k)$, $(h, k \pm b)$

$(2 \pm 5, -3)$; $(-3, -3)$, $(7, -3)$

$(2, -3 \pm 3)$; $(2, -6)$, $(2, 0)$



37. $4x^2 - 4x + 8y^2 + 48y = -57$

$4(x^2 - x) + 8(y^2 + 6y) = -57$

$4(x^2 - x + \frac{1}{4}) + 8(y^2 + 6y + 9) =$
 $-57 + 4(\frac{1}{4}) + 72$

$4(x - \frac{1}{2})^2 + 8(y + 3)^2 = 16$

$\frac{(x - \frac{1}{2})^2}{4} + \frac{(y + 3)^2}{2} = 1$

center: $(h, k) = (\frac{1}{2}, -3)$

$a = \sqrt{4} = 2$; $b = \sqrt{2}$;

$c = \sqrt{4 - 2} = \sqrt{2}$

major axis parallel to x -axis

foci: $(h \pm c, k) = (\frac{1}{2} \pm \sqrt{2}, -3)$

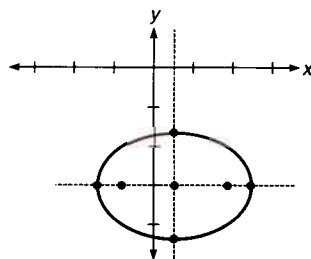
end points of major/minor axes:

$(h \pm a, k)$

$(\frac{1}{2} \pm 2, -3) = (-1\frac{1}{2}, -3)$ and $(2\frac{1}{2}, -3)$

$(h, k \pm b)$

$(\frac{1}{2}, -3 \pm \sqrt{2}) \approx (0.5, -4.4)$, $(0.5, -1.6)$



47. foci: $(-2,0)$ and $(2,0)$; one y -intercept at 3

The equation is of this form.

$$[1] \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$\pm b$ are the y -intercepts: $b = 3$

distance to the foci: $c = 2$

The foci are on the major axis, which is parallel to the x -axis,

so $a > b$.

$$c = \sqrt{a^2 - b^2}$$

$$2 = \sqrt{a^2 - 3^2}$$

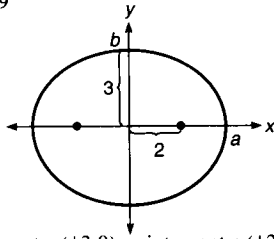
$$4 = a^2 - 9$$

$$13 = a^2$$

Replace $a^2 = 13$, $b^2 = 9$ in equation

[1].

$$\frac{x^2}{13} + \frac{y^2}{9} = 1$$



51. x -intercepts: $(\pm 3, 0)$; y -intercepts: $(\pm 2, 0)$

The equation is of this form.

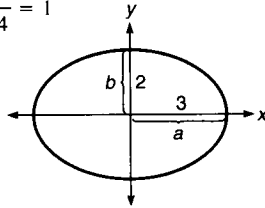
$$[1] \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

x -intercept: $a = 3$

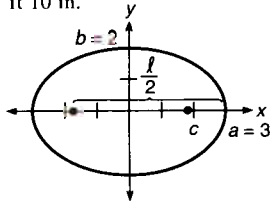
y -intercept: $b = 2$

Replace a and b in [1].

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$



54. We can see that $a = 3$, $b = 2$, so $c = \sqrt{a^2 - b^2} = \sqrt{5}$. Thus, the equation is $\frac{x^2}{9} + \frac{y^2}{4} = 1$, and the foci should be at $(\pm\sqrt{5}, 0)$. The length of the string l is $2(3 + \sqrt{2}) = 6 + 2\sqrt{2} \approx 8$ ft 10 in.



60. $a = 3$, $b = 5$, $c = 4$;

$$b > a \text{ so } e = \frac{c}{b} = \frac{4}{5}$$

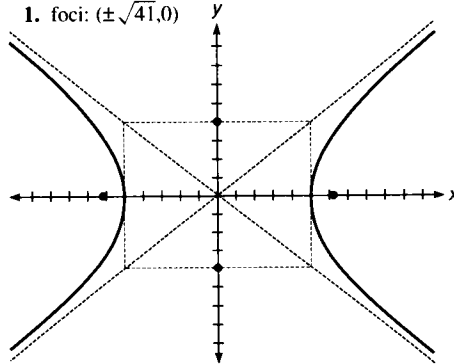
62. $a = 3$, $b = 2$, $c = \sqrt{5}$;

$$a > b \text{ so } e = \frac{c}{a} = \frac{\sqrt{5}}{3}$$

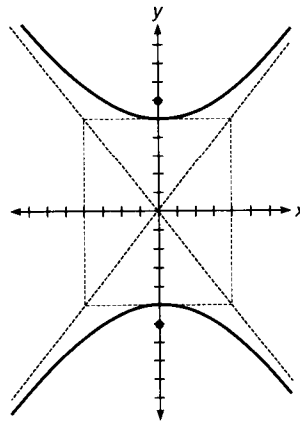
Exercise 11-3

Answers to odd-numbered problems

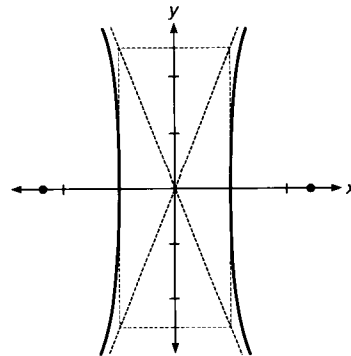
1. foci: $(\pm\sqrt{41}, 0)$



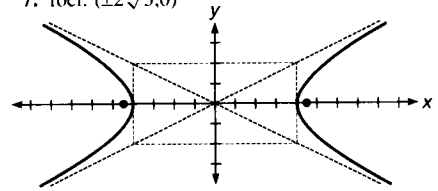
3. foci: $(0, \pm\sqrt{41})$



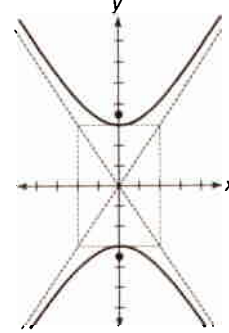
5. foci: $(\pm\sqrt{7}, 0)$



7. foci: $(\pm 2\sqrt{5}, 0)$

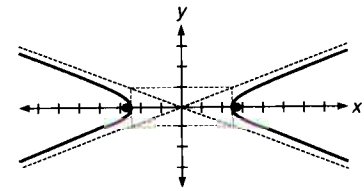


9. foci: $(0, \pm\sqrt{13})$



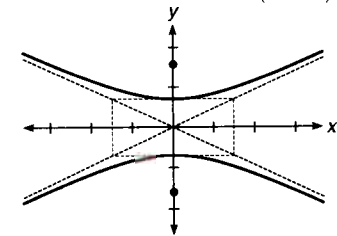
11. $\frac{x^2}{\frac{25}{4}} - y^2 = 1$; foci: $(\pm\frac{\sqrt{29}}{2}, 0)$;

end points of major axis: $(\pm 2\frac{1}{2}, 0)$

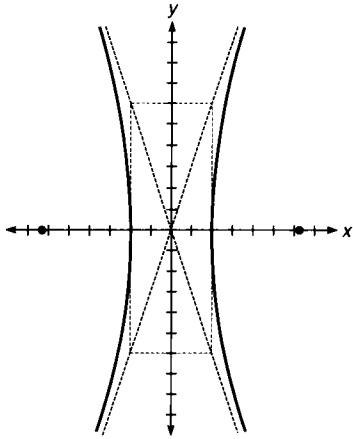


13. $\frac{y^2}{\frac{1}{2}} - \frac{x^2}{2} = 1$; foci: $(0, \pm\frac{\sqrt{10}}{2})$;

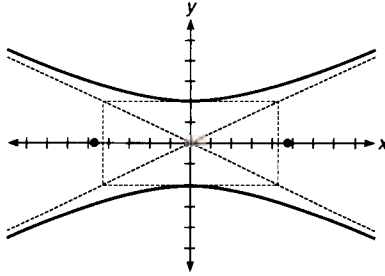
end points of major axis: $(0, \pm\frac{\sqrt{2}}{2})$



15. $\frac{x^2}{4} - \frac{y^2}{36} = 1$; foci: $(\pm 2\sqrt{10}, 0)$

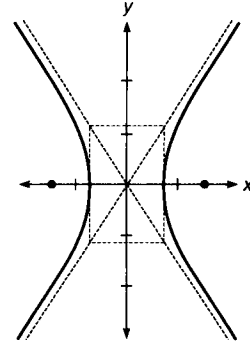


19. $\frac{y^2}{4} - \frac{x^2}{18} = 1$; foci: $(0, \pm\sqrt{22})$

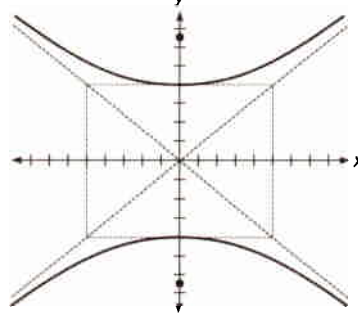


23. $\frac{x^2}{\frac{1}{2}} - \frac{y^2}{\frac{4}{3}} = 1$; foci: $(\pm\frac{\sqrt{66}}{6}, 0)$;

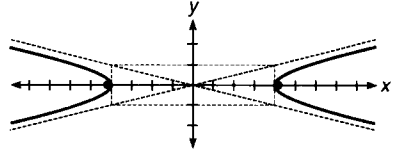
end points of major axis: $(\pm\frac{\sqrt{2}}{2}, 0)$



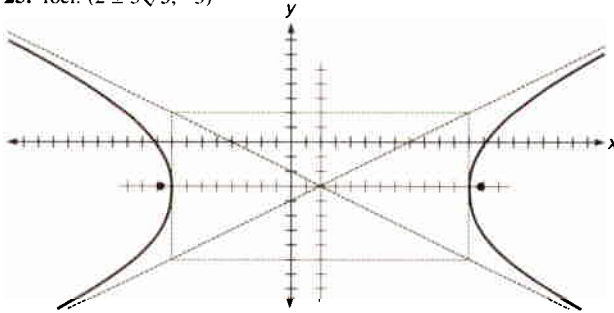
21. $\frac{y^2}{16} - \frac{x^2}{25} = 1$; foci: $(0, \pm\sqrt{41})$



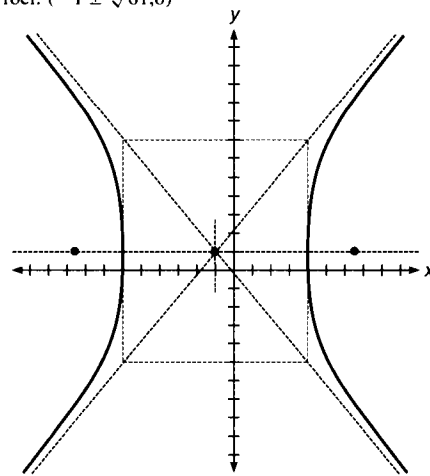
17. $\frac{x^2}{16} - y^2 = 1$; foci: $(\pm\sqrt{17}, 0)$



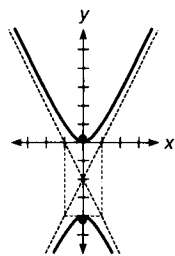
25. foci: $(2 \pm 5\sqrt{5}, -3)$



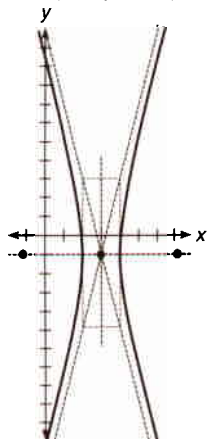
29. foci: $(-1 \pm \sqrt{61}, 0)$



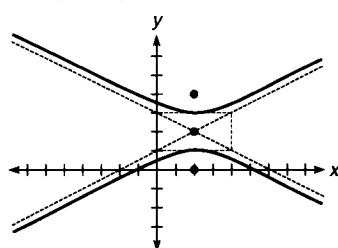
27. foci: $(0, -2 \pm \sqrt{5})$



31. foci: $(3 \pm \sqrt{17}, -1)$

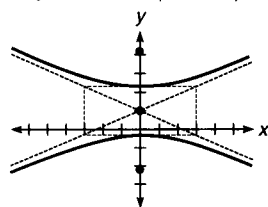


33. foci: $(2, 2 \pm \sqrt{5})$



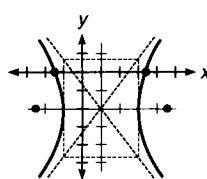
35. $\frac{(y-1)^2}{\frac{25}{16}} - \frac{x^2}{9} = 1$;

foci: $(1, -2\frac{1}{4}), (1, 4\frac{1}{4})$; end points of major axis: $(0, 2\frac{1}{4}), (0, -\frac{1}{4})$



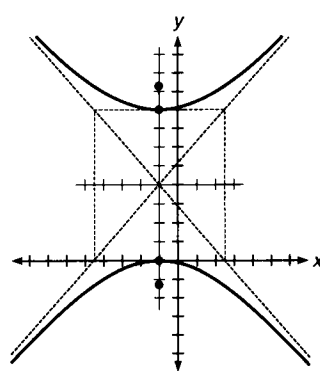
37. $\frac{(x-1)^2}{4} - \frac{(y+2)^2}{8} = 1$;

foci: $(1 \pm 2\sqrt{3}, -2)$

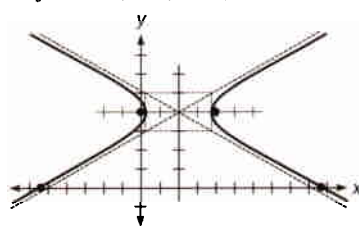


39. $\frac{(y-4)^2}{16} - \frac{(x+1)^2}{12} = 1$;

foci: $(-1, 4 \pm 2\sqrt{7})$

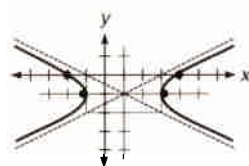


41. $\frac{(x-2)^2}{3} - (y-4)^2 = 1$; end points of major axis: $(2 \pm \sqrt{3}, -1)$



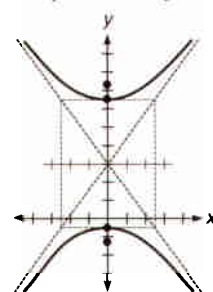
43. $\frac{(x-1)^2}{4} - (y+1)^2 = 1$;

foci: $(1 \pm \sqrt{5}, -1)$



45. $\frac{(y-3)^2}{12} - \frac{x^2}{6} = 1$; foci: $(0, 3 \pm 3\sqrt{2})$;

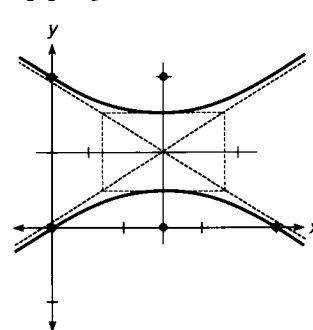
end points of major axis: $(0, 3 \pm 2\sqrt{3})$



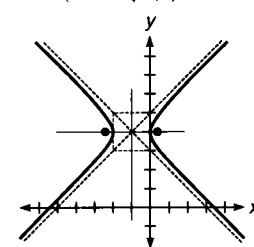
47. $\frac{(y-1)^2}{\frac{1}{4}} - \frac{(x-\frac{3}{2})^2}{\frac{3}{4}} = 1$; foci: $(1\frac{1}{2}, 2)$,

$(1\frac{1}{2}, 0)$; end points of major axis:

$(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 1)$

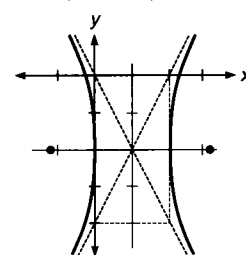


49. $(x+1)^2 - (y-4)^2 = 1$; hyperbola; foci: $(-1 \pm \sqrt{2}, 4)$

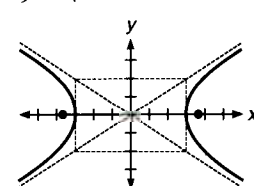


51. $(x-1)^2 - \frac{(y+2)^2}{4} = 1$; hyperbola;

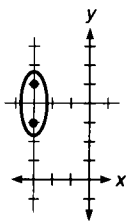
foci: $(1, -2 \pm \sqrt{5})$



53. $\frac{x^2}{9} - \frac{y^2}{4} = 1$; foci: $(\pm\sqrt{13}, 0)$; hyperbola

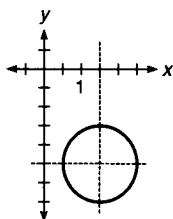


55. $(x+3)^2 + \frac{(y-4)^2}{2} = 1$; ellipse

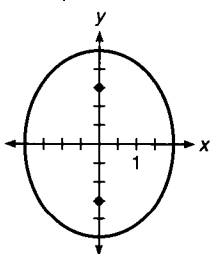


57. $(x - \frac{3}{2})^2 + (y + \frac{5}{2})^2 = 1$; circle;

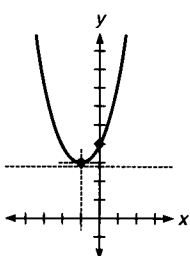
center: $(\frac{3}{2}, -\frac{5}{2})$



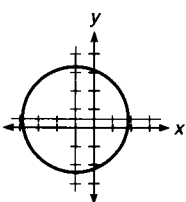
59. $\frac{x^2}{4} + \frac{y^2}{\frac{25}{4}} = 1$



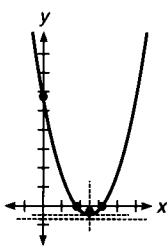
61. $y = (x+1)^2 + 3$; parabola;
focus: $(-1, 3\frac{1}{4})$



63. $(x+1)^2 + (y - \frac{1}{2})^2 = 8$; circle

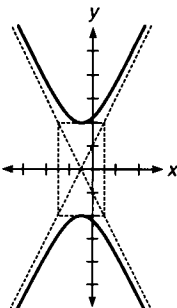


65. $y = (x - \frac{5}{2})^2 + \frac{1}{2}$; focus: $(2\frac{1}{2}, -\frac{1}{4})$



67. $\frac{x^2}{625} - \frac{y^2}{3,600} = 1$

69. $\frac{y^2}{4} - (x + \frac{1}{2})^2 = 1$



71. $(\sqrt{(x+c)^2 + y^2})^2 = (\sqrt{(x-c)^2 + y^2} \pm 2a)^2$

$$(x+c)^2 + y^2 = [(x-c)^2 + y^2] \pm 4a\sqrt{(x-c)^2 + y^2} + 4a^2$$

$$x^2 + 2cx + c^2 + y^2 = x^2 - 2cx + c^2 + y^2 \pm 4a\sqrt{(x-c)^2 + y^2} + 4a^2$$

$$4cx - 4a^2 = \pm 4a\sqrt{(x-c)^2 + y^2}$$

$$cx - a^2 = \pm a\sqrt{(x-c)^2 + y^2}$$

$$c^2x^2 - 2a^2cx + a^4 = a^2[(x-c)^2 + y^2]$$

$$c^2x^2 - 2a^2cx + a^4 = a^2x^2 - 2a^2cx + a^2c^2 + a^2y^2$$

$$c^2x^2 - a^2x^2 - a^2y^2 = a^2c^2 - a^4$$

$$(c^2 - a^2)x^2 - a^2y^2 = a^2(c^2 - a^2)$$

Let $b^2 = c^2 - a^2$; this is valid since $|c| > |a|$, so $c^2 > a^2$, and $c^2 - a^2 > 0$.

$$\frac{b^2x^2}{a^2b^2} - \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2}$$

$$\frac{b^2x^2}{a^2b^2} - \frac{a^2y^2}{a^2b^2} = \frac{a^2b^2}{a^2b^2}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

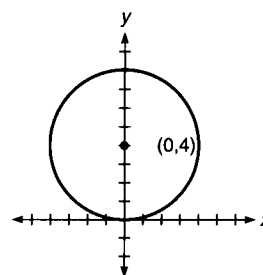
Solutions to skill and review problems

1. $x^2 + y^2 - 8y = 0$

$$x^2 + y^2 - 8y + 16 = 16$$

$$x^2 + (y-4)^2 = 16$$

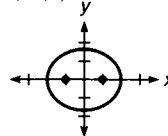
Circle; center is (0,4), radius is 4.



2. $3x^2 + 4y^2 = 12$

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

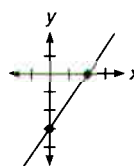
Ellipse; $a = 2$, $b = \sqrt{3}$, $c = 1$; foci: $(\pm 1, 0)$



3. $3x - 2y = 6$

Straight line; x-intercept is (2,0);

y-intercept is (0,-3).



4. $2x^2 - 4x + 4y^2 = 2$

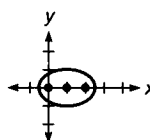
$$x^2 - 2x + 2y^2 = 1$$

$$x^2 - 2x + 1 + 2y^2 = 1 + 1$$

$$(x-1)^2 + 2y^2 = 2$$

$$\frac{(x-1)^2}{2} + y^2 = 1$$

Ellipse; center (1,0); $a = \sqrt{2}$, $b = 1$,
 $c = 1$; foci: $(1 \pm 1, 0) = (0,0), (2,0)$



5. $y = x^2 - 6x - 8$

$y = x^2 - 6x + 9 - 8 - 9$

$y = (x - 3)^2 - 17$

Parabola; vertex: $(3, -17)$; intercepts:

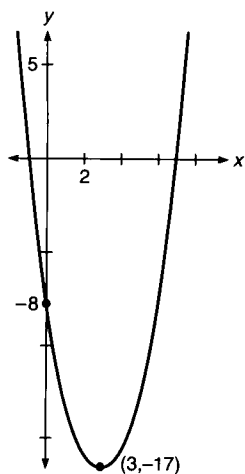
$x = 0: y = -8; (0, -8)$

$y = 0: 0 = (x - 3)^2 - 17$

$(x - 3)^2 = 17$

$x - 3 = \pm\sqrt{17}$

$x = 3 \pm \sqrt{17}; (-1.1, 0), (7.1, 0)$



36. $\frac{25(x-1)^2}{36} - \frac{9(y+1)^2}{4} = 1$

$\frac{(x-1)^2}{\frac{36}{25}} - \frac{(y+1)^2}{\frac{4}{9}} = 1$

$a = \frac{6}{5}, b = \frac{2}{3}$

$c = \sqrt{\frac{36}{25} + \frac{4}{9}} = \sqrt{\frac{424}{225}} = \frac{2\sqrt{106}}{15}$

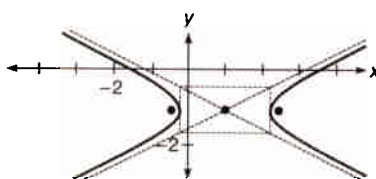
center: $(1, -1)$

foci: $\left(1 \pm \frac{2\sqrt{106}}{15}, -1\right) \approx (-0.4, -1),$

$(2.4, -1)$

end points of major axis: $\left(-\frac{1}{5}, -1\right),$

$\left(\frac{11}{5}, -1\right)$



39. $4x^2 + 8x - 3y^2 + 24y + 4 = 0$

$4(x^2 + 2x) - 3(y^2 - 8y) = -4$

$4(x^2 + 2x + 1) - 3(y^2 - 8y + 16) = -4 + 4(1) - 3(16)$

$-4 + 4(1) - 3(16)$

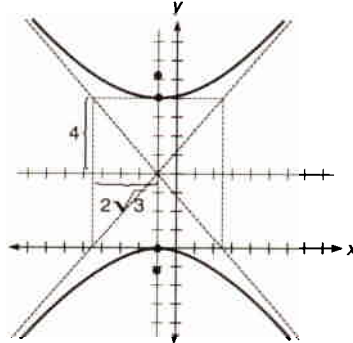
$4(x+1)^2 - 3(y-4)^2 = -48$

$3(y-4)^2 - 4(x+1)^2 = 48$

$\frac{(y-4)^2}{16} - \frac{(x+1)^2}{12} = 1$

center $(-1, 4)$

$c = \sqrt{16 + 12} = 2\sqrt{7}$; foci $(-1, 4 \pm 2\sqrt{7})$

 $a = 4, b = 2\sqrt{3}$; end points of major axis: $(-1, 0), (-1, 8)$ 

Solutions to trial exercise problems

23. $8x^2 - 3y^2 = 4$

$2x^2 - \frac{3y^2}{4} = 1$

$\frac{x^2}{\frac{1}{2}} - \frac{y^2}{\frac{4}{3}} = 1$

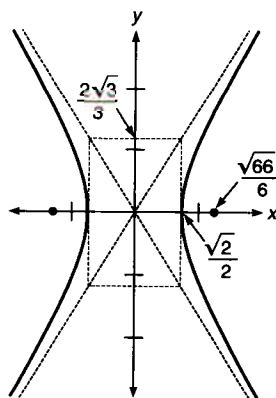
$c = \sqrt{\frac{1}{2} + \frac{4}{3}} = \sqrt{\frac{11}{6}}$

$= \sqrt{\frac{66}{36}} = \frac{\sqrt{66}}{6}$

foci: $\left(\pm \frac{\sqrt{66}}{6}, 0\right)$

$a = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

$b = \sqrt{\frac{4}{3}} = \frac{2\sqrt{3}}{3}$

end points of major axis: $\left(\pm \frac{\sqrt{2}}{2}, 0\right)$ 

65. $4y = 4x^2 - 20x + 23$; parabola since the equation is quadratic in only one variable.

$y - \frac{23}{4} = x^2 - 5x$

$y - \frac{23}{4} + \frac{25}{4} = x^2 - 5x + \frac{25}{4}$

$y + \frac{1}{2} = \left(x - \frac{5}{2}\right)^2$

center: $\left(2\frac{1}{2}, -\frac{1}{2}\right)$

$\frac{1}{4p} = 1, p = \frac{1}{4}$; focus:

$\left(2\frac{1}{2}, -\frac{1}{2} + \frac{1}{4}\right) = \left(2\frac{1}{2}, -\frac{1}{4}\right)$

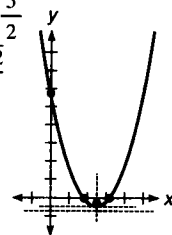
intercepts:

$x = 0: 4y = 23, y = \frac{23}{4}$

$y = 0: \frac{1}{2} = \left(x - \frac{5}{2}\right)^2$

$\pm \frac{\sqrt{2}}{2} = x - \frac{5}{2}$

$x = \frac{5}{2} \pm \frac{\sqrt{2}}{2}$



68. We know that $c = \frac{80}{2} = 40$,

and $2a = 60$, so $a = 30$.

$$c^2 = a^2 + b^2$$

$$40^2 = 30^2 + b^2$$

$$700 = b^2$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{900} - \frac{y^2}{700} = 1$$

70. $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$$\frac{y^2}{a^2} = \frac{x^2}{b^2} + 1$$

$$y^2 = \frac{a^2}{b^2}x^2 + a^2$$

$$y^2 \approx \frac{a^2}{b^2}x^2, \text{ as } x \text{ gets larger and larger.}$$

$$y \approx \pm \frac{a}{b}x$$

Exercise 11-4

Answers to odd-numbered problems

1. $(-2, -3), (3, 7)$

3. $\left(\frac{1 + \sqrt{33}}{4}, \frac{15 - \sqrt{33}}{8}\right),$
 $\left(\frac{1 - \sqrt{33}}{4}, \frac{15 + \sqrt{33}}{8}\right)$

5. $(-1, 1), (2\frac{1}{2}, 9\frac{3}{4})$

7. $(1, -6), (3, -4)$

9. $(-1 + \sqrt{11}, 1 + \sqrt{11}),$
 $(-1 - \sqrt{11}, 1 - \sqrt{11})$

11. $\left(\frac{1 - \sqrt{7}}{4}, \frac{-1 - \sqrt{7}}{2}\right),$
 $\left(\frac{1 + \sqrt{7}}{4}, \frac{-1 + \sqrt{7}}{2}\right)$

13. $(0, 1), (\frac{2}{3}, -\frac{1}{3})$

15. $(-\frac{1}{3}, -1\frac{2}{3}), (1, 1)$

17. $(\frac{1}{2}, -1\frac{1}{2})$

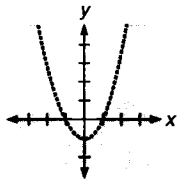
19. $(2, \sqrt{3}), (2, -\sqrt{3}), (-2, \sqrt{3}), (-2, -\sqrt{3})$

21. $\left(\frac{4\sqrt{7}}{7}, \frac{3\sqrt{14}}{7}\right), \left(\frac{4\sqrt{7}}{7}, -\frac{3\sqrt{14}}{7}\right),$
 $\left(-\frac{4\sqrt{7}}{7}, \frac{3\sqrt{14}}{7}\right), \left(-\frac{4\sqrt{7}}{7}, -\frac{3\sqrt{14}}{7}\right)$

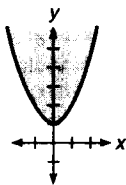
23. $5x^2 - 10x + 5y^2 - 20y - 56 = 0$

25. $(x - 2)^2 + (y - 5)^2 = 32$

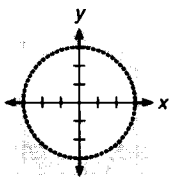
27.



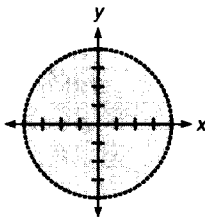
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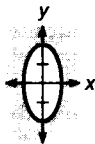
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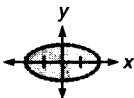
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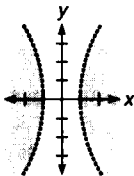
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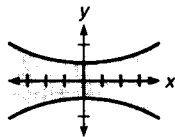
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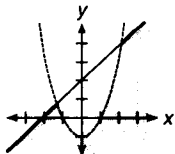
39.



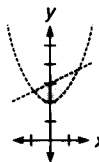
41.



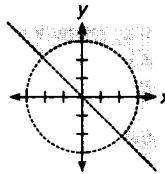
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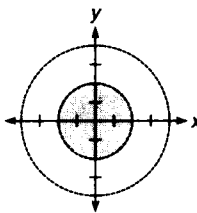
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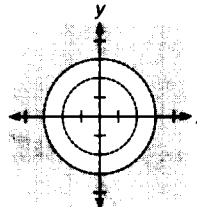
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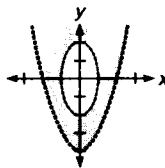
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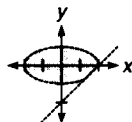
51.



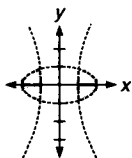
53.



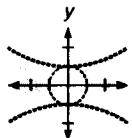
55.



57.



59.



61. 133 feet

$$\begin{aligned}
 63. \text{ a. } (5+x)(3-y) &\geq 15 \\
 15 - 5y + 3x - xy &\geq 15 \\
 -5y - xy &\geq -3x \\
 5y + xy &\leq 3x \\
 y(5+x) &\leq 3x \\
 y &\leq \frac{3x}{x+5}
 \end{aligned}$$

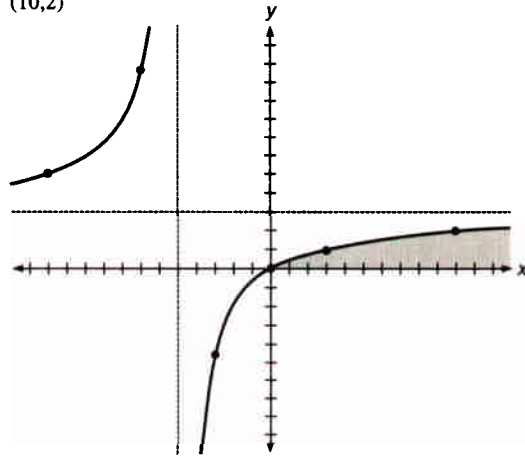
Note that we divided an inequality by $x+5$; we know that $x > 0$ so that $x+5 > 0$ also. Thus, the direction of the inequality is not affected.

b. We graph the equality

$$y = \frac{3x}{x+5} = 3 - \frac{15}{x+5}$$

Horizontal asymptote: $y = 3$ Vertical asymptote: $x = -5$

Intercepts: Origin

Additional points: $(-12, 5.1)$, $(-7, 10.5)$, $(-3, -4.5)$, $(3, 1.1)$, $(10, 2)$ 

Solutions to skill and review problems

1. $y - 3x = -9$

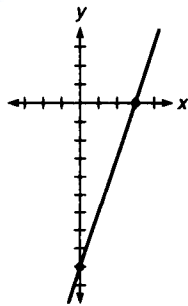
Straight line

Intercepts:

$x = 0: y - 0 = -9; (0, -9)$

$y = 0: 0 - 3x = -9$

$x = 3; (3, 0)$



2. $y - 3x^2 = -9$

$y = 3x^2 - 9$

Parabola

Vertex: $(0, -9)$

Intercepts:

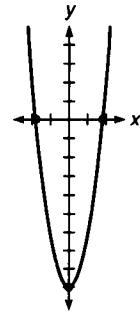
$x = 0: y = 0 - 9 = -9; (0, -9)$

$y = 0: 0 = 3x^2 - 9$

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}; (\pm\sqrt{3}, 0)$



3. $y^2 - 3x^2 = 9$

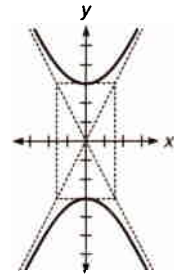
Hyperbola

Intercepts:

$x = 0: y^2 = 9$

$y = \pm 3; (0, \pm 3)$

$y = 0: -3x^2 = 9; \text{No real solution.}$



4. $y^2 + 3x^2 = 9$

Ellipse

Intercepts:

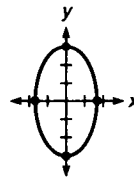
$x = 0: y^2 = 9$

$y = \pm 3; (0, \pm 3)$

$y = 0: 3x^2 = 9$

$x^2 = 3$

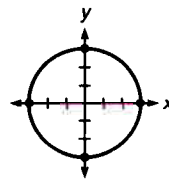
$x = \pm\sqrt{3}; (\pm\sqrt{3}, 0)$



5. $3y^2 + 3x^2 = 9$

$y^2 + x^2 = 3$

Divide each member by 3.

Circle; center at origin, radius = $\sqrt{3}$.

6. $y = 2x^5 + x^4 - 10x^3 - 5x^2 + 8x + 4$

Possible rational zeros: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$.

We use synthetic division to find rational zeros.

	2	1	-10	-5	8	4
		2	3	-7	-12	-4
1	2	3	-7	-12	-4	0

	2	3	-7	-12	-4
		4	14	14	4
2	2	7	7	2	0

	2	7	7	2
		-4	-6	-2
-2	2	3	1	0

$$y = (x-1)(x-2)(x+2)(2x^2+3x+1)$$

$$y = (x-1)(x-2)(x+2)(2x+1)(x+1)$$

Intercepts:

$$x = 0: y = 4; (0, 4)$$

$$y = 0: 0 = (x-1)(x-2)(x+2)$$

$$(2x+1)(x+1)$$

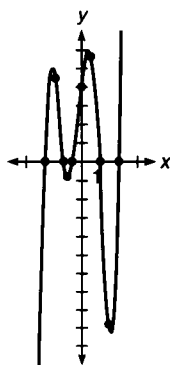
$$x = 1, 2, -2, -\frac{1}{2}, -1$$

$$(1, 0), (2, 0), (-2, 0), (-\frac{1}{2}, 0),$$

$$(-1, 0)$$

Additional points: $(-1.5, 4.4),$

$(-0.75, -0.75), (0.5, 5.6), (1.5, -8.8)$



Solutions to trial exercise problems

5. $y = 3x^2 - 2x - 4$

$$y = x^2 + x + 1$$

$$3x^2 - 2x - 4 = x^2 + x + 1$$

$$2x^2 - 3x - 5 = 0$$

$$(2x-5)(x+1) = 0$$

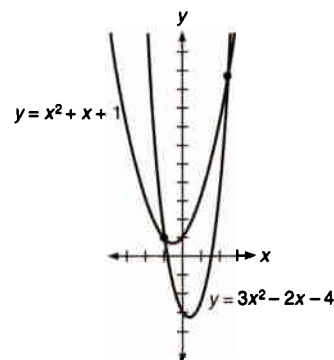
$$x = \frac{5}{2} \text{ or } -1$$

$$y = x^2 + x + 1$$

$$x = -1: y = (-1)^2 + (-1) + 1 = 1$$

$$x = \frac{5}{2}: y = (\frac{5}{2})^2 + \frac{5}{2} + 1 = \frac{25}{4} + \frac{10}{4} + \frac{4}{4} = \frac{39}{4}$$

The points are $(-1, 1)$ and $(\frac{5}{2}, \frac{39}{4})$.



19. $x^2 - y^2 = 1$ so $x^2 = y^2 + 1$

$$2y^2 - x^2 = 2 \text{ so } x^2 = 2y^2 - 2$$

$$2y^2 - 2 = y^2 + 1$$

$$y^2 = 3$$

$$y = \pm\sqrt{3}$$

$$x^2 = y^2 + 1$$

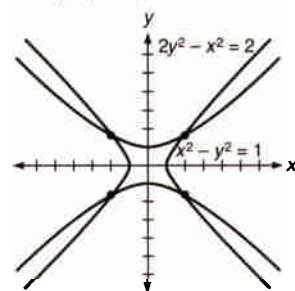
$$x^2 = 3 + 1$$

$x = \pm 2$, so the points of intersection

are $(2, \sqrt{3}), (2, -\sqrt{3}),$

$(-2, \sqrt{3}), (-2, -\sqrt{3}),$ or about $(2, \pm 1.7)$

and $(-2, \pm 1.7).$



23. Let L represent the line $y = \frac{1}{2}x - 3$.

Let (a, b) be the point where the circle is tangent to the line L . Let L' be the line which passes through $(1, 2)$ and the point (a, b) .

Since the slope of L is $\frac{1}{2}$, the slope of L' is -2 (section 3-2). Using $m = -2$ and the point $(1, 2)$ we can find the equation of L' to be $y = -2x + 4$.

We can now find the point (a, b) by solving the system of equations

$$y = \frac{1}{2}x - 3$$

$$y = -2x + 4$$

$$\frac{1}{2}x - 3 = -2x + 4$$

$$x - 6 = -4x + 8$$

$$5x = 14$$

$$x = \frac{14}{5}$$

$$y = -2x + 4 = -2(\frac{14}{5}) + 4 = -\frac{8}{5}$$

Thus, (a, b) is $(\frac{14}{5}, -\frac{8}{5})$.

Now find the distance between $(1, 2)$ and $(\frac{14}{5}, -\frac{8}{5})$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(\frac{14}{5} - 1)^2 + (-\frac{8}{5} - 2)^2}$$

$$= \sqrt{(\frac{9}{5})^2 + (-\frac{18}{5})^2}$$

$$= \sqrt{\frac{405}{25}} = \frac{9}{5}\sqrt{5}$$

This is the radius of the circle. Thus,

the center of the circle is $(h, k) = (1, 2)$

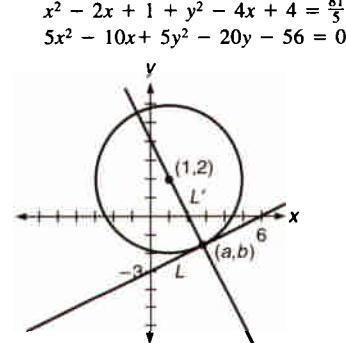
and $r = \frac{9}{5}\sqrt{5}$.

$$\text{Circle: } (x - h)^2 + (y - k)^2 = r^2$$

$$(x - 1)^2 + (y - 2)^2 = (\frac{9}{5}\sqrt{5})^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = \frac{81}{5}$$

$$5x^2 - 10x + 5y^2 - 20y - 56 = 0$$



24. The circles have equations $(x + 3)^2 + (y - 3)^2 = 9$ or $x^2 + 6x + y^2 - 6y + 9 = 0$ and $x^2 + y^2 = 25$. To find where they intersect we need to solve one of them for y and substitute into the other equation. Using $x^2 + y^2 = 25$ we obtain $y^2 = 25 - x^2$ and $y = \pm\sqrt{25 - x^2}$.

We can see that $y > 0$ for the points that interest us, so we will use $y = \sqrt{25 - x^2}$. Substituting these values for y into the first equation we obtain $x^2 + 6x + (25 - x^2) - 6(\sqrt{25 - x^2}) + 9 = 0$

$$6x + 34 = 6\sqrt{25 - x^2}$$

$$3x + 17 = 3\sqrt{25 - x^2}$$

Now square both sides.

$$9x^2 + 102x + 289 = 9(25 - x^2)$$

$$18x^2 + 102x + 64 = 0$$

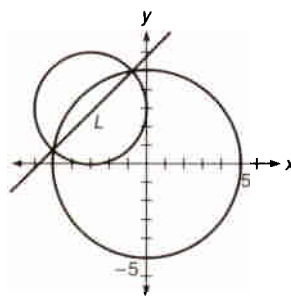
$$9x^2 + 51x + 32 = 0$$

$$x = \frac{-17 \pm \sqrt{161}}{6} \approx -0.71857, -4.9481$$

Note that we are keeping the first five nonzero digits in each value for now so that our final answer will have two-place accuracy.

To find the y -values we use $y = \sqrt{25 - x^2}$. From the last result we compute y and find that it is 0.71857 and 4.9481. These are just the absolute values of the x -values, which is not

surprising given the symmetry of the points, as seen in the graph. Thus the two points of intersection are $(-0.71857, 4.9481)$, $(-4.9481, 0.71857)$.



We can see from the graph or by computation that the slope of the line we want is 1, so using $y = x + b$ we compute b from either of the points we have. Using the first we obtain $4.9481 = -0.71857 + b$, so $b \approx 5.67$, to two decimal places. We can also see by some retracing of our steps that b is exactly

$$\frac{17 + \sqrt{161}}{6} - \frac{-17 + \sqrt{161}}{6} = \frac{17}{3}.$$

Thus, an approximate equation of the line is $y = x + 5.67$, and an exact solution is $y = x + 5\frac{2}{3}$.

25. The circle has equation

$$(x - 2)^2 + (y - 5)^2 = r^2$$

The circle touches the line $y = -x - 1$ at one point (since it is tangent to it); at this point, y may be replaced by $-x - 1$:

$$(x - 2)^2 + (y - 5)^2 = r^2$$

$$(x - 2)^2 + ((-x - 1) - 5)^2 = r^2$$

$$2x^2 + 8x + 40 - r^2 = 0$$

Now apply the quadratic formula with

$$a = 2, b = 8, \text{ and } c = 40 - r^2:$$

$$x = \frac{-8 \pm \sqrt{64 - 4(2)(40 - r^2)}}{4}$$

$$= \frac{-8 \pm \sqrt{8r^2 - 256}}{4}$$

$$= -2 \pm \frac{1}{4}\sqrt{4(2r^2 - 64)}$$

$$= -2 \pm \frac{1}{2}\sqrt{2r^2 - 64}$$

We know that where the line touches the circle there is only one point, and therefore one value of x . This happens only if $2r^2 - 64$ is zero.

$$2r^2 - 64 = 0$$

$$2r^2 = 64$$

$$r^2 = 32$$

Thus, we learn the value of r^2 , and so the equation of the circle is

$$(x - 2)^2 + (y - 5)^2 = 32$$

34. $4x^2 + y^2 < 4$

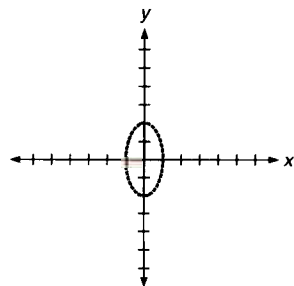
Graph the ellipse $4x^2 + y^2 = 4$; use $(0, 0)$ as a test point.

$$4x^2 + y^2 < 4$$

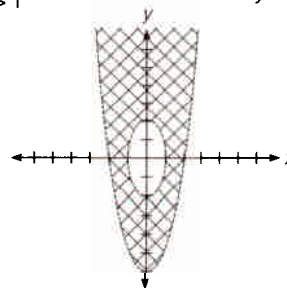
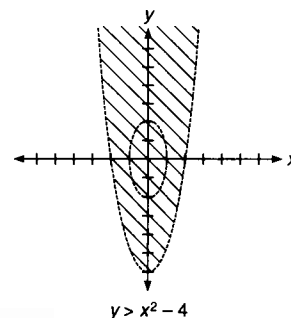
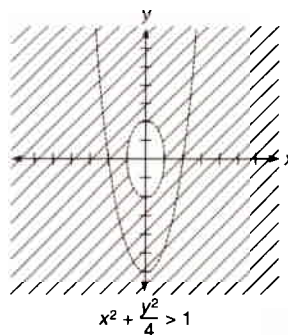
$$4(0) + 0 < 4$$

$$0 < 4$$

True, so the solution is the part of the plane that contains the origin.



- 53.



61. Since z is the time it takes to fall to the bottom of the well we know that $s = 16z^2$. Since the time to come back up is $3 - z$ seconds we know that $s = 1,100(3 - z)$. Thus,

$$s = 16z^2$$

$$s = 1,100(3 - z)$$

so

$$16z^2 = 1,100(3 - z)$$

$$16z^2 + 1,100z - 3,300 = 0$$

$$4z^2 + 275z - 825 = 0$$

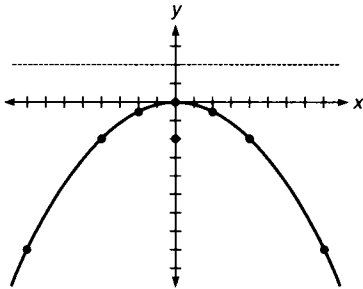
$$z = \frac{-275 \pm \sqrt{(-275)^2 - 4(4)(-825)}}{2(4)}$$

$$z = \frac{-275 \pm \sqrt{88,825}}{8} \approx -71.6, 2.879$$

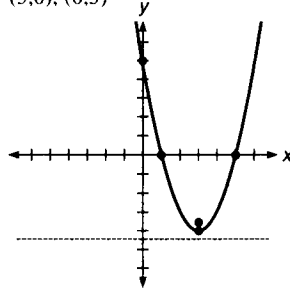
Ignoring the negative value for time, we find that it takes 2.879 seconds for the rock to fall. At this point s is computed as $s = 16(2.879^2) \approx 132.6$ feet. It takes the remaining $3 - 2.879$ or 0.121 seconds for the sound to travel back up the well, so $s = 1,100(0.121) \approx 133.1$ feet. Thus, both calculations show a depth of the well of 133 feet, to the nearest foot. (The results will be the same if more decimal places are used in the approximation of z).

Chapter 11 review

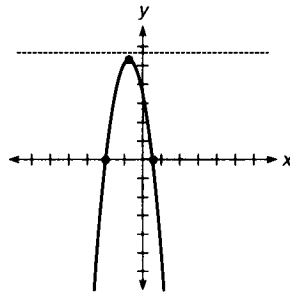
1. vertex: (0,0); focus: (0,-2); directrix: $y = 2$; all intercepts at the origin



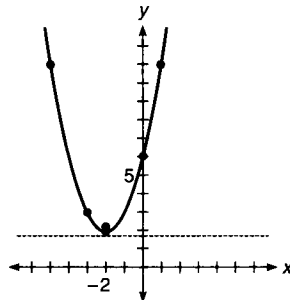
2. vertex: (3,-4); focus is at $(3, -3\frac{3}{4})$; directrix is $y = -4\frac{1}{4}$; intercepts: (1,0), (5,0), (0,5)



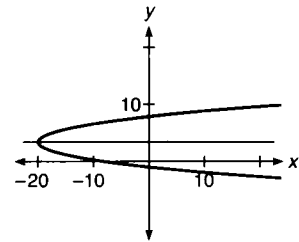
3. vertex: $(-\frac{2}{3}, 5\frac{1}{3})$; focus: $(-\frac{2}{3}, 5\frac{1}{4})$; directrix: $y = 5\frac{5}{12}$; intercepts: $(\frac{2}{3}, 0)$, (-2,0), (0,4)



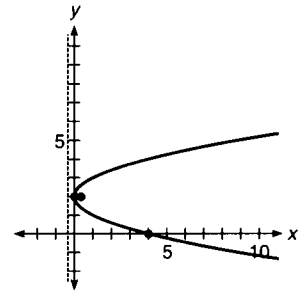
4. vertex: (-2,2); focus: $(-2, 2\frac{1}{4})$; directrix: $y = 1\frac{3}{4}$; intercepts: (0,6)



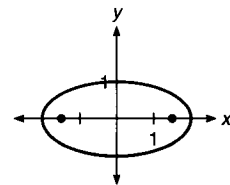
5. vertex: $(-20\frac{1}{4}, 3\frac{1}{2})$; focus: $(-20, 3\frac{1}{2})$; directrix: $x = -20\frac{1}{2}$; intercept: (0,-1), (0,8), (-8,0)



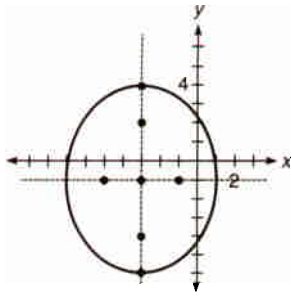
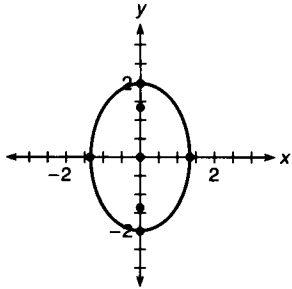
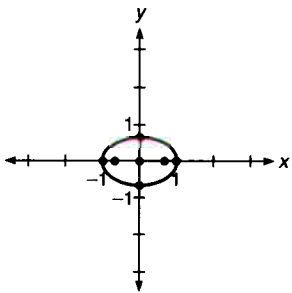
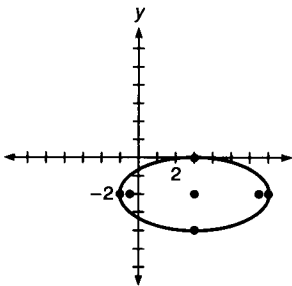
6. vertex: (0,2); focus: $(\frac{1}{4}, 2)$; directrix: $x = -\frac{1}{4}$; intercept: (0,2), (4,0)



7. $y = -\frac{1}{10}(x-1)^2 - \frac{1}{2}$
 8. $y = \frac{1}{2}(x+3)^2 - \frac{3}{2}$
 9. $y = -(x-2)^2 - 1$
 10. $y = \frac{1}{4}(x+4)^2 + 1$
 11. $y = 4(x-3)^2 - 1$
 12. $w = 16\sqrt{10}$
 13. $d = 9\frac{3}{8}$
 14. $h = \frac{25}{16}$
 15. foci: $(-\sqrt{3}, 0)$, $(\sqrt{3}, 0)$



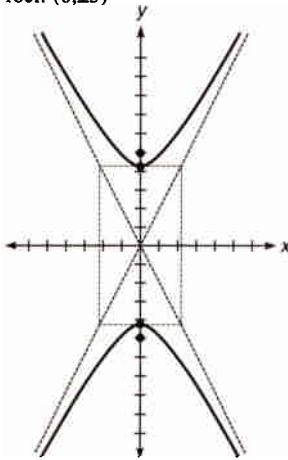
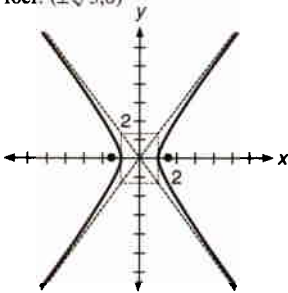
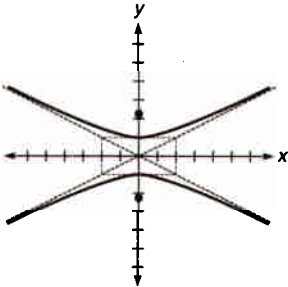
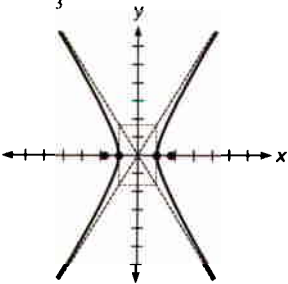
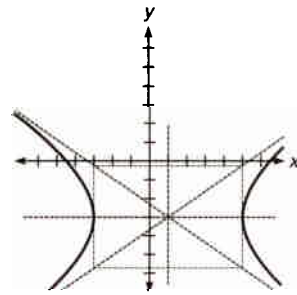
16.

17. foci: $(0, \pm\sqrt{2})$; minor axis: $(\pm\sqrt{2}, 0)$ 18. foci: $(\pm\frac{\sqrt{2}}{2}, 0)$; minor axis: $(0, \pm\frac{\sqrt{2}}{2})$ 19. foci: $(3 \pm 2\sqrt{3}, -2)$ 

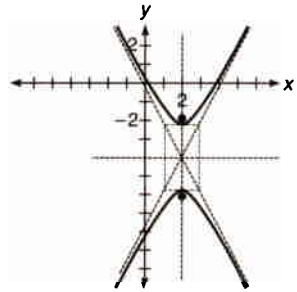
20. $\frac{x^2}{16} + \frac{y^2}{7} = 1$

21. $\frac{x^2}{9} + \frac{y^2}{16} = 1$

22. $\frac{x^2}{16} + \frac{y^2}{7} = 1$

23. foci: $(0, \pm 5)$ 24. foci: $(\pm\sqrt{3}, 0)$ 25. $y^2 - \frac{x^2}{4} = 4$; foci at $(0, \pm\sqrt{5})$ 26. $x^2 - \frac{y^2}{8} = 1$; foci: $(\pm\frac{\sqrt{33}}{3}, 0)$ 27. foci: $(1 \pm 2\sqrt{6}, -3)$ 

28. $\frac{(y+4)^2}{3} - (x-2)^2 = 1$



29. hyperbola; $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{\frac{9}{2}} = 1$

30. ellipse; $\frac{y^2}{4} + \frac{x^2}{9} = 1$

31. ellipse; $\frac{(x+3)^2}{2} + \frac{(y-3)^2}{4} = 1$

32. circle; $(x - \frac{3}{2})^2 + y^2 = \frac{39}{4}$

33. parabola; $y = (x + \frac{3}{2})^2 - \frac{25}{4}$

34. ellipse; $\frac{(x+4)^2}{36} + \frac{(y-\frac{1}{2})^2}{9} = 1$

35. parabola; $x = (y - \frac{5}{2})^2 - \frac{1}{2}$

36. $(0, 4)$ and $(3\frac{2}{3}, 6\frac{4}{9})$

37. $(0, -1)$ and $(-\frac{8}{9}, \frac{7}{9})$

38. $(-\frac{1}{2} + \frac{1}{2}\sqrt{6}, \frac{3}{2} + \frac{1}{2}\sqrt{6})$
and $(-\frac{1}{2} - \frac{1}{2}\sqrt{6}, \frac{3}{2} - \frac{1}{2}\sqrt{6})$

39. $(-\frac{2}{13} + \frac{4}{13}\sqrt{231}, -\frac{40}{13} + \frac{2}{13}\sqrt{231})$ and
 $(-\frac{2}{13} - \frac{4}{13}\sqrt{231}, -\frac{40}{13} - \frac{2}{13}\sqrt{231})$

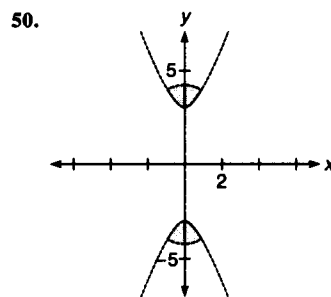
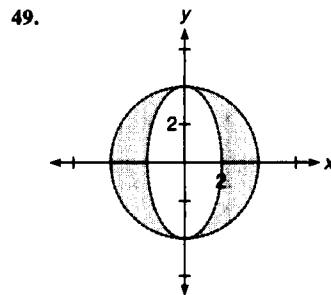
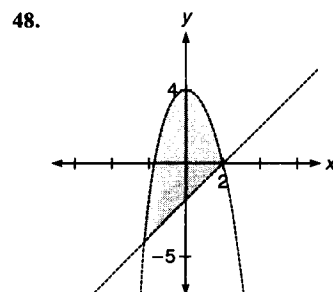
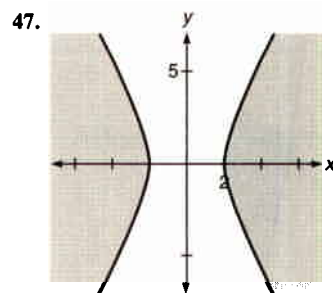
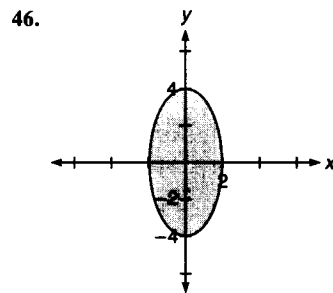
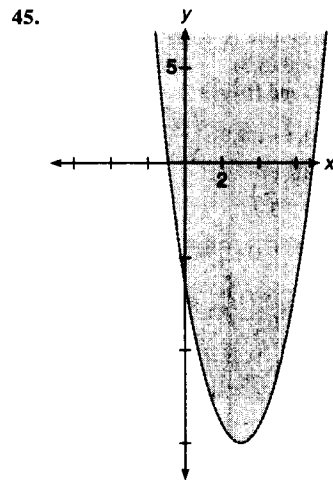
40. $(\frac{39}{31} + \frac{5}{31}\sqrt{41}, \frac{63}{31} + \frac{20}{31}\sqrt{41})$ and
 $(\frac{39}{31} - \frac{5}{31}\sqrt{41}, \frac{63}{31} - \frac{20}{31}\sqrt{41})$

41. $(1\frac{1}{13}, -\frac{14}{13})$

42. $(-4, 7)$ and $(1, -3)$

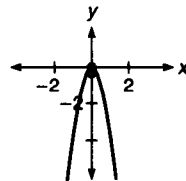
43. $(3, 16)$ and $(-1, 0)$

44. $(x+2)^2 + (y-3)^2 = \frac{121}{10}$

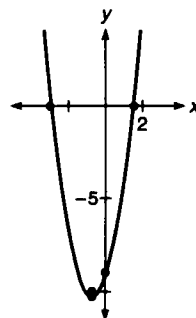


Chapter 11 test

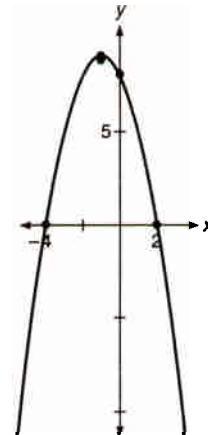
1. all intercepts and vertex at the origin;
focus: $(0, -\frac{1}{16})$; directrix: $y = \frac{1}{16}$



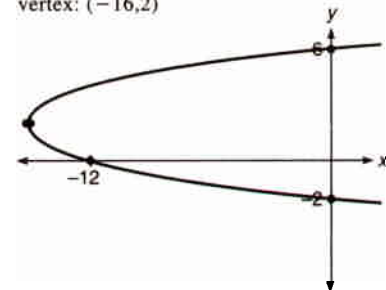
2. intercepts: $(-3, 0)$, $(1\frac{1}{2}, 0)$, $(0, -9)$;
focus: $(-\frac{3}{4}, -10)$; directrix: $y = -10\frac{1}{4}$; vertex: $(-\frac{3}{4}, -10\frac{1}{8})$



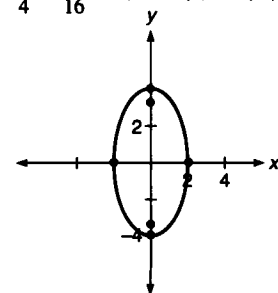
3. intercepts: $(-4, 0)$, $(2, 0)$, $(0, 8)$;
focus: $(-1, 8\frac{3}{4})$; directrix: $y = 9\frac{1}{4}$;
vertex: $(-1, 9)$



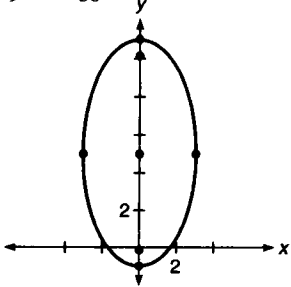
4. intercepts: $(0, -2)$, $(0, 6)$, $(-12, 0)$;
focus: $(-15\frac{3}{4}, 2)$; directrix: $x = -16\frac{1}{4}$;
vertex: $(-16, 2)$



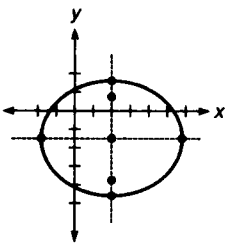
5. $y = \frac{1}{2}(x - 1)^2 + 2\frac{1}{2}$
6. $y = -\frac{1}{8}(x + 2)^2$
7. $w = 8\sqrt{6}$
8. $h = 2\frac{1}{2}$
9. $\frac{x^2}{4} + \frac{y^2}{16} = 1$; foci: $(0, 2 \pm \sqrt{3})$



10. $\frac{x^2}{9} + \frac{(y-5)^2}{36} = 1$; foci: $(0, 5 \pm 3\sqrt{3})$



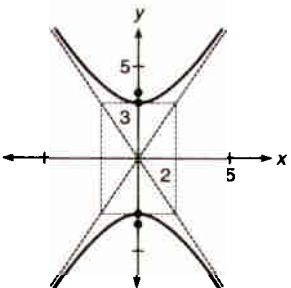
11. $\frac{(x-2)^2}{15} + \frac{(y+\frac{3}{2})^2}{10} = 1$; ends of minor axis: $(2, -1\frac{1}{2} \pm \sqrt{10})$; ends of major axis: $(2 \pm \sqrt{15}, -1\frac{1}{2})$; foci: $(2 \pm \sqrt{5}, -1\frac{1}{2})$



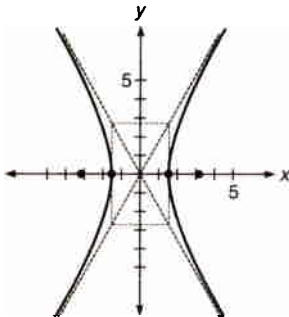
12. $\frac{x^2}{25} + \frac{y^2}{16} = 1$

13. $\frac{x^2}{64} + \frac{y^2}{48} = 1$

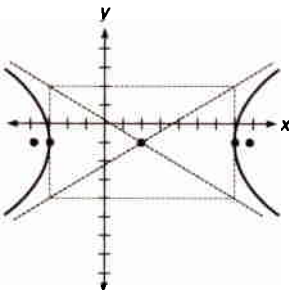
14. foci: $(0, \pm\sqrt{13})$



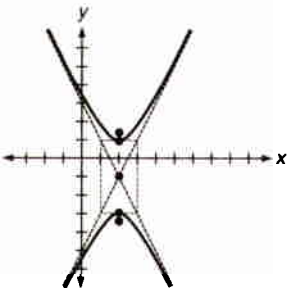
15. $\frac{x^2}{2} - \frac{y^2}{8} = 1$; ends of major axis: $(\pm\sqrt{2}, 0)$; foci: $(\pm\sqrt{10}, 0)$



16. $\frac{(x-2)^2}{25} - \frac{(y+1)^2}{9} = 1$; foci: $(2 \pm \sqrt{34}, -1)$



17. $\frac{(y+1)^2}{4} - (x-2)^2 = 1$; foci: $(2, -1 \pm \sqrt{5})$



18. straight line; $4x + 20y - 23 = 0$

19. hyperbola; $\frac{y^2}{2} - \frac{x^2}{6} = 1$

20. degenerate ellipse; actually just the point $(0, 3)$; $2x^2 + (y-3)^2 = 0$

21. hyperbola; $\frac{(y-2)^2}{\frac{5}{4}} - \frac{(x+\frac{1}{2})^2}{\frac{5}{4}} = 1$

22. circle; $\frac{(x-\frac{3}{2})^2}{\frac{9}{4}} + \frac{y^2}{\frac{9}{4}} = 1$

23. circle; $(x+4)^2 + (y-2)^2 = 40$

24. parabola; $y = (x + \frac{3}{2})^2 - \frac{33}{4}$

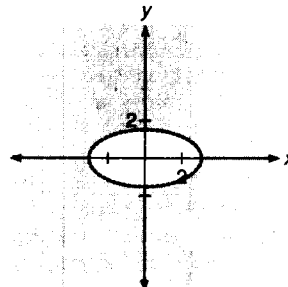
25. $(1, 3)$ and $(3, 7)$

26. $(0, -1)$ and $(1\frac{1}{2}, \frac{1}{2})$

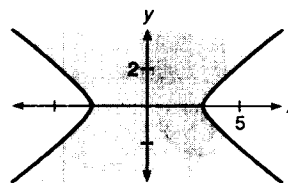
27. $(\sqrt{2}, 0)$, $(-\sqrt{2}, 0)$, $(\frac{\sqrt{39}}{3}, \frac{7}{3})$, $(-\frac{\sqrt{39}}{3}, \frac{7}{3})$

28. $(x+1)^2 + (y-3)^2 = \frac{49}{5}$

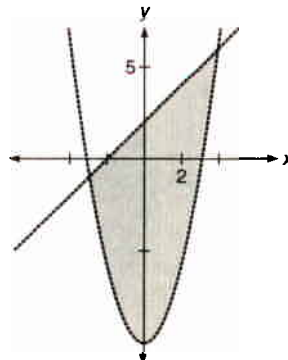
29.



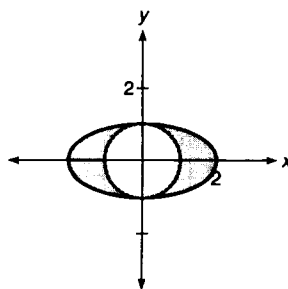
30.



31.



32.



33. outer ellipse: $\frac{x^2}{64} + \frac{y^2}{16} = 1$
 inner ellipse: $\frac{x^2}{4} + \frac{y^2}{16} = 1$
 circle: $x^2 + y^2 = 16$

Chapter 12

Exercise 12-1

Answers to odd-numbered problems

1. $-\frac{1}{2}, 2, \frac{9}{2}, 7, \dots$
 3. 1, 0, -1, 0, \dots
 5. 3, 3, 3, 3, \dots
 7. -1, -2, -1, 2, \dots
 9. $\frac{1}{2}, \frac{\sqrt{2}}{3}, \frac{\sqrt{3}}{4}, \frac{2}{5}, \dots$
 11. -4, 0, 6, 14, \dots
 13. $3n - 1$ 15. $4n - 24$
 17. $(-1)^n \left(\frac{1}{n}\right)$ 19. $\frac{n}{5n + 1}$
 21. $(-1)^{n+1} \left(\frac{n^2}{(n+1)^2}\right)$
 23. $(-1)^{n+1}$ 25. approximately 950
 27. arithmetic sequence; $d = 2\frac{1}{2}$
 29. neither 31. arithmetic sequence, $d = 0$, and geometric sequence, $r = 1$
 33. neither 35. neither 37. neither
 39. arithmetic; $d = 3$ 41. arithmetic; $d = 4$ 43. neither 45. neither
 47. neither 49. geometric; $r = -1$
 51. 61 53. 33 55. $34\frac{2}{7}$
 57. 202 59. 14 61. 16
 63. -8 65. $-\frac{3}{4}$ 67. $\frac{3}{2}$
 69. $\frac{5}{27}$ 71. 1,125 73. $\frac{5}{8}$
 75. \$26,000; \$28,080; \$30,326.40; \$32,752.51; \$35,372.71; \$38,202.53
 77. 23.5% 79. a. 21, 27, 33, 39
 b. yes c. $d_n = 6n + 15$; 135
 81. Yes. We know that $a_n = a_1 + (n-1)d_a$ for some constant d_a , and that $b_n = b_1 + (n-1)d_b$ for some constant d_b .
 Thus,

$$c_n = a_n + b_n$$

$$= a_1 + (n-1)d_a + b_1 + (n-1)d_b$$

$$= a_1 + b_1 + (n-1)(d_a + d_b)$$
 By definition $a_1 + b_1 = c_1$, and let $d_c = d_a + d_b$, a constant, so that $c_n = c_1 + (n-1)d_c$, which is an arithmetic sequence.
 83. No. Let a be the arithmetic sequence 1, 2, 3, 4, \dots and b the arithmetic sequence 2, 4, 6, 8, \dots . Then c is

the sequence 2, 8, 18, 32, \dots , which is not an arithmetic sequence.

85. a. $\frac{3}{2}, \frac{9}{2}, \frac{27}{2}, \frac{81}{2}$
 b. yes c. $d_n = \frac{1}{2}(3^n)$; $d_5 = \frac{243}{2}$
 87. No. Let a be the geometric sequence $1, \frac{1}{2}, \frac{1}{4}, \dots$ and b be the geometric sequence 1, 2, 4, \dots . Then c is the sequence $2, 2\frac{1}{2}, 4\frac{1}{4}, \dots$, and

$$\frac{c_2}{c_1} = \frac{5}{4}, \text{ while } \frac{c_3}{c_2} = \frac{17}{10}, \text{ so there is no constant ratio.}$$

 89. Yes. Let

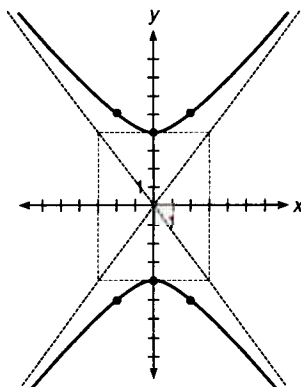
$$c_n = (a_n)(b_n)$$

$$= [a_1(r_a)^{n-1}][b_1(r_b)^{n-1}]$$

$$= (a_1b_1)(r_ar_b)^{n-1}$$
 since $a_1 \neq 0, b_1 \neq 0, r_a \neq 0, r_b \neq 0$, then $a_1b_1 \neq 0$, and $r_ar_b \neq 0$, so c_n is a geometric sequence.
 91. All of them. Observe that the same sequence of numbers can be generated by different values of a_1 and r .
 93. a. $b_n = \frac{1}{2}n^2 - \frac{9}{2}n + 13, b_4 = 3$
 b. $b_n = -\frac{1}{6}n^2 - \frac{1}{2}n + \frac{11}{3}, b_4 = -1$
 c. $b_n = \frac{5}{2}n^2 - \frac{19}{2}n + 10, b_4 = 12$

Solutions to skill and review problems

1. $3 + 6 + 9 + \dots + 3n = 231$
 $3(1 + 2 + 3 + \dots + n) = 3(77)$
 $1 + 2 + 3 + \dots + n = 77$.
 Thus the sum is 77.
 2. $(1 - 5) + (5 - 9) + (9 - 13) + \dots + (81 - 85)$
 $1 - 5 + 5 - 9 + 9 - 13 + \dots + 77 - 81 + 81 - 85$
 $1 - 85$
 -84
 3. $x_1 + x_2 + x_3 + \dots + x_n = 420$
 $3(x_1 + x_2 + x_3 + \dots + x_n) = 3(420)$
 $3x_1 + 3x_2 + 3x_3 + \dots + 3x_n = 1260$
 4. $(a_1 + a_2 + a_3 + \dots + a_n) + (b_1 + b_2 + b_3 + \dots + b_n) = 500 + 200$
 $(a_1 + b_1) + (a_2 + b_2) + (a_3 + b_3) + \dots + (a_n + b_n) = 700$
 5. $\frac{y^2}{16} - \frac{x^2}{9} = 1$



6. $|3 - \frac{1}{2}x| > 12$
 $3 - \frac{1}{2}x > 12$ or $3 - \frac{1}{2}x < -12$
 $-\frac{1}{2}x > 9$ or $-\frac{1}{2}x < -15$
 $-2(-\frac{1}{2}x) < -2(9)$ or $-2(-\frac{1}{2}x) > -2(-15)$
 $x < -18$ or $x > 30$
 $x < -18$ or $x > 30$

Solutions to trial exercise problems

15. -20, -16, -12, \dots
 $-20 + 0(4), -20 + 1(4), -20 + 2(4),$
 \dots
 $-20 + (n-1)(4)$
 $-20 + 4n - 4$
 $4n - 24$
 19. $\frac{1}{6}, \frac{2}{11}, \frac{3}{16}, \frac{4}{21}, \dots$

$$\frac{1}{1(5) + 1}, \frac{2}{2(5) + 1}, \frac{3}{3(5) + 1}, \dots$$

$$\frac{4}{4(5) + 1}, \dots, \frac{n}{5n + 1}$$

25. The sequence 300, 400, 530, 710, . . . is definitely not an arithmetic sequence since the difference between terms is increasing. We therefore guess that it is a geometric sequence. The ratios of successive terms is $\frac{400}{300} = 1\frac{1}{3} \approx 1.33$, $\frac{530}{400} = 1\frac{13}{40} \approx 1.325$, $\frac{710}{530} = 1\frac{18}{53} \approx 1.34$. It seems reasonable to assume a constant ratio of $1\frac{1}{3}$, and therefore to estimate the next measurement as $710(\frac{4}{3}) = 947$, or about 950.

30. Geometric with ratio $\frac{1}{2}$ since each term is the previous term multiplied by $\frac{1}{2}$.

35. $\frac{1}{2}, \frac{\sqrt{2}}{3}, \frac{\sqrt{3}}{4}, \frac{2}{5}, \dots$
 $\frac{\sqrt{2}}{3} - \frac{1}{2} = \frac{2\sqrt{2}-3}{6}$, and
 $\frac{\sqrt{3}}{4} - \frac{\sqrt{2}}{3} = \frac{3\sqrt{3}-4\sqrt{2}}{12}$, so there is
 no constant difference. $\frac{\frac{\sqrt{2}}{3}}{\frac{1}{2}} = \frac{2\sqrt{2}}{3}$,

$$\text{and } \frac{\frac{\sqrt{3}}{4}}{\frac{\sqrt{2}}{3}} = \frac{3\sqrt{3}}{4\sqrt{2}} = \frac{3\sqrt{6}}{8}, \text{ so there is no}$$

constant ratio. Thus this sequence is neither arithmetic nor geometric.

41. $-20, -16, -12, \dots$
 arithmetic; $d = 4$

55. $a_{15} = a_1 + (15-1)d$
 $40 = -40 + 14d$
 $d = \frac{40}{7}$
 so $a_{14} = -40 + 13(\frac{40}{7}) = 34\frac{2}{7}$

61. $a_{15} = a_1 + 14d$
 $49 = a_1 + 14d$
 $a_{28} = a_1 + 27d$
 $88 = a_1 + 27d$
 Thus, $a_1 = 49 - 14d$
 and $a_1 = 88 - 27d$
 so $49 - 14d = 88 - 27d$
 $13d = 39$
 $d = 3$
 $a_1 = 88 - 27d$
 so $a_1 = 88 - 81 = 7$.
 Thus, $a_4 = a_1 + 3d = 7 + 3(3) = 16$.

$$70. a_3 = \frac{1}{3} = a_1 r^2, a_6 = -\frac{1}{81} = a_1 r^5,$$

$$\text{so } a_1 = \frac{1}{3r^2} \text{ and}$$

$$a_1 = -\frac{1}{81r^5}, \text{ so } \frac{1}{3r^2} = -\frac{1}{81r^5},$$

$$\text{so } 3r^2 = -81r^5 \text{ (divide by } r^2), 3 =$$

$$-81r^3, -\frac{1}{27} = r^3, r = -\frac{1}{3}.$$

$$a_1 = \frac{1}{3r^2} = \frac{1}{3(-\frac{1}{3})^2} = 3.$$

Thus, we know a_1 and r . $a_2 = a_1 r$
 $= 3(-\frac{1}{3}) = -1$.

74. The length of each swing forms a geometric sequence with $a_1 = 20$ and $r = 0.95$.

a. $a_4 = 20(0.95)^3 \approx 17.1$ inches

b. $a_8 = 20(0.95)^7 \approx 14.0$ inches

90. Every fifth roll of film is developed free. Let n = number of rolls developed, then a_n , the average cost for n rolls, is the ratio of total cost for n rolls to n :

$\frac{\text{total cost to develop } n \text{ rolls}}{\text{number of rolls}}$. This value is indicated in the following table.

Number of rolls n	Cost for roll	Total cost	a_n	Form of a_n	Value of $\left[\frac{n}{5}\right]$
1	5	5	$\frac{5}{1}$	$\frac{5(n-0)}{n}$	0
2	5	10	$\frac{10}{2}$	$\frac{5(n-0)}{n}$	0
3	5	15	$\frac{15}{3}$	$\frac{5(n-0)}{n}$	0
4	5	20	$\frac{20}{4}$	$\frac{5(n-0)}{n}$	0
5	0	20	$\frac{20}{5}$	$\frac{5(n-1)}{n}$	1
6	5	25	$\frac{25}{6}$	$\frac{5(n-1)}{n}$	1
7	5	30	$\frac{30}{7}$	$\frac{5(n-1)}{n}$	1
8	5	35	$\frac{35}{8}$	$\frac{5(n-1)}{n}$	1
9	5	40	$\frac{40}{9}$	$\frac{5(n-1)}{n}$	1
10	0	40	$\frac{40}{10}$	$\frac{5(n-2)}{n}$	2
11	5	45	$\frac{45}{11}$	$\frac{5(n-2)}{n}$	2

The numerators in the a_n column are of the form $\frac{5(n-i)}{n}$, where i is the quotient, without

the remainder, of $n \div 5$. This value is $\left[\frac{n}{5}\right]$. Thus, $a_n = \frac{5\left(n - \left[\frac{n}{5}\right]\right)}{n}$.

Exercise 12-2

Answers to odd-numbered problems

1. $5 + 9 + 13 + 17$
3. $6 + 12 + 20 + 30$ 5. $\frac{3}{4} + \frac{4}{5}$
7. $-\frac{4}{3} + \frac{2}{3} - \frac{4}{9} + \frac{1}{3} - \frac{4}{15} + \frac{2}{9}$
9. $1 + (1 + 4) + (1 + 4 + 9) + (1 + 4 + 9 + 16)$ 11. 1,584
13. -570 15. $-45\frac{1}{2}$ 17. -294
19. 418 21. -490 23. $132\frac{1}{2}$
25. 246 27. 1,365 29. $-\frac{369}{256}$
31. 129 33. $22\frac{163}{804}$ 35. 6,560
37. $121\frac{1}{3}$ 39. $\frac{2,062}{3,125}$ 41. $5\frac{85}{256}$
43. $1\frac{1}{2}$ 45. $\frac{1}{3}$ 47. $-\frac{2}{5}$
49. not defined 51. 9 53. $\frac{3}{5}$ 55. $\frac{2}{9}$
57. $\frac{28}{99}$ 59. $\frac{98}{111}$ 61. $\frac{5,155}{9,999}$
63. $\frac{3,401}{9,900}$ 65. $\frac{1,987}{4,950}$
67. 400 inches 69. 1,024 feet
71. 7 seconds 73. 6 hours
75. $2^{64} - 1 \approx 1.8 \times 10^{19}$ grains of wheat.
(This is more wheat than has ever existed.)
77. 39 boxes 79. 37% 81. \$8,031.25
83. \$250,000 85. 5,050 87. 300 miles
89. 3 91. About 6.1 days

Solutions to skill and review problems

1. $a_n = a_1 + (n - 1)d$; $a_1 = 3$, $d = 5$,
 $n = 33$
 $a_{33} = 3 + 32(5) = 163$
2. $a_n = a_1 r^{n-1}$; $a_1 = 3$, $r = 5$, $n = 5$
 $a_5 = 3 \cdot 5^4 = 1,875$
3. $\left| \frac{x+2}{x} \right| < 4$

Nonlinear inequality; use the critical point/test point method.

Critical points:

- a. Solve the corresponding equality.

$$\left| \frac{x+2}{x} \right| = 4$$

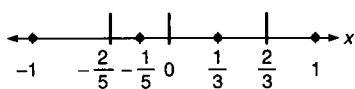
$$\frac{x+2}{x} = 4 \text{ or } \frac{x+2}{x} = -4$$

$$x+2 = 4x \quad x+2 = -4x$$

$$2 = 3x \quad 5x = -2$$

$$\frac{2}{3} = x \quad x = -\frac{2}{5}$$

- b. Find zeros of denominators. $x = 0$
Critical points are $-\frac{2}{5}$, 0 , $\frac{2}{3}$.



We will use test points of -1 , $-\frac{1}{5}$, $\frac{1}{3}$, 1 .

$$\left| \frac{x+2}{x} \right| < 4$$

$$x = -1: |-1| < 4; \text{ true}$$

$$x = -\frac{1}{5}: |-9| < 4; \text{ false}$$

$$x = \frac{1}{3}: |7| < 4; \text{ false}$$

$$x = 1: |3| < 4; \text{ true}$$

$$x < -\frac{2}{5} \text{ or } x > \frac{2}{3}$$

4. $f(x) = x^2 + 5x - 6$

Parabola; complete the square.

$$y = x^2 + 5x + \frac{25}{4} - 6 - \frac{25}{4},$$

$$\text{since } \frac{1}{2} \cdot 5 = \frac{5}{2} \text{ and } \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

$$y = \left(x + \frac{5}{2}\right)^2 - \frac{49}{4}$$

$$\text{vertex: } \left(-2\frac{1}{2}, -12\frac{1}{4}\right)$$

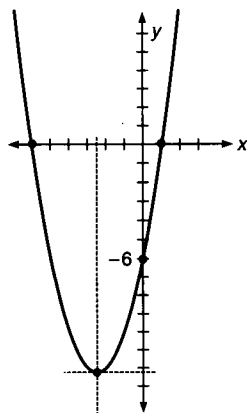
intercepts:

$$x = 0: f(0) = -6; (0, -6)$$

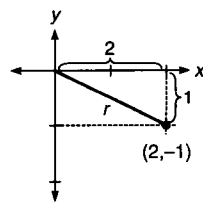
$$y = 0: 0 = x^2 + 5x - 6$$

$$0 = (x+6)(x-1)$$

$$x = -6 \text{ or } 1; (-6, 0), (1, 0)$$



5. Use $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. To find r , find the distance from the origin $(0, 0)$ to $(2, -1)$. This can be done by the distance formula or a simple sketch (see the figure) where we see that $r^2 = 2^2 + 1^2 = 5$.
 $(x - 2)^2 + (y - (-1))^2 = 5$
 $(x - 2)^2 + (y + 1)^2 = 5$



6. $f(x) = x^3 - 3x^2 + x + 2$

Possible zeros are $\pm 1, \pm 2$. Synthetic division shows that 2 is a zero, so $x - 2$ is a factor.

$$y = (x - 2)(x^2 - x - 1)$$

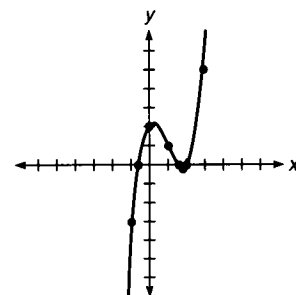
The zeros of $x^2 - x - 1$ are $\frac{1 \pm \sqrt{5}}{2} \approx$

$-0.6, 1.6$ (from the quadratic formula).

Thus, x-intercepts are $(2, 0)$, $(-0.6, 0)$, $(1.6, 0)$.

y-intercept: $f(0) = 2$; $(0, 2)$

Additional points: $(-1, -3)$, $(1, 1)$, $(1.8, -0.09)$, $(3, 5)$



Solutions to trial exercise problems

9. $\sum_{k=1}^1 k^2 + \sum_{k=1}^2 k^2 + \sum_{k=1}^3 k^2 + \sum_{k=1}^4 k^2$

$$1 + (1 + 4) + (1 + 4 + 9)$$

$$+ (1 + 4 + 9 + 16)$$

15. $-8, -7\frac{1}{4}, -6\frac{1}{2}, \dots, 1$

This is an arithmetic sequence with $a_1 = -8$ and $d = \frac{3}{4}$. Using $a_n = a_1 +$

$(n - 1)d$ we obtain

$$1 = -8 + (n - 1)\left(\frac{3}{4}\right)$$

$$9 = \frac{3}{4}(n - 1)$$

$$\frac{4}{3}(9) = n - 1$$

$$n = 13$$

Thus, there are 13 terms.

$$S_{13} = \frac{13}{2}(-8 + 1) = \frac{13}{2}(-7)$$

$$= -\frac{91}{2} = -45\frac{1}{2}$$

$$-2\left(1 - \left(-\frac{2}{3}\right)^4\right) - 2\left(1 - \frac{16}{81}\right)$$

30. $S_4 = \frac{1 - \left(-\frac{2}{3}\right)^4}{1 - \left(-\frac{2}{3}\right)} = \frac{\frac{5}{81}}{\frac{5}{3}}$

$$= \frac{3}{5}(-2)\left(\frac{65}{81}\right) = -\frac{26}{27}$$

33. $29\frac{17}{256}$ 35. $-61\frac{1}{1,024}$

45. $1 + 2 + 3 + \cdots + n$ is an arithmetic sequence with $a_1 = 1$, $a_n = n$, and $n =$

$$n. S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}(1 + n) = \frac{n(n+1)}{2}.$$

47. $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, and

$$\binom{n}{n-k} = \frac{n!}{(n-k)!(n-(n-k))!} = \frac{n!}{(n-k)!k!}, \text{ so } \binom{n}{k} = \binom{n}{n-k}$$

49. Using $(x+y)^n = \sum_{i=0}^n \binom{n}{i} x^{n-i} y^i$ with $x = y = 1$ we obtain

$$2^n = (1+1)^n = \sum_{i=0}^n \binom{n}{i} 1^{n-i} 1^i = \sum_{i=0}^n \binom{n}{i}$$

Solutions to skill and review problems

1. $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$

$$1 + 2 + \cdots + n + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{(n+1)(n+2)}{2}$$

2. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} + \frac{1}{(n+1)(n+2)} = \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n+1}{n+2}$$

3. $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots + \frac{1}{3^n}$

This is a geometric series; $a_1 = r = \frac{1}{3}$. We want S_n .

$$S_n = a_n \left(\frac{1-r^n}{1-r} \right) = \frac{1}{3} \left(\frac{1 - (\frac{1}{3})^n}{1 - \frac{1}{3}} \right) = \frac{1}{3} \left(\frac{1 - \frac{1}{3^n}}{\frac{2}{3}} \right) =$$

$$\frac{1}{3} \cdot \frac{3}{2} \left(\frac{1}{1} - \frac{1}{3^n} \right) = \frac{1}{2} \left(\frac{3^n - 1}{3^n} \right) = \frac{3^n - 1}{2(3^n)}$$

4. $2 + 4 + \cdots + 240$. Divide by 2;

$1 + 2 + \cdots + 120$ shows that $n = 120$.

This is an arithmetic series with $a_1 = 2$, $d = 2$, $n = 120$.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{120} = \frac{120}{2}(2 + 240) = 14,520.$$

5. $\frac{x-1}{2} - \frac{x-1}{3} = x$

$$6\left(\frac{x-1}{2}\right) - 6\left(\frac{x-1}{3}\right) = 6x$$

$$3(x-1) - 2(x-1) = 6x$$

$$-\frac{1}{5} = x$$

6. $f(x) = (x-1)^3 - 1$

This is the graph of $y = x^3$, shifted right

1 unit and down 1 unit. Thus the

“origin” is shifted to $(1, -1)$.

Intercepts:

$$x = 0: f(0) = (-1)^3 - 1 = -2; (0, -2)$$

$$y = 0: 0 = (x-1)^3 - 1$$

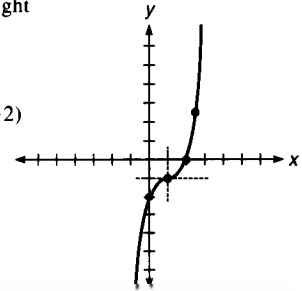
$$1 = (x-1)^3$$

$$\sqrt[3]{1} = x-1$$

$$1 = x-1$$

$$2 = x; (2, 0)$$

Additional points: $(1, -1)$, $(2.5, 2.4)$



Solutions to trial exercise problems

13. $(a^3b^2 - 2c)^7 =$

$$\binom{7}{0}(a^3b^2)^7(-2c)^0 + \binom{7}{1}(a^3b^2)^6(-2c)^1 + \binom{7}{2}(a^3b^2)^5(-2c)^2 + \binom{7}{3}(a^3b^2)^4(-2c)^3$$

$$+ \binom{7}{4}(a^3b^2)^3(-2c)^4 + \binom{7}{5}(a^3b^2)^2(-2c)^5 + \binom{7}{6}(a^3b^2)^1(-2c)^6 + \binom{7}{7}(a^3b^2)^0(-2c)^7 =$$

$$a^{21}b^{14} + 7(a^{18}b^{12})(-2c) + 21(a^{15}b^{10})(4c^2) + 35(a^{12}b^8)(-8c^3) + 35(a^9b^6)(16c^4)$$

$$+ 21(a^6b^4)(-32c^5) + 7(a^3b^2)(64c^6) + (-128c^7) =$$

$$a^{21}b^{14} - 14a^{18}b^{12}c + 84a^{15}b^{10}c^2 - 280a^{12}b^8c^3 + 560a^9b^6c^4 - 672a^6b^4c^5 + 448a^3b^2c^6 - 128c^7$$

19. Let $i = 3$:

$$\binom{22}{3} p^{22-3} (-3q)^3 = 1,540 p^{19} (-27) q^3 = -41,580 p^{19} q^3$$

25. $\sum_{i=1}^9 (3 - 4i + i^2)$

$$= \sum_{i=1}^9 3 - 4 \sum_{i=1}^9 i + \sum_{i=1}^9 i^2$$

$$= 9(3) - 4 \left(\frac{9(10)}{2} \right) + \frac{9(10)(19)}{6} = 132$$

$$33. \sum_{i=1}^4 [i^2 - (\frac{1}{4})^i] = \sum_{i=1}^4 i^2 - \sum_{i=1}^4 (\frac{1}{4})^i$$

The second expression is a geometric series, $a_1 = r = \frac{1}{4}$;

$$\begin{aligned} S_n &= a_n \left(\frac{1 - r^n}{1 - r} \right) \\ &= \frac{4(5)(9)}{6} - \frac{1}{4} \left(\frac{1 - (\frac{1}{4})^4}{1 - \frac{1}{4}} \right) \\ &= 30 - \frac{95}{256} = 29\frac{171}{256} \end{aligned}$$

$$\begin{aligned} 39. \sum_{i=1}^k (2i^2 + 5i - 12) \\ &= 2 \sum_{i=1}^k i^2 + 5 \sum_{i=1}^k i - \sum_{i=1}^k 12 = 2 \cdot \frac{k(k+1)(2k+1)}{6} + 5 \cdot \frac{k(k+1)}{2} - 12 \cdot k \\ &= \frac{2}{3}k^3 + \frac{7}{2}k^2 - \frac{55}{6}k \end{aligned}$$

Exercise set 12-4

Answers to odd-numbered problems

1. Show true for $n = 1$: $2(1) = 1(1 + 1)$; $2 = 2 \checkmark$

Find goal statement:

$$2 + 4 + 6 + \cdots + 2(k+1) = (k+1)[(k+1) + 1] = (k+1)(k+2)$$

Assume true for $n = k$:

$$2 + 4 + 6 + \cdots + 2k = k(k+1)$$

$$\begin{aligned} 2 + 4 + 6 + \cdots + 2k + 2(k+1) &= k(k+1) + 2(k+1) \\ &= (k+1)(k+2) \checkmark \end{aligned}$$

3. Show true for $n = 1$: $(5(1) - 1) = \frac{1(5(1) + 3)}{2}$; $4 = 4 \checkmark$

Find goal statement:

$$4 + 9 + 14 + \cdots + (5(k+1) - 1) = \frac{(k+1)(5(k+1) + 3)}{2} = \frac{(k+1)(5k+8)}{2}$$

Assume true for $n = k$:

$$4 + 9 + 14 + \cdots + (5k - 1) = \frac{k(5k+3)}{2}$$

$$\begin{aligned} 4 + 9 + 14 + \cdots + (5k - 1) + (5(k+1) - 1) &= \frac{k(5k+3)}{2} + (5(k+1) - 1) \\ &= \frac{5k^2 + 13k + 8}{2} = \frac{(k+1)(5k+8)}{2} \checkmark \end{aligned}$$

5. Show true for $n = 1$: $(4(1) - 3) = 2(1)^2 - 1$; $1 = 1 \checkmark$

Find the goal statement:

$$1 + 5 + 9 + \cdots + (4(k+1) - 3) = 2(k+1)^2 - (k+1) = 2k^2 + 3k + 1$$

Assume true for $n = k$:

$$1 + 5 + 9 + \cdots + (4k - 3) = 2k^2 - k$$

$$\begin{aligned} 1 + 5 + 9 + \cdots + (4k - 3) + (4(k+1) - 3) &= 2k^2 - k + (4(k+1) - 3) \\ &= 2k^2 + 3k + 1 \checkmark \end{aligned}$$

7. Show true for
- $n = 1$
- :

$$\frac{1^2(1+1)(1+2)}{6} = \frac{1(1+1)(1+2)(1+3)(4(1)+1)}{120}; 1 = 1 \checkmark$$

Find goal statement:

$$1 + 8 + 30 + 80 + \dots + \frac{(k+1)^2((k+1)+1)((k+1)+2)}{6} \\ = \frac{(k+1)((k+1)+1)((k+1)+2)((k+1)+3)(4(k+1)+1)}{120} = \frac{(k+1)(k+2)(k+3)(k+4)(4k+5)}{120}$$

Assume true for $n = k$:

$$1 + 8 + 30 + 80 + \dots + \frac{k^2(k+1)(k+2)}{6} = \frac{k(k+1)(k+2)(k+3)(4k+1)}{120} \\ 1 + 8 + 30 + 80 + \dots + \frac{k^2(k+1)(k+2)}{6} + \frac{(k+1)^2((k+1)+1)((k+1)+2)}{6} \\ = \frac{k(k+1)(k+2)(k+3)(4k+1)}{120} + \frac{(k+1)^2((k+1)+1)((k+1)+2)}{6} \\ = \frac{k(k+1)(k+2)(k+3)(4k+1) + 20(k+1)(k+1)(k+2)(k+3)}{120} \\ = \frac{(k+1)(k+2)(k+3)[k(4k+1) + 20(k+1)]}{120} = \frac{(k+1)(k+2)(k+3)[4k^2 + 21k + 20]}{120} = \frac{(k+1)(k+2)(k+3)(k+4)(4k+5)}{120} \checkmark$$

9. Show true for
- $n = 1$
- :
- $\frac{1}{2^1} = \frac{2^1 - 1}{2^1}$
- ;
- $\frac{1}{2} = \frac{1}{2} \checkmark$

$$\text{Find goal statement: } \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^{k+1}} = \frac{2^{k+1} - 1}{2^{k+1}}$$

Assume true for $n = k$:

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^k} = \frac{2^k - 1}{2^k} \\ \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^k} + \frac{1}{2^{k+1}} = \frac{2^k - 1}{2^k} + \frac{1}{2^{k+1}} = \frac{2(2^k - 1)}{2(2^k)} + \frac{1}{2^{k+1}} = \frac{2^{k+1} - 1}{2^{k+1}} \checkmark$$

11. Show true for
- $n = 1$
- :
- $1^3 + 2 = 3$
- , which is divisible by 3.
- \checkmark

Find goal statement: $(k+1)^3 + 2(k+1)$ is divisible by 3.Assume true for $n = k$: $k^3 + 2k$ is divisible by 3.

Examine goal statement:

$$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 5k + 3 = k^3 + 2k + 3k^2 + 3k + 3 = [k^3 + 2k] + [3(k^2 + k + 1)]$$

We know 3 divides $k^3 + 2k$.We can see that 3 divides $3(k^2 + k + 1)$.Therefore, 3, divides their sum $[k^3 + 2k] + [3(k^2 + k + 1)]$.As shown above, $[k^3 + 2k] + [3(k^2 + k + 1)] = (k+1)^3 + 2(k+1)$.Thus, 3 divides $(k+1)^3 + 2(k+1)$. \checkmark

13. Show true for
- $n = 1$
- :

$$\frac{1}{(2(1)-1)(2(1)+1)} = \frac{1}{2(1)+1}; \frac{1}{3} = \frac{1}{3} \checkmark$$

Find the goal statement:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2(k+1)-1)(2(k+1)+1)} = \frac{k+1}{2(k+1)+1} = \frac{k+1}{2k+3}$$

Assume true for $n = k$:

$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1} \\ \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)} \\ = \frac{k}{2k+1} + \frac{1}{(2(k+1)-1)(2(k+1)+1)} = \frac{2k^2 + 3k + 1}{(2k+1)(2k+3)} = \frac{k+1}{2k+3} \checkmark$$

15. Show true for
- $n = 1$
- :
- $2(3^0) = 3^1 - 1$
- ;
- $2 = 2 \checkmark$

Find goal statement:

$$2 + 6 + 18 + \cdots + 2(3^{(k+1)-1}) = 3^{k+1} - 1$$

Assume true for $n = k$:

$$2 + 6 + 18 + \cdots + 2(3^{k-1}) = 3^k - 1$$

$$\begin{aligned} 2 + 6 + 18 + \cdots + 2(3^{k-1}) + 2(3^{(k+1)-1}) &= 3^k - 1 + 2(3^{(k+1)-1}) \\ &= 3^k - 1 + 2(3^k) \\ &= 3(3^k) - 1 \\ &= 3^{k+1} - 1 \checkmark \end{aligned}$$

17. Show true for
- $n = 1$
- :
- $\frac{1}{2^{-3}} = \frac{2^1 - 1}{2^{-3}}$
- ;
- $8 = 8 \checkmark$

Find goal statement:

$$8 + 4 + 2 + \cdots + \frac{1}{2^{(k+1)-4}} = \frac{2^{k+1} - 1}{2^{(k+1)-4}}$$

Assume true for $n = k$:

$$8 + 4 + 2 + \cdots + \frac{1}{2^{k-4}} = \frac{2^k - 1}{2^{k-4}}$$

$$\begin{aligned} 8 + 4 + 2 + \cdots + \frac{1}{2^{k-4}} + \frac{1}{2^{(k+1)-4}} &= \frac{2^k - 1}{2^{k-4}} + \frac{1}{2^{(k+1)-4}} \\ &= \frac{2}{2} \cdot \frac{2^k - 1}{2^{k-4}} + \frac{1}{2^{k-3}} = \frac{2^{k+1} - 1}{2^{k-3}} = \frac{2^{k+1} - 1}{2^{(k+1)-4}} \checkmark \end{aligned}$$

19. Show true for
- $n = 1$
- :
- $\frac{1}{(3(1) - 2)(3(1) + 1)} = \frac{1}{3(1) + 1}$
- ;
- $\frac{1}{4} = \frac{1}{4} \checkmark$

Find goal statement:

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3(k+1) - 2)(3(k+1) + 1)} = \frac{k+1}{3(k+1) + 1} = \frac{k+1}{3k+4}$$

Assume true for $n = k$:

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3k - 2)(3k + 1)} = \frac{k}{3k + 1}$$

$$\begin{aligned} \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \cdots + \frac{1}{(3k - 2)(3k + 1)} + \frac{1}{(3(k+1) - 2)(3(k+1) + 1)} \\ = \frac{k}{3k + 1} + \frac{1}{(3(k+1) - 2)(3(k+1) + 1)} = \frac{k}{(3k + 1)(3k + 4)} + \frac{1}{(3k + 1)(3k + 4)} = \frac{(3k + 1)(k + 1)}{(3k + 1)(3k + 4)} = \frac{k + 1}{3k + 4} \checkmark \end{aligned}$$

21. Show true for
- $n = 1$
- :
- $\frac{1}{1 \cdot 2 \cdot 3} = \frac{1(4)}{4(2)(3)}$
- ;
- $\frac{1}{6} = \frac{1}{6} \checkmark$

Find goal statement:

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{(k+1)((k+1)+1)((k+1)+2)} = \frac{(k+1)((k+1)+3)}{4((k+1)+1)((k+1)+2)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

Assume true for $n = k$:

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$$

$$\begin{aligned} \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \frac{1}{3 \cdot 4 \cdot 5} + \cdots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} \\ = \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)} = \frac{k(k+3)^2}{4(k+1)(k+2)(k+3)} + \frac{4}{4(k+1)(k+2)(k+3)} = \frac{k^3 + 6k^2 + 9k + 4}{4(k+1)(k+2)(k+3)} \end{aligned}$$

Using both the goal statement and the rational zero theorem as a guide we factor the numerator.

$$= \frac{(k+1)^2(k+4)}{4(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)} \checkmark$$

23. Show true for $n = 1$: $\frac{1}{1 \cdot 4 \cdot 7} = \frac{1(8)}{8(4)(7)}$; $\frac{1}{28} = \frac{1}{28} \checkmark$

Find goal statement:

$$\frac{1}{1 \cdot 4 \cdot 7} + \frac{1}{4 \cdot 7 \cdot 10} + \frac{1}{7 \cdot 10 \cdot 13} + \cdots + \frac{1}{(3(k+1)-2)(3(k+1)+1)(3(k+1)+4)}$$

$$= \frac{(k+1)(3(k+1)+5)}{8(3(k+1)+1)(3(k+1)+4)} = \frac{(k+1)(3k+8)}{8(3k+4)(3k+7)}$$

Assume true for $n = k$:

$$\frac{1}{1 \cdot 4 \cdot 7} + \frac{1}{4 \cdot 7 \cdot 10} + \frac{1}{7 \cdot 10 \cdot 13} + \cdots + \frac{1}{(3k-2)(3k+1)(3k+4)} = \frac{k(3k+5)}{8(3k+1)(3k+4)}$$

$$\frac{1}{1 \cdot 4 \cdot 7} + \frac{1}{4 \cdot 7 \cdot 10} + \frac{1}{7 \cdot 10 \cdot 13} + \cdots + \frac{1}{(3k-2)(3k+1)(3k+4)} + \frac{1}{(3k+1)(3k+4)(3k+7)}$$

$$= \frac{k(3k+5)}{8(3k+1)(3k+4)} + \frac{1}{(3k+1)(3k+4)(3k+7)} = \frac{9k^3 + 36k^2 + 35k + 8}{8(3k+1)(3k+4)(3k+7)}$$

Use the goal statement to help us factor the numerator.

$$= \frac{(k+1)(3k+8)(3k+1)}{8(3k+1)(3k+4)(3k+7)} = \frac{(k+1)(3k+8)}{8(3k+4)(3k+7)} \checkmark$$

25. Goal statement:

$$1 + 3 + 5 + \cdots + (2(k+1) - 1) = \frac{(k+1)^2 + (k+1)}{2} = \frac{k^2 + 3k + 2}{2}$$

Assume true for $n = k$, then add the next term to both members.

$$1 + 3 + 5 + \cdots + (2k - 1) = \frac{k^2 + k}{2}$$

$$1 + 3 + 5 + \cdots + (2k - 1) + (2(k+1) - 1) = \frac{k^2 + k}{2} + (2(k+1) - 1)$$

The left side is now the left side of the goal statement; we must show that the right side is the same as the right side of the goal statement.

$$= \frac{k^2 + k}{2} + \frac{2(2k+1)}{2} = \frac{k^2 + 5k + 2}{2}$$

This expression is clearly not the same as the goal expression.

27. Given the sum of the first n terms of an arithmetic series

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n, S_n = \frac{n}{2}(a_1 + a_n).$$

a. $a_1 = 1, a_n = (2n - 1)$, so $S_n = \frac{n}{2}[1 + (2n - 1)] = \frac{n}{2}(2n) = n^2$

b. $a_1 = 4, a_n = (6n - 2)$, so $S_n = \frac{n}{2}[4 + (6n - 2)] = \frac{n}{2}(6n + 2) = n(3n + 1) = 3n^2 + n$

Solutions to skill and review problems

1. $1 + 4 + 7 + \cdots + (3n - 2) + [3(n+1) - 2]$

$$= \frac{n(3n-1)}{2} + [3(n+1) - 2]$$

$$= \frac{n(3n-1)}{2} + (3n+1)$$

$$= \frac{(n+1)(3n+2)}{2}$$

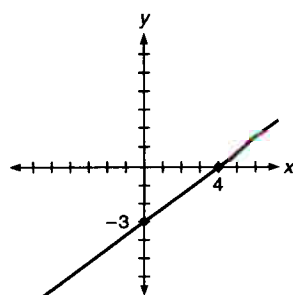
2. $3x - 4y = 12$

Straight line.

Intercepts:

$$x = 0: y = -3; (0, -3)$$

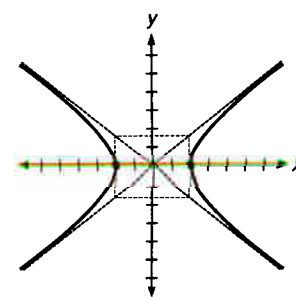
$$y = 0: x = 4; (4, 0)$$



3. $3x^2 - 4y^2 = 12$

$$\frac{x^2}{4} - \frac{y^2}{3} = 1$$

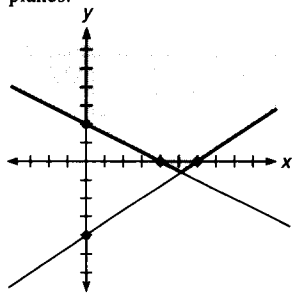
$$a = 2, b = \sqrt{3}, c = \sqrt{7}$$



4. $2x - 3y \leq 12$

$x + 2y \geq 4$

Graph the straight lines $2x - 3y = 12$ and $x + 2y = 4$. Use $(0,0)$ as a test point to find the appropriate half-planes.



5. $2x - 1 > \frac{5}{x + 1}$

This is a nonlinear inequality. Use the critical point/test point method.

Critical points

Solve the corresponding equality:

$$2x - 1 = \frac{5}{x + 1}$$

$$(2x - 1)(x + 1) = 5$$

$$2x^2 + x - 6 = 0$$

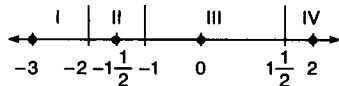
$$(2x - 3)(x + 2) = 0$$

$$x = \frac{3}{2} \text{ or } -2$$

Find zeros of denominators:

$$x + 1 = 0; x = -1$$

Critical points are $-2, -1, 1\frac{1}{2}$.



Use $-3, -1.5, 0, 2$ for test points.

$$2x - 1 > \frac{5}{x + 1}$$

$$x = -3: -7 > -2\frac{1}{2}; \text{ false}$$

$$x = -1.5: -4 > -10; \text{ true}$$

$$x = 0: -1 > 5; \text{ false}$$

$$x = 2: 3 > 1\frac{2}{3}; \text{ true}$$

Thus the solution is intervals II and IV: $-2 < x < -1$ or $x > 1\frac{1}{2}$.

Solutions to trial exercise problems

12. Show that $(1 + a)^n \geq 1 + na$ for any natural number n , assuming $a \geq 0$.

Show true for $n = 1$:

$$1 + a \geq 1 + a \quad \checkmark$$

Find the goal statement:

Replace n by $k + 1$

$$[2] \quad (1 + a)^{k+1} \geq 1 + (k + 1)a$$

Assume true for $n = k$:

(Replace n by k)

$$[1] \quad (1 + a)^k \geq 1 + ka$$

We can achieve the left side of statement [2] by multiplying both members of statement [1] by $(1 + a)$. We know $1 + a$ is nonnegative, which is important when multiplying the members of an inequality.

We start with statement [1], which we know to be true.

$$[1] \quad (1 + a)^k \geq 1 + ka$$

$$(1 + a)(1 + a)^k \geq (1 + a)(1 + ka)$$

$$(1 + a)^{k+1} \geq 1 + ka + a + ka^2$$

$$\text{Now, } 1 + ka + a + ka^2 \geq 1 +$$

$$ka + a \text{ (since } k > 0, a^2 \geq 0), \text{ so}$$

$$(1 + a)^{k+1} \geq 1 + ka + a + ka^2$$

$$\geq 1 + ka + a, \text{ so}$$

$$[2] \quad (1 + a)^{k+1} \geq 1 + ka + a \text{ is true. } \checkmark$$

28. Observe that the statement about finding the light coin among five coins includes the assumption that the light coin is actually among the five coins. This is the only basis for selecting the fifth coin when we reject four of the coins.

In this light consider six coins. We group the six coins into two groups: 5 coins and 1 coin. On the basis of our hypothesis we can actually find the light coin among the five coins in two weighings only if we already know that the light coin is among these five coins. Unfortunately the light coin could be in the group which contains one coin. Thus, in the case of six coins we do not meet the hypothesis we required for five coins.

Ironically, two weighings will suffice to find the light coin among six coins, or even seven coins, but not necessarily using the method suggested

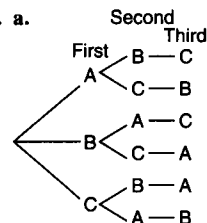
above for six coins (try to see how).

However, two weighings will not suffice for eight coins, which could be shown by trying all possible combinations of weighings of up to eight coins, remembering that no purpose is served by weighing unequal numbers of coins.

Exercise 12-5

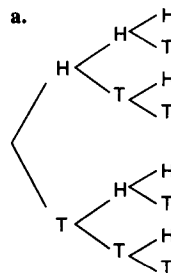
Answers to odd-numbered problems

1. a.



b. ABC, ACB, BAC, BCA, CBA, CAB

3. a.



b. HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

5. 24 7. 16,807 9. 180

11. 256 13. 5,040 15. 30,240

17. 220 19. 360 21. 720

23. 13,366,080

25. 20 27. By definition, ${}_nP_r = \frac{n!}{(n-r)!}$

$= \frac{n!}{(n-1)(n-2) \cdots (n-(r-1))}$ so ${}_nP_n = \frac{n!}{n(n-1)(n-2) \cdots (n-(n-1))} = \frac{n!}{n!} = 1$

29. 504 31. 210 33. 5,040

35. 990 37. 1,816,214,400

39. 9,979,200 41. 83,160 43. 3,003

45. 56 47. 91 49. By definition, ${}_nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!} = 1$

51. 4,845 53. 495 55. 28 57. 35

59. 1,001 61. a. 24,310 b. 272
 c. 8,821,612,800 d. 6,188 63. 1,024
 65. a. 60 b. 36 c. 6 d. 6
 67. 90 69. a. 179,520 b. 924
 c. 593,775 71. 32,640 73. 169,344

$$75. \binom{n}{r} \binom{r}{k} = \frac{n!}{r!(n-r)!} \cdot \frac{r!}{k!(r-k)!} = \frac{n!}{(n-r)!k!(r-k)!} = \frac{\binom{n}{k} \binom{n-k}{r-k}}{k!(n-k)!} = \frac{n!}{(r-k)![(n-k)-(r-k)]!} = \frac{n!}{k!(r-k)!(n-r)!}$$

77. Let $n = 10$, $r = 3$.
 $nP_r = n(n-1)(n-2) \cdots (n-[r-1])$
 $n - (r-1) = 8$, so nP_r becomes ${}_{10}P_3$
 $= 10 \cdot 9 \cdot 8$.
 $\frac{n!}{(n-r)!}$ becomes $\frac{10!}{(10-3)!}$
 $= \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!} = 10 \cdot 9 \cdot 8$.

79. $\frac{nP_r}{r!} = \frac{n(n-1)(n-2) \cdots (n-[r-1])}{r!}$,
 so
 $\frac{{}_{10}P_3}{3!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$
 Next, $\frac{n!}{r!(n-r)!}$
 $\frac{10!}{3! \cdot 7!} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \cdot 2 \cdot 1 \cdot 7!}$
 $= \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1}$

Solutions to skill and review problems

1. Infinite geometric series; $a = r = \frac{1}{2}$.
 Since
 $|r| < 1$, $S_\infty = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$.

2. Arithmetic series; $a_1 = 2$, $d = 5$. To find out how many terms proceed as follows:
 $2 + 7 + 12 + 17 + \cdots + 97$
 Subtract 2 from each term.
 $0 + 5 + 10 + 15 + \cdots + 95$
 Divide each term by 5.
 $0 + 1 + 2 + 3 + \cdots + 19$
 There are thus 20 terms.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{20} = \frac{20}{2}(2 + 97) = 10(99) = 990$$

3. $\left| \frac{2-3x}{4} \right| \leq 10$
 $-10 \leq \frac{2-3x}{4} \leq 10$
 $-40 \leq 2-3x \leq 40$
 $-42 \leq -3x \leq 38$
 $\frac{-42}{-3} \geq \frac{-3x}{-3} \geq \frac{38}{-3}$
 $14 \geq x \geq -12\frac{2}{3}$
 $-12\frac{2}{3} \leq x \leq 14$

4. $f(x) = \frac{x^2-1}{x^2-4}$
 $y = \frac{x^2-1}{x^2-4} = \frac{x^2-1}{(x-2)(x+2)}$
 $= 1 + \frac{3}{x^2-4}$ (Long division.)

Horizontal asymptote: $y = 1$
 Vertical asymptotes at $x = \pm 2$
 Intercepts:

$$x = 0: y = \frac{-1}{-4} = \frac{1}{4}; \left(0, \frac{1}{4}\right)$$

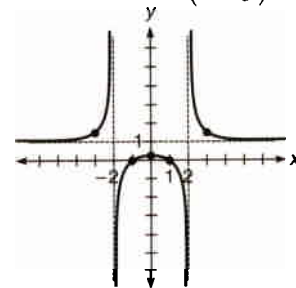
$$y = 0: 0 = \frac{x^2-1}{x^2-4}$$

$$0 = x^2 - 1$$

$$1 = x^2$$

$$\pm 1 = x; (\pm 1, 0)$$

$$\text{Additional points: } \left(\pm 3, 1\frac{3}{5}\right)$$



$$5. f(x) = (x-2)^2(x+1)^3$$

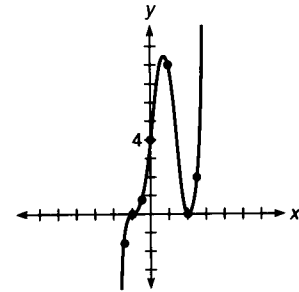
Intercepts:

$$x = 0: y = (-2)^2(1)^3 = 4; (0, 4)$$

$$y = 0: 0 = (x-2)^2(x+1)^3$$

$$x = -1 \text{ or } 2; (-1, 0), (2, 0)$$

Additional points: $(-1.5, -1.53)$, $(-0.5, 0.8)$, $(1, 8)$, $(2.25, 2.15)$



$$6. x + \frac{1}{x} = 3$$

$$x^2 + 1 = 3x$$

$$x^2 - 3x + 1 = 0$$

$$x = \frac{3 \pm \sqrt{5}}{2}$$

$$7. \frac{x+y}{x} = \frac{x-y}{2}$$

$$2(x+y) = x(x-y)$$

$$2x + 2y = x^2 - xy$$

$$xy + 2y = x^2 - 2x$$

$$y(x+2) = x^2 - 2x$$

$$y = \frac{x^2 - 2x}{x+2}$$

Solutions to trial exercise problems

8. 12 choices for the entrance. After this choice is made there are 11 choices left for the exit: $12 \cdot 11 = 132$
 29. Order is important, so we are counting permutations: ${}_9P_3 = 504$
 36. There are eight lots and eight houses, so it is ${}_8P_8 = 8! = 40,320$. (The side of the street they are on is irrelevant. To see this, try a smaller example, say three houses on three lots—put the lots anywhere you want!)
 42. We are interested only in the number of ways to list aabbbbcccc, which is $\frac{11!}{2!4!5!} = 6,930$.
 56. Once five players are picked for one team, the remaining five are on the other team. The order of selection is not important, so there are ${}_{10}C_5 = 252$ ways.

62. a. Three males and four females are seven people. Thus they can sit in $7! = 5,040$ different orders.

b. A female must sit first and last to have alternation. There are $4!$ ways to sit the females and $3!$ ways to sit the males. These orderings of females and males can be selected in $4! \cdot 3! = 144$ ways.

c. We have seven people, of which a group of four and a group of three are indistinguishable, so there are $\frac{7!}{4!3!} =$

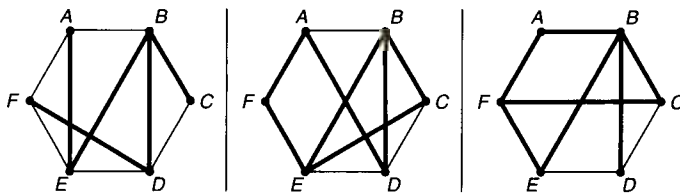
35 distinguishable ways to order them.

64. a. We are not forbidden to repeat digits, so for each of the three digits there are five choices: $5 \cdot 5 \cdot 5 = 125$.
b. A three-digit odd number, with digits selected from this set of digits, ends in 1, 3, or 5. Thus there are only 3 choices for the last digit: $5 \cdot 5 \cdot 3 = 75$. c. $2 \cdot 5 \cdot 2 = 20$ d. $3 \cdot 3 \cdot 3 = 27$
66. There are ${}_8C_4$ ways to choose the males and ${}_6C_4$ ways to choose the females. For each of the male groups we can choose any of the female groups. Thus there are ${}_8C_4 \cdot {}_6C_4 = 1,050$ ways to select the groups.

72. ${}_5C_3 \cdot {}_4C_2 \cdot {}_3C_2 = 180$

76.
$$\begin{aligned} & \binom{n}{r+1} + \binom{n}{r} \\ &= \frac{n!}{(r+1)!(n-(r+1))!} + \frac{n!}{r!(n-r)!} \\ &= \frac{n!}{(r+1)!(n-r-1)!} + \frac{n!}{r!(n-r)!} \\ &= \frac{(n-r)n!}{(r+1)!(n-r)!(n-r-1)!} + \frac{(r+1)n!}{(r+1)!(n-r)!} \\ &= \frac{(n-r)n!}{(r+1)!(n-r)!} + \frac{(r+1)n!}{(r+1)!(n-r)!} \\ &= \frac{(n-r)n! + (r+1)n!}{(r+1)!(n-r)!} \\ &= \frac{n![(n-r) + (r+1)]}{(r+1)!(n-r)!} \\ &= \frac{n!(n+1)}{(r+1)!(n-r)!} = \frac{(n+1)!}{(r+1)!(n-r)!} \\ &\text{and } \binom{n+1}{r+1} = \frac{(n+1)!}{(r+1)!(n-(r+1))!} \\ &= \frac{(n+1)!}{(r+1)!(n-r)!} \end{aligned}$$

80. a. In each of the three situations shown find a group of three people who either mutually know each other or who mutually do not know each other.



A, C, and D are mutual strangers in the leftmost figure; B, C, and E are mutual acquaintances in the central figure; and A, D, and E are mutual strangers in the rightmost figure.

- b. How many groups of three are there, given six people?

There are ${}_6C_3 = 20$ groups, each of which would have to be checked to see if all knew each other or if all were mutual strangers.

Exercise 12–6

Answers to odd-numbered problems

1. $\frac{1}{2}$ 3. $\frac{3}{4}$ 5. $\frac{1}{8}$
7. $\frac{1}{2}$ 9. $\frac{3}{8}$ 11. $\frac{5}{16}$
13. $\frac{1}{13}$ 15. $\frac{1}{4}$ 17. $\frac{6}{13}$
19. $\frac{1}{26}$ 21. $\frac{1}{13}$ 23. $\frac{4}{13}$
25. $\frac{7}{13}$ 27. $\frac{12}{13}$ 29. $\frac{6}{13}$
31. $\frac{3}{4}$ 33. $\frac{1}{2}$ 35. $\frac{1}{38}$
37. $\frac{18}{19}$ 39. 0 41. $\frac{9}{19}$
43. $\frac{7}{12}$ 45. 0 47. $\frac{5}{12}$
49. 0.0113 51. 0.1697 53. 0.0253
55. 0.2215 57. 0.0003 59. 0.1496
61. 0.0036 63. 0.0095 65. a. $\frac{2}{25}$
b. 0.4105 67. $\frac{1}{4}$ 69. $\frac{1}{6}$
71. 0.3543 73. 0.777 75. 0.777
77. 0.39

Solutions to skill and review problems

1. $S = \frac{1}{2}[a - b(a + c)]$
 $2S = a - b(a + c)$
 $2S = a - ab - bc$
 $2S + bc = a - ab$
 $2S + bc = a(1 - b)$
 $\frac{2S + bc}{1 - b} = a$

2. $f(x) = x^3 - x^2 - x$
 $f(a - 1)$
 $= (a - 1)^3 - (a - 1)^2 - (a - 1)$
 $= (a^3 - 3a^2 + 3a - 1) -$
 $(a^2 - 2a + 1) - (a - 1)$
 $= a^3 - 4a^2 + 4a - 1$

3. $y = \frac{1 - 2x}{3}$
 $x = \frac{1 - 2y}{3}$
 $3x = 1 - 2y$
 $2y = 1 - 3x$
 $y = \frac{1 - 3x}{2}$
 $f^{-1}(x) = \frac{1 - 3x}{2}$

4. $y = \log_4 64$
 $4^y = 64$
 $y = 3$

5. $\log(x + 3) + \log(x - 1) = 1$
 $\log[(x + 3)(x - 1)] = 1$
 $\log(x^2 + 2x - 3) = 1$
 $x^2 + 2x - 3 = 10^1$
 $x^2 + 2x - 13 = 0$

By quadratic formula:

$$x = -1 \pm \sqrt{14} \approx -4.7, 2.7$$

We select the positive solution since $\log(x - 1)$ is not defined for $x < 1$, since then $x - 1$ is negative.

$$x = -1 + \sqrt{14}$$

6. $f(x) = \sqrt{x-3} - 1$

This is $y = \sqrt{x}$ shifted right 3 units and down 1 unit. Thus the "origin" shifts to (3, -1).

Intercepts:

$x = 0$: $y = \sqrt{-3} - 1$; No real solution so no y -intercept.

$y = 0$: $0 = \sqrt{x-3} - 1$

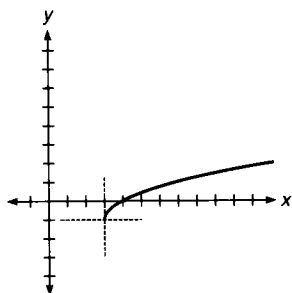
$1 = \sqrt{x-3}$

Square both sides.

$1 = x - 3$

$4 = x$; (4, 0)

Additional points: (3, -1), (7, 1), (12, 2)



Solutions to trial exercise problems

7. $A = \{HTT, THT, TTH, TTT\}$;

$P(A) = \frac{n(A)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

19. There are two red sevens. $P(\text{red seven})$

$= \frac{\text{number of red sevens}}{\text{number of cards}} = \frac{2}{52} = \frac{1}{26}$

26. $P(\text{from 5 through 8, inclusive, or a club}) = P(5, 6, 7, 8) + P(\text{club}) -$

$P(5, 6, 7, \text{ or } 8 \text{ of clubs}) =$

$\frac{16}{52} + \frac{13}{52} - \frac{4}{52} = \frac{25}{52}$

32. $P(\text{not a heart}) = 1 - P(\text{heart})$

$= 1 - \frac{13}{52} = 1 - \frac{1}{4} = \frac{3}{4}$

38. $P(\text{a black or green number}) = P(\text{black}) + P(\text{green}) = \frac{9}{19} + \frac{1}{19} = \frac{10}{19}$

46. $P(\text{not white}) = 1 - P(\text{white})$

$= 1 - \frac{8}{24} = 1 - \frac{1}{3} = \frac{2}{3}$

50. ${}_{10}C_3 \cdot {}_8C_3 = 6,720$ so out of the ${}_{18}C_6$

possible shipments, exactly 6,720 contain 3 new and 3 remanufactured alternators. $P(3 \text{ new and } 3$

$\text{manufactured}) = \frac{6,720}{{}_{18}C_6} = \frac{6,720}{18,564}$

≈ 0.3620

56. $P(\text{none of the cards are face cards})$

$= \frac{\text{number of ways to choose five non face cards}}{\text{number of ways to choose five cards}}$

$= \frac{{}_{40}C_5}{{}_{52}C_5} \approx 0.2532$

60. $P(\text{three clubs and two hearts})$

$= \frac{{}_{13}C_3 \cdot {}_{13}C_2}{{}_{52}C_5} = \frac{22,308}{2,598,960} \approx 0.0086$

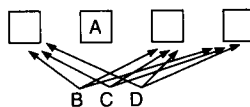
66. a. $\frac{\text{number infected units}}{\text{total number of units}} = \frac{3}{100}$

b. $\frac{\text{number of ways to choose 4 units out of 97 uninfected units}}{\text{number of ways to choose 4 units out of all 100 units}}$

$= \frac{{}_{97}C_4}{{}_{100}C_4}$

$= \frac{3,464,840}{3,921,225} \approx 0.8836$

68. There are $4! = 24$ possible orders in which to see the four patients. This is the sample space. We need to determine in how many permutations A is in position 2. There are $3!$ ways to position patients B, C, and D in the three remaining slots. Thus the probability of one of these orders of patient selection is $\frac{3!}{4!} = \frac{1}{4}$.



74. This is 1 less the sum of the probabilities of 0 and 1 failures.

$n = \frac{t}{\text{MTBF}} = \frac{3,000}{1,000} = 3$

$P(\geq 2, 3,000) = 1 - P(0, 3,000) - P(1, 3,000)$

$= 1 - \frac{e^{-3} \cdot 3^0}{0!} - \frac{e^{-3} \cdot 3^1}{1!}$

$\approx 1 - 0.0498 - 0.1494 \approx 0.801$

78. Of the 999,500 virus-free people, 0.998

$\cdot 999,500 = 997,501$ will test negative.

This means that $999,500 - 997,501 = 1,999$ virus-free individuals will falsely test positive. Of the 500 with the virus, $0.983 \cdot 500 = 492$ will test positive.

Thus there are 2,492 positives. Of these the probability of having the

virus is $\frac{492}{2,491} \approx 0.198$. In other words,

if a person tests positive there is about a 20% chance of having the virus.

Exercise 12-7

Answers to odd-numbered problems

1. 3, 8, 13, 18, 23; $a_n = 5n + 3$

3. 5, 10, 20, 40, 80; $a_n = 5 \cdot 2^n$

5. -2, 3, 0, 9, 18; $a_n = \frac{1}{4}(3^n) - \frac{9}{4}(-1)^n$

7. 3, 1, 15, 49, 207; $a_n = \frac{4}{5}(4^n) + \frac{11}{5}(-1)^n$

9. -2, 4, 0, 24, 72; $a_n = \left(\frac{14 - 2\sqrt{33}}{2\sqrt{33}} \right) \left(\frac{3 + \sqrt{33}}{2} \right)^n + \left(\frac{-14 - 2\sqrt{33}}{2\sqrt{33}} \right) \left(\frac{3 - \sqrt{33}}{2} \right)^n$

11. $a_n = 3^n$

13. $a_n = 4 - n$

15. $a_n = \frac{1}{2}(-1)^n + n + \frac{1}{2}$

17. With a recursive definition, to compute a_n we need to first find some or all of the previous terms, a_0, a_1, \dots, a_{n-1} .

19. An arithmetic sequence is a sequence in which $a_{n+1} - a_n = d$ for all n in the domain of the sequence and for some real number d . For the sequence given here we have $a_n = a_{n-1} + 3$ if $n > 0$, so that $a_n - a_{n-1} = 3$ for $n > 0$, or $a_{n-1} - a_n = 3$ for $n > 1$. It is easy to verify that $a_{n+1} - a_n = 3$ for $n = 1$ and $n = 0$, also, so that it is true that $a_{n+1} - a_n = 3$ for all n in the domain of the sequence.

21. -2 or 6

23. The statement we wish to prove is that $a_n = 3^n$ for all $n \in \mathbb{N}$,where $a_n = \begin{cases} 1 & \text{if } n = 0, 3 \text{ if } n = 1 \\ 2a_{n-1} + 3a_{n-2} & \text{if } n > 1 \end{cases}$ Show true for $n = 0, 1$: $a_0 = 1 = 3^0$,
so $a_1 = 3 = 3^1 \checkmark$ Find goal statement: (Replace n by $k + 1$): $a_{k+1} = 3^{k+1}$ Assume true for $n = k, k > 1$: $a_k = 3^k$ for all $n \leq k$, where $k > 1$.

Assume this statement is true.

 $a_{k+1} = 2a_k + 3a_{k-1}$ Definition of a_{k+1} for $k > 1$. $a_k = 3^k$

Assumed true above.

 $a_{k-1} = 3^{k-1}$ True because $k - 1 < k$ Replace a_k, a_{k-1} by $3^k, 3^{k-1}$: $a_{k+1} = 2(3^k) + 3(3^{k-1})$ $= 2(3^k) + 3^k$, since $3(3^{k-1}) = 3^k$ $= 3^k[2 + 1]$ $= 3^k(3) = 3^{k+1} \checkmark$ **Solutions to skill and review problems**1. $a_n = a_1 + (n - 1)d$ $a_1 = 4$; $a_n = 4 + (n - 1)d$ $a_{58} = 67$: $67 = 4 + 57d$, so $d = \frac{63}{57} = \frac{21}{19}$ a_{96} : $a_{96} = 4 + 95(\frac{21}{19}) = 109$ 2. a. There are four choices for the first part, then three choices remain for the second, two choices remain for the third, and there is then one choice for the fourth part: $4 \cdot 3 \cdot 2 \cdot 1 = 24$.b. ${}_6P_4 = 360$ ways c. $5 \cdot 5 \cdot 5 \cdot 5 = 5^4 = 625$ 3. $x^2 - 10x - 18 > 6$

Critical points:

 $x^2 - 10x - 18 = 6$ $x^2 - 10x - 24 = 0$ $(x - 12)(x + 2) = 0$ $x = -2$ or 12 (Not part of solution set.)Test points: $-3, 0, 13$,

 $x^2 - 10x - 18 > 6$ -3 : $21 > 6$ (true) 0 : $-18 > 6$ (false) 13 : $21 > 6$ (true)Solution: $x < -2$ or $x > 12$ 4. This is a geometric series with $a_1 = 1$,

$$r = \frac{1}{3}, n = 6: S_n = a_1 \frac{1 - r^n}{1 - r}$$

$$S_6 = 1 \frac{1 - \left(\frac{1}{3}\right)^6}{1 - \frac{1}{3}} = \frac{364}{243} = 1.50$$

Solutions to trial exercise problems5. $a_n = \begin{cases} -2 & \text{if } n = 0, 3 \text{ if } n = 1 \\ 2a_{n-1} + 3a_{n-2} & \text{if } n > 1 \end{cases}$
 $-2, 3, 2(3) + 3(-2) = 0, 2(0) + 3(3) = 9, 2(9) + 3(0) = 18, \dots$ or $-2, 3, 0, 9, 18, \dots$

This sequence is neither geometric nor arithmetic, so we try a recurrence relation.

$$a_n = 2a_{n-1} + 3a_{n-2}$$

$$a_n - 2a_{n-1} - 3a_{n-2} = 0$$

$$x^n - 2x^{n-1} - 3x^{n-2} = 0$$

Replace n by 2.

$$x^2 - 2x - 3 = 0$$
, so $x = 3$ or -1 .

Then $a_n = A(3^n) + B(-1)^n$. We find A and B from a_0 and a_1 .

$$n = 0: a_0 = -2 = A + B$$

$$n = 1: a_1 = 3 = 3A - B$$

Solving (for example, by adding the two equations we find $1 = 4A$) we find

$$A = \frac{1}{4}, B = -\frac{9}{4}$$
, so a general term is

$$a_n = \frac{1}{4}(3^n) - \frac{9}{4}(-1)^n$$

11. $a_n = \begin{cases} 1 & \text{if } n = 0, 3 \text{ if } n = 1 \\ 6a_{n-1} - 9a_{n-2} & \text{if } n > 1 \end{cases}$

$$a_n = 6a_{n-1} - 9a_{n-2}$$

$$a_n - 6a_{n-1} + 9a_{n-2} = 0$$

$$x^n - 6x^{n-1} + 9x^{n-2} = 0$$

$$x^2 - 6x + 9 = 0$$

$$x = 3$$
 (multiplicity 2)

$$a_n = A(3^n) + Bn(3^n)$$

$$n = 0: a_0 = 1 = A$$

$$n = 1: a_1 = 3 = 3A + 3B$$
, so $B = 0$.

Thus, $a_n = 3^n$.16. $a_n = \begin{cases} 1 & \text{if } n = 0, 1 \text{ if } n = 1, 3 \text{ if } n = 2 \\ 6a_{n-1} - 12a_{n-2} + 8a_{n-3} & \text{if } n > 2 \end{cases}$

$$a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$$

$$a_n - 6a_{n-1} + 12a_{n-2} - 8a_{n-3} = 0$$

$$x^n - 6x^{n-1} + 12x^{n-2} - 8x^{n-3} = 0$$

let $n = 3$

$$x^3 - 6x^2 + 12x - 8 = 0$$

The rational zero theorem

(section 4-2) tells us that the only rational zeros are $\pm 1, \pm 2, \pm 4$, and ± 8 .

Synthetic division or trial and error tells us that 2 is a zero, so the equation

is $(x - 2)(x^2 - 4x + 4) = 0$, or $(x - 2)^3 = 0$, so 2 is the only zero, and has multiplicity 3.

$$a_n = A(2^n) + Bn(2^n) + Cn^2(2^n)$$

$$n = 0: a_0 = 1 = A$$

$$n = 1: a_1 = 1 = 2A + 2B + 2C$$

$$n = 2: a_2 = 3 = 4A + 8B + 16C$$

$$A = 1, B = -\frac{7}{8}, C = \frac{3}{8}$$
, so

$$a_n = 2^n - \frac{7}{8}n(2^n) + \frac{3}{8}n^2(2^n)$$
 or

$$2^n\left(\frac{3}{8}n^2 - \frac{7}{8}n + 1\right)$$

20. A geometric sequence is a sequence in

which $\frac{a_{n+1}}{a_n} = r$ for all n in the domainof the sequence and for some real number r . For this sequence, $a_n =$

$$3a_{n-1}$$
 for $n > 0$, so $\frac{a_n}{a_{n-1}} = 3$ for $n > 0$,

or $\frac{a_{n+1}}{a_n} = 3$ for $n > 1$. It can be

verified that this is also true for $n = 0$ and $n = 1$.

21. For the given sequence,

$$a_2 = 2a_1 + 3a_0 = 2A + 6$$
. We thus

know that $\frac{a_1}{a_0} = \frac{A}{2}$, and $\frac{a_2}{a_1} = \frac{2A + 6}{A}$.

We want these ratios to be equal, so we solve

$$\frac{A}{2} = \frac{2A + 6}{A}$$

$$A^2 = 4A + 12$$

$$A^2 - 4A - 12 = 0$$

 $A = -2$ or 6 . Both of these values do produce geometric sequences.**Chapter 12 review**1. 4, 10, 16, 22 2. $0, 3\frac{1}{2}, 8\frac{2}{3}, 15\frac{3}{4}$ 3. 0, 1, 4, 9 4. $a_n = 3 + (n - 1)(4)$ 5. $a_n = -200 + (n - 1)(40)$ 6. $a_n = \frac{n + 1}{n}$ 7. 650 cars8. geometric sequence; $r = 4$ 9. neither 10. arithmetic sequence;
 $d = 6$ 11. 1712. $c_n = a_n + 2b_n$

$$= [a_1 + (n - 1)d_a] + 2[b_1 + (n - 1)d_b]$$

$$= a_1 + 2b_1 + (n - 1)d_a + 2(n - 1)d_b$$

$$= (a_1 + 2b_1) + (n - 1)(d_a + 2d_b)$$

$$= c_1 + (n - 1)d_c$$

Thus, c is an arithmetic sequence, and $c_1 = a_1 + 2b_1$ and $d_c = d_a + 2d_b$.

13. 64 14. $1\frac{3}{5}$ 15. 0.01 16. 3

17. 10

$$\begin{aligned}
 18. c_n &= a_n(2b_n) \\
 &= (a_1r_a^{n-1})(2b_1r_b^{n-1}) \\
 &= 2a_1b_1(r_ar_b)^{n-1} \\
 &= c_1r_c^{n-1}
 \end{aligned}$$

(Replace a_1b_1 by c_1 , r_ar_b by r_c .)Thus, c is a geometric sequence in which c_1 is a_1b_1 and r is r_ar_b .

19. a. $\frac{16}{3}$ b. $\frac{64}{27}$ c. $12 \cdot (\frac{2}{3})^n$

20. $5 + 7 + 9 + 11$ 21. $\frac{3}{2} + \frac{5}{3} + \frac{7}{4} +$

$\frac{9}{5} + \frac{11}{6}$ 22. $-4 + 9 - 16 + 25$

23. $0 + (0 + 1) + (0 + 1 + 2)$

24. 867 25. -570 26. -22

27. 100 28. $80\frac{2}{3}$ 29. $\frac{220}{243}$ 30. $7\frac{7}{8}$

31. 44,286 32. $\frac{1}{2}$ 33. 4 34. $1\frac{4}{5}$

35. $\frac{32}{99}$ 36. $\frac{104}{333}$ 37. $\frac{29}{90}$ 38. 72 meters

39. about 5 hours 40. 220 41. 1,330

42. $\frac{1}{6}(n^3 + 3n^2 + 2n)$

43. $16x^4 - 32x^3y + 24x^2y^2 - 8xy^3 + y^4$

44. $a^{10}b^5 - 15a^8b^4 + 90a^6b^3 - 270a^4b^2 + 405a^2b - 243$ 45. $70,000a^6b^{65}$

46. 400 47. 1,530 48. $54\frac{122}{243}$

49. $\frac{1}{3}(k^3 + 2k)$

50. $\binom{n}{0} = \frac{n!}{(n-0)!0!} = \frac{n!}{n!(1)} = 1$

51. Case $n = 1$: $3(1) + 1 = \frac{1(3(1) + 5)}{2}$

$4 = 4 \checkmark$

Case $n = k$: $4 + 7 + 10 + \cdots + (3k + 1) = \frac{k(3k + 5)}{2}$
(Assume true up to some k .)

Case $n = k + 1$: $4 + 7 + 10 + \cdots + (3(k + 1) + 1)$
 $= \frac{(k + 1)(3(k + 1) + 5)}{2}$
 $= \frac{(k + 1)(3k + 8)}{2}$ (Goal statement).

Proof for $n = k$:

$4 + 7 + 10 + \cdots + (3k + 1) + (3(k + 1) + 1)$

$= \frac{k(3k + 5)}{2} + (3(k + 1) + 1)$

$= \frac{k(3k + 5)}{2} + \frac{2(3k + 4)}{2}$

$= \frac{3k^2 + 11k + 8}{2}$

$= \frac{(k + 1)(3k + 8)}{2}$ Right side of goal statement. \checkmark

52. Case $n = 1$: $1^2 = \frac{1(1 + 1)(2(1) + 1)}{6}$

$1 = 1 \checkmark$

Case $n = k$: $1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k + 1)(2k + 1)}{6}$
(Assume true.)

Case $n = k + 1$: $1^2 + 2^2 + 3^2 + \cdots + (k + 1)^2$
 $= \frac{(k + 1)((k + 1) + 1)(2(k + 1) + 1)}{6}$
 $= \frac{(k + 1)(k + 2)(2k + 3)}{6}$ (Goal statement.)

Proof for $n = k$:

$1^2 + 2^2 + 3^2 + \cdots + k^2 + (k + 1)^2$

$= \frac{k(k + 1)(2k + 1)}{6} + (k + 1)^2$

$= \frac{k(k + 1)(2k + 1) + 6(k + 1)^2}{6}$

$= \frac{(k + 1)[k(2k + 1) + 6(k + 1)]}{6}$

$= \frac{(k + 1)(2k^2 + 7k + 6)}{6}$

$= \frac{(k + 1)(2k + 3)(k + 2)}{6} \checkmark$

53. Case $n = 1$: $1^3 - 1 = 0$, which is divisible by 3: $0 = 3 \cdot 0$.Case $n = k$: Assume $k^3 - k = 3n$ for some natural number n .Case $n = k + 1$: $(k + 1)^3 - (k + 1)$

$= (k + 1)[(k + 1)^2 - 1]$

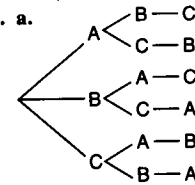
$= k^3 + 3k^2 + 2k$

$= k^3 - k + 3k^2 + 3k$

$= 3n + 3k^2 + 3k$

$= 3(n + k^2 + k) \checkmark$

54. a.



b. ABC, ACB, BAC, BCA, CAB, CBA

55. 24 56. 90 57. 6,561

58. 1,014 59. 90 60. 336

61. 132 62. 116,280

63. 30 64. 153

65. ${}_nC_k = \frac{n!}{k!(n - k)!}$

${}_nC_{n-k} = \frac{n!}{(n - k)![n - (n - k)]!} = \frac{n!}{(n - k)!k!}$

66. 220 67. 15 68. 35

69. 24 70. a. 455 b. 5,005

c. 210 d. 1,816,214,400

71. a. 40,320 b. 1,152 c. 70

72. 295,245 73. a. 1,296 b. 648

c. 108 d. 27 74. a. 360

b. 180 c. 60 d. 6

75. 56 76. 2,520 77. a. 26,400

b. 1,134 c. 74,613 78. 4,480

79. $\frac{1}{4}$ 80. $\frac{1}{16}$ 81. $\frac{1}{13}$

82. $\frac{1}{4}$ 83. $\frac{7}{13}$ 84. $\frac{1}{26}$

85. $\frac{4}{13}$ 86. $\frac{11}{26}$ 87. $\frac{3}{4}$

88. $\frac{5}{7}$ 89. $\frac{4}{7}$ 90. $\frac{5}{7}$

91. $\frac{5}{11}$ 92. a. $\frac{1}{33}$ b. $\frac{1}{33}$

93. 0.00050 94. 0.0253

95. 0.0253 96. 0.3251

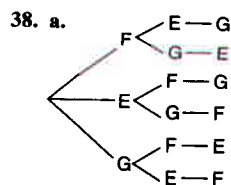
97. 0.00000007 98. $\frac{1}{3}$

99. 2, 8, 14, 20, 26; $a_n = 6n + 2$
 100. 3, 6, 12, 24, 48; $a_n = 3 \cdot 2^n$
 101. 2, 3, 8, 19, 46; $a_n = \frac{1 + 2\sqrt{2}}{2\sqrt{2}}(1 + \sqrt{2})^n - \frac{1 - 2\sqrt{2}}{2\sqrt{2}}(1 - \sqrt{2})^n$
 102. 2, 3, 4, 5, 6; $a_n = n + 2$
 103. 1, 3, 8, 20, 48; $a_n = 2^n + \frac{n}{2}(2^n)$ or $\left(\frac{n}{2} + 1\right)2^n$

Chapter 12 test

1. 2, -1, 0, 1 2. $0, 1\frac{1}{2}, 2\frac{2}{3}, 3\frac{3}{4}$
 3. $4n + 2$ 4. $\frac{n+2}{n}$ 5. 28 6. $6\frac{2}{3}$
 7. Let $A = 2, 6, 10, 14, \dots$ be an arithmetic sequence. Then, if $b_n = 3a_n$, $B = 6, 18, 30, 42, \dots$, which seems to be an arithmetic sequence. Thus, we shall try to show that B is always arithmetic.
 $b_n = 3a_n$
 $= 3(a_1 + (n-1)d_a)$
 $= 3a_1 + (n-1)(3d_a)$
 $= b_1 + (n-1)d_b$
 Thus B is an arithmetic sequence, where $b_1 = 3a_1$ and $d_b = 3d_a$.
 8. -72 9. $6\frac{1}{4}$
 10. Consider the geometric sequence $A = 1, 2, 4, 8, \dots$. Then $B = 2, 3, 5, 9, \dots$, and since the ratio of successive elements is not constant, this is not a geometric sequence. Thus, we cannot conclude that B , where $b_n = a_n + 1$, is necessarily geometric.
 11. a. 16 ft b. $\frac{64}{9}$ ft c. $24(\frac{2}{3})^{n-1}$ ft
 12. $1 + \frac{4}{5} + \frac{3}{5} + \frac{8}{17}$
 13. $-4 + 1 + 0 + 1 - 4$
 14. $1 + (1+4) + (1+4+9)$
 15. 22 16. 2,684 17. $518\frac{1}{3}$
 18. $5\frac{31}{16}$ 19. -170
 20. $\frac{42,753}{100,000} \approx 0.42753$ 21. not defined
 22. $13\frac{1}{2}$ 23. $\frac{3}{11}$ 24. $\frac{13}{30}$
 25. 280 meters 26. 8 years 27. 70

28. n
 29. $x^8 - 12x^6y + 54x^4y^2 - 108x^2y^3 + 81y^4$
 30. 81,081 a^8b^{33} 31. 1,771
 32. $14\frac{683}{1,024}$ 33. $k^2 + 2k$ 34. 12
 35. Case $n = 1$: $(4(1) + 1) = 2(1^2) + 3(1)$
 $5 = 5 \checkmark$
 Case $n = k$: $5 + 9 + 13 + \dots + (4k + 1) = 2k^2 + 3k$
 Case $n = k + 1$: $5 + 9 + 13 + \dots + (4(k+1) + 1)$
 $= 2(k+1)^2 + 3(k+1)$ (Goal)
 $= 2k^2 + 7k + 5$ (Right member expanded.)
 Proof for $n = k + 1$:
 $5 + 9 + 13 + \dots + (4k + 1) + (4(k+1) + 1)$
 $= 2k^2 + 3k + (4(k+1) + 1)$
 $= 2k^2 + 7k + 5 \checkmark$
 36. Case $n = 1$: $\frac{3}{2^0} = \frac{6(2^1 - 1)}{2^1}$; $3 = 3 \checkmark$
 Case $n = k$: $3 + \frac{3}{2} + \frac{3}{4} + \dots + \frac{3}{2^{k-1}}$
 $= \frac{6(2^k - 1)}{2^k}$
 Case $n = k + 1$: $3 + \frac{3}{2} + \frac{3}{4} + \dots + \frac{3}{2^{k+1-1}}$
 $= \frac{6(2^{k+1} - 1)}{2^{k+1}}$ (Goal)
 Proof for $n = k + 1$:
 $3 + \frac{3}{2} + \frac{3}{4} + \dots + \frac{3}{2^{k-1}} + \frac{3}{2^k} =$
 $\frac{6(2^k - 1)}{2^k} + \frac{3}{2^k}$
 $= \frac{6(2^k) - 3}{2^k} = \frac{6(2^k) - 3}{2^k} \cdot \frac{2}{2} =$
 $\frac{6(2)(2^k) - 6}{2^{k+1}} = \frac{6(2^{k+1}) - 6}{2^{k+1}} =$
 $\frac{6(2^{k+1} - 1)}{2^{k+1}} \checkmark$
 37. $n = 1$: $1^2 + 7 + 12 = 20$, which is divisible by 2. \checkmark
 Assume true for $n = k$; that is, $k^2 + 7k + 12 = 2m$ for some integer m .
 For $n = k + 1$: $(k+1)^2 + 7(k+1) + 12$
 $= k^2 + 9k + 20 = k^2 + 7k + 12 + 2k + 8$
 $= 2m + 2k + 8 = 2(m + k + 8) \checkmark$



b. FEG, FGE, EFG, EGF, GFE, GEF

39. 120 40. 1,048,576 41. 24,336
 42. 2,730 43. 210 44. 4,989,600
 45. a. $26! = 4.0329 \times 10^{26}$
 b. 1.28×10^{18} years. (It is thought the universe is less than 20 billion years old; this is 20×10^9 years.) 46. 27,405
 47. $\frac{n(n-1)}{2}$ 48. 45 49. 253
 50. 96 51. 167,960 52. 380
 53. 6.09×10^{10} 54. 72,930
 55. 860,160 56. 720 57. 276
 58. a. 216,000 b. 7,776 59. $\frac{3}{8}$
 60. $\frac{1}{26}$ 61. $\frac{8}{13}$ 62. $\frac{12}{13}$
 63. $\frac{8}{11}$ 64. 0.54 65. $\frac{1}{190}$
 66. $\frac{{}_nC_r}{{}_nC_{r-1}} = \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-(r-1))!}} = \frac{n!}{r!(n-r)!} \cdot \frac{(r-1)!(n-(r-1))!}{n!} = \frac{(r-1)!(n-r+1)!}{r!(n-r)!} = \frac{(r-1)!(n-r+1)(n-r)!}{r!(n-r)!} = \frac{(r-1)(n-r+1)}{r} = \frac{n-r+1}{r}$
 Note that $[n - (r-1)]! = [n - (r-1)][n - (r-1) - 1]! = [n - (r-1)][n - r]!$
 $\frac{{}_nC_r}{{}_nC_{r-1}} = \frac{(r-1)![n - (r-1)][n - r]!}{r(r-1)!(n-r)!} = \frac{n - (r-1)}{r}$
 67. 5, 7, 9, 11, 13; $a_n = 5 + 2n$
 68. 2, 6, 18, 54, 162; $a_n = 2(3^n)$
 69. 2, 4, 6, 8, 10; $a_n = 2(n+1)$
 70. 2, 3, 7, 18, 47; $a_n = \left(\frac{3}{2} + \frac{\sqrt{5}}{2}\right)^n + \left(\frac{3}{2} - \frac{\sqrt{5}}{2}\right)^n$

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HARD

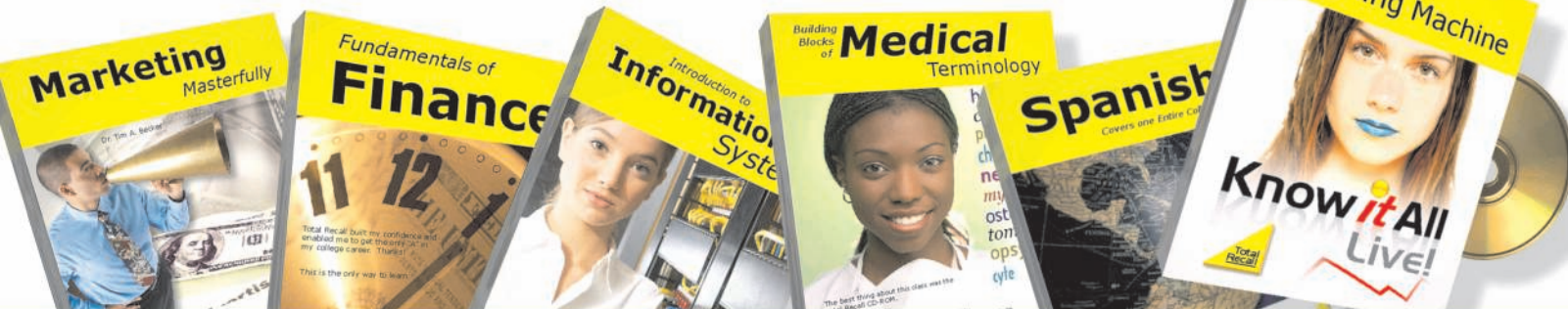
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